Chapter 1

(1.1) As in Example 1.3, assume outgoing plane wave fields in each region. To get J_{sx}, we need H_y, since $\hat{n}\times(\bar{H}_2-\bar{H}_1)=\bar{J}_s$ $(\hat{n}=\hat{z})$. Then we must have E_x to get $\bar{S}=\bar{E}\times\bar{H}^*=\pm S\hat{z}$. So the form of the fields must be,

for $z<0$, $\bar{E}_1=\hat{x}Ae^{jk_0z}$ $\qquad$ for $z>0$, $\bar{E}_2=\hat{x}Be^{-jkz}$

$\bar{H}_1=\dfrac{-\hat{y}}{\eta_0}Ae^{jk_0z}$ $\qquad$ $\bar{H}_2=\hat{y}\dfrac{B}{\eta}e^{-jkz}$

with $k_0=\omega\sqrt{\mu_0\epsilon_0}$, $k=\omega\sqrt{\mu_0\epsilon_0\epsilon_r}$, $\eta_0=\sqrt{\mu_0/\epsilon_0}$, $\eta=\sqrt{\mu_0/\epsilon_0\epsilon_r}$, and A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$(\bar{E}_2-\bar{E}_1)\times\hat{n}=0 \quad\Rightarrow\quad A=B$$
$$\hat{z}\times(\bar{H}_2-\bar{H}_1)=\bar{J}_s \quad\Rightarrow\quad -\left(\frac{B}{\eta}+\frac{A}{\eta_0}\right)=J_0$$

$$\therefore\quad A=B=\frac{-J_0\eta\eta_0}{\eta+\eta_0}$$

(1.2)

$$\nabla \times \bar{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi}\right)$$

$$\nabla \times \nabla \times \bar{E} = \hat{\rho}\left[\frac{-1}{\rho^2}\frac{\partial^2 E_\rho}{\partial \phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial \phi}\right]$$

$$+\hat{\phi}\left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_z}{\partial \phi \partial z} - \frac{\partial^2 E_\phi}{\partial \rho^2} - \frac{1}{\rho}\frac{\partial E_\phi}{\partial \rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial \phi} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial \phi \partial \rho}\right]$$

$$+\hat{z}\left[-\frac{\partial^2 E_z}{\partial \rho^2} - \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z} - \frac{1}{\rho}\frac{\partial E_z}{\partial \rho}\right]$$

$$\nabla(\nabla \cdot \bar{E}) = \hat{\rho}\left[\frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial \rho} - \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial \phi} - \frac{E_\rho}{\rho^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial^2 E_z}{\partial \phi \partial z} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial \rho \partial \phi} + \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial \phi}\right]$$

$$+\hat{z}\left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z}\right]$$

If we apply ∇^2 to the cylindrical components of $\bar{E}$ we get:

$$\nabla^2 \bar{E} \overset{?}{=} \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z \qquad \text{(THIS IS NOT A VALID STEP!)}$$

$$= \hat{\rho}\left[\frac{1}{\rho}\frac{\partial E_\rho}{\partial \rho} + \frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\rho}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial z^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial E_\phi}{\partial \rho} + \frac{\partial^2 E_\phi}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\partial^2 E_\phi}{\partial z^2}\right]$$

$$+\hat{z}\left[\frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2}\right]$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla \times \nabla \times \bar{E}$ and $\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2 \bar{E} \neq \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z$. The $\hat{z}$ components are equal.

1.3

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 1 & -2j & 0 \\ 2j & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 3-8j \\ 12+6j \\ 24 \end{bmatrix} \quad \checkmark$$

1.4 $\bar{S} = \bar{E} \times \bar{H}^* = E_0 H_0 \hat{z} \neq 0.$

The problem here is that Poynting's theorem requires a closed surface integral for a meaningful interpretation in terms of power flow. If we calculate $\oint \bar{S}\, d\bar{s}$ over the closed surface of a cube bounded by the magnet faces and the capacitor plates, we will get zero, since $\hat{n} = \hat{z}$ on one side of the cube, and $\hat{n} = -\hat{z}$ on the opposite side. Since S is a constant, these terms cancel.

1.5 Let $\bar{E} = A(\hat{x} - j\hat{y})e^{-jk_0 z} + B(\hat{x} + j\hat{y})e^{-jk_0 z}$,

where A is the amplitude of the RHCP component, and B is the amplitude of the LHCP component. Equating this expression to the given linearly polarized field gives,

$\hat{x}$: $\quad A + B = E_0$

$\hat{y}$: $\quad -jA + jB = 2E_0$

Solving for A, B gives

$$A = \left(\tfrac{1}{2} + j\right) E_0$$

$$B = \left(\tfrac{1}{2} - j\right) E_0$$

Any linearly polarized wave can be decomposed into the sum of two circularly polarized waves.

(1.6) From eq. (1.76),

$$\bar{H} = \frac{1}{\eta_0}\hat{n} \times \bar{E} \quad , \quad \bar{E} = \bar{E}_0 e^{[illegible]}$$

$$\bar{S} = \bar{E} \times \bar{H}^* = \frac{1}{\eta_0}\bar{E} \times \hat{n} \times \bar{E}^*$$

$$= \frac{1}{\eta_0}\left[(\bar{E}\cdot\bar{E}^*)\hat{n} - (\bar{E}\cdot\hat{n})\bar{E}^*\right] \qquad \text{(from B.5)}$$

Since $\bar{k}\cdot\bar{E}_0 = k_0\hat{n}\cdot\bar{E}_0 = 0$ from (1.69) and [illegible], we have

$$\bar{S} = \frac{\hat{n}}{\eta_0}\bar{E}\cdot\bar{E}^* = \frac{\hat{n}}{\eta_0}|E_0|^2 \quad [illegible] \checkmark$$

(1.7) Writing general plane wave fields in [illegible] region:

$\bar{E}^i = \hat{x}\, e^{-jk_0 z}$ $\qquad \bar{H}^i = \frac{\hat{y}}{\eta_0} e^{-jk_0 z}$ $\qquad$ for $z<0$

$\bar{E}^r = \hat{x}\,\Gamma e^{jk_0 z}$ $\qquad \bar{H}^r = \frac{-\hat{y}}{\eta_0}\Gamma e^{jk_0 z}$ $\qquad$ for $z<0$

$\bar{E}^s = \hat{x}(Ae^{-jkz} + Be^{jkz})$ $\qquad \bar{H}^s = \frac{\hat{y}}{\eta}(Ae^{-jkz} - [illegible])$ $\qquad$ for $0<z<d$

$\bar{E}^t = \hat{x}\,T\, e^{-jk_0(z-d)}$ $\qquad \bar{H}^t = \frac{\hat{y}}{\eta_0}T e^{-j[illegible]}$ $\qquad$ for $z>d$

Now match E_x and H_y at $z=0$ and $z=d$ [illegible] four equations for Γ, T, A, B:

$z=0$: $\qquad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma) = [illegible]$

$z=d$: $\qquad j(-A+B) = T \qquad \frac{j}{\eta}(-A-B) = [illegible] \qquad$ (since $d = \lambda_0/4\sqrt{\epsilon_r}$)

Solving for Γ gives

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2} \checkmark$$

1.8 The incident, reflected, and transmitted fields can be written as,

$$\bar{E}^i = E_o(\hat{x} - j\hat{y})e^{-jk_o z} \qquad \bar{H}^i = j\frac{E_o}{\eta_o}(\hat{x} - j\hat{y})e^{-jk_o z} \qquad (RHCP)$$

$$\bar{E}^r = E_o\Gamma(\hat{x} - j\hat{y})e^{jk_o z} \qquad \bar{H}^r = -j\frac{E_o}{\eta_o}\Gamma(\hat{x} - j\hat{y})e^{jk_o z} \qquad (LHCP)$$

$$\bar{E}^t = E_o T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \bar{H}^t = j\frac{E_o}{\eta}T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad (RHCP)$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_o}{\eta + \eta_o}, \qquad T = \frac{2\eta}{\eta + \eta_o}$$

The Poynting vectors are: $\qquad (\hat{x} - j\hat{y}) \times (\hat{x} - j\hat{y})^* = 2j\hat{z}$

For $z<0$: $\bar{S}^- = (\bar{E}^i + \bar{E}^r) \times (\bar{H}^i + \bar{H}^r)^* = \dfrac{2\hat{z}|E_o|^2}{\eta_o}\left(1 - |\Gamma|^2 + \Gamma e^{2jk_o z} + \Gamma^* e^{-2jk_o z}\right)$ ✓

For $z>0$: $\bar{S}^+ = \bar{E}^t \times \bar{H}^{t*} = \dfrac{2\hat{z}|E_o|^2|T|^2}{\eta^*}e^{-2\alpha z}$ ✓

at $z=0$,

$$\bar{S}^- = \frac{2\hat{z}|E_o|^2}{\eta_o}\left(1 - |\Gamma|^2 + \Gamma - \Gamma^*\right) = \frac{2\hat{z}|E_o|^2}{\eta_o}(1+\Gamma)(1-\Gamma^*)$$ ✓

$$\bar{S}^+ = 2\hat{z}|E_o|^2 \frac{4\eta}{|\eta + \eta_o|^2} \qquad \left(\text{using } T = \frac{2\eta}{\eta + \eta_o}\right)$$

$$= \frac{2\hat{z}|E_o|^2}{\eta_o}\left(\frac{2\eta}{\eta + \eta_o}\right)\left(\frac{2\eta_o}{\eta + \eta_o}\right)^* = \frac{2\hat{z}|E_o|^2}{\eta_o}(1+\Gamma)(1-\Gamma^*)$$ ✓

Thus $\bar{S}^- = \bar{S}^+$ at $z=0$, and power is conserved.

1.9 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = 2\pi j f\sqrt{\mu_0\epsilon_0}\sqrt{5-j2} = j\frac{2\pi(1000)}{300}\sqrt{5.385\angle -22^\circ}$$

$$= 48.5\angle 79^\circ = 9.25 + j47.6 = \alpha + j\beta \quad (\text{neper/m, rad/m})$$

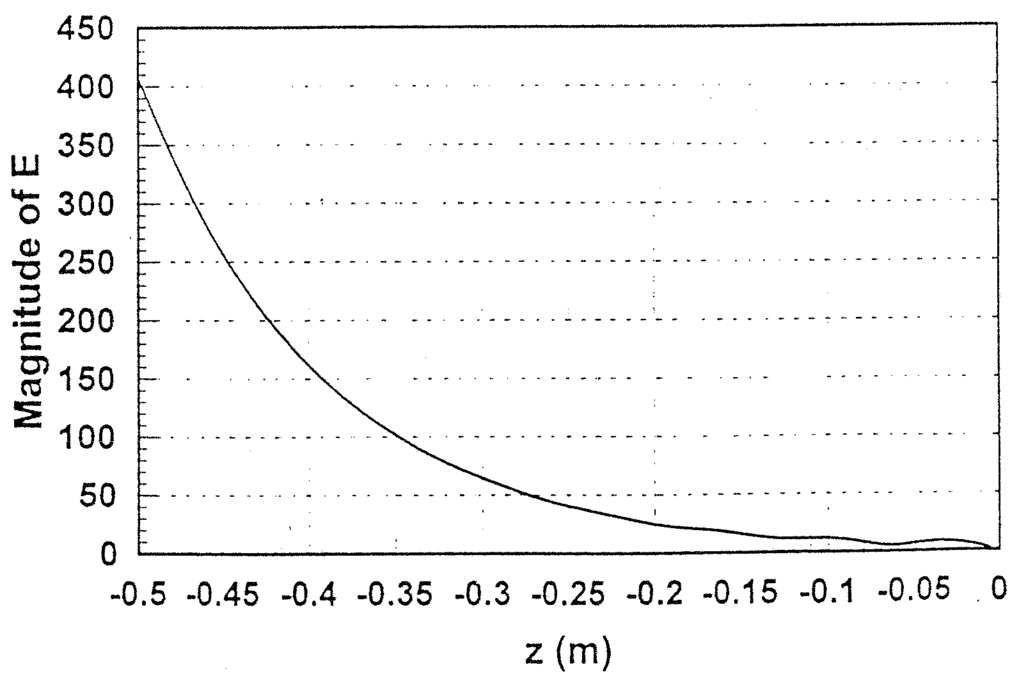
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{\cancel{j\omega}\sqrt{\cancel{\mu_0}\epsilon_0}}{\cancel{j\omega}\sqrt{\cancel{\mu_0}\epsilon}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32\angle -11^\circ} = 163\angle 11^\circ \ \Omega$$

$$\Gamma = -1$$

For $z<0$, $\bar{E} = \bar{E}^i + \bar{E}^r = 4\hat{x}\,(e^{-\gamma z} - e^{\gamma z})$

$$|\bar{E}| = 4\left|e^{-\alpha z}e^{-j\beta z} - e^{\alpha z}e^{j\beta z}\right|$$

$|\bar{E}|$ vs z is plotted below.



1.10 In this problem we are only interested in an "order-of-magnitude" estimate for the sheet thickness, so we will ignore reflections at the two interfaces between air and copper. (The more accurate solution can be saved for an exam!)

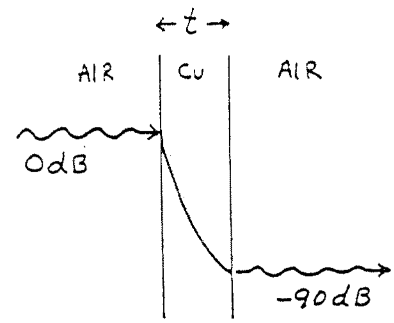
From Table 1.1, the attenuation constant is,

$\alpha = \frac{1}{\delta_s} = \sqrt{\frac{\omega \mu \sigma}{2}}$. From Appendix F, $\sigma_{cu} = 5.8 \times 10^7$ S/m.

To obtain 90 dB attenuation, $20 \log e^{-\alpha t} = -90\text{dB}$,

or $e^{-\alpha t} = 3.16 \times 10^{-5} \Rightarrow \alpha t = 10.4$

f	α (1/m)	t (mm)
1 MHz	1.51×10^4	0.68
1 GHz	4.79×10^5	0.022 ✓
100 GHz	4.79×10^6	0.0022



(1.11) From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = j\,\frac{2\pi(3000)}{300}\sqrt{3(1-j.1)} = 5.435 + j\,108.964 = \alpha + j\beta \quad m^{-1}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r(1-j.1)}} = 217.121\angle 2.855^\circ$$

(a) $$P_i = Re\left\{\frac{|\bar{E}_i(z=0)|^2}{\eta^*}\right\} = 46.000 \quad W/m^2 \quad \checkmark$$

$\Gamma = -1$ at $z = l = 20$ cm

$$\bar{E}_r = \Gamma\,\bar{E}_i(z=l)\,e^{\gamma(z-l)} = -100\hat{x}\,e^{-2\gamma l}e^{\gamma z}$$

$$P_r = Re\left\{\frac{|\bar{E}_r(z=0)|^2}{\eta^*}\right\} = 0.595 \quad W/m^2 \quad \checkmark$$

(b) $$\bar{E}_t = \bar{E}_i + \bar{E}_r$$

$$\bar{E}_t(z=0) = 100\hat{x}(1 - e^{-2\gamma l}) \qquad H_t(z=0) = \frac{100\hat{y}}{\eta}(1 + e^{-2\gamma l})$$

$$P_{in} = Re\left\{\bar{E}_t \times \bar{H}_t^* \cdot \hat{z}\right\} = 45.584 \quad W/m^2$$

But $P_i - P_r = 45.405\ W/m^2 \neq P_{in}$. This is because P_i and P_r individually are not physically meaningful in a lossy medium.

The above values were computed entirely using a FORTRAN program, with 6-digit precision. The error between $P_i - P_r$ and P_{in} is only about 0.4% – this could be made more significant if the loss were increased.

(1.12) This current sheet will generate obliquely propagating plane waves. From (1.132)–(1.133), assume

$$\left.\begin{aligned}\bar{E}_1 &= A(\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1)\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)} \\ \bar{H}_1 &= \frac{-A}{\eta_0}\hat{y}\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)}\end{aligned}\right\} \text{ for } z<0$$

$$\left.\begin{aligned}\bar{E}_2 &= B(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \\ \bar{H}_2 &= \frac{B}{\eta}\hat{y}\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)}\end{aligned}\right\} \text{ for } z>0$$

with $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $k = \sqrt{\epsilon_r}\,k_0$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\eta = \eta_0/\sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z}\times(\bar{E}_2 - \bar{E}_1) = 0 \implies A\cos\theta_1 e^{-jk_0 x\sin\theta_1} - B\cos\theta_2 e^{-jkx\sin\theta_2} = 0$$

$$\hat{z}\times(\bar{H}_2 - \bar{H}_1) = J_s \implies \frac{A}{\eta_0} e^{-jk_0 x\sin\theta_1} + \frac{B}{\eta} e^{-jkx\sin\theta_2} = -J_0 e^{-j\beta x}$$

For phase matching we must have $k_0\sin\theta_1 = k\sin\theta_2 = \beta$

$$\therefore\ \theta_1 = \sin^{-1}\beta/k_0 \checkmark \qquad \theta_2 = \sin^{-1}\beta/k \quad (\text{must have } \beta < k_0)$$

Then, $\quad A\cos\theta_1 = B\cos\theta_2 \quad , \quad \dfrac{A}{\eta_0} + \dfrac{B}{\eta} = -J_0$

$$A = \frac{-J_0\eta\eta_0\cos\theta_2}{\eta\cos\theta_2 + \eta_0\cos\theta_1}, \quad B = \frac{-J_0\eta\eta_0\cos\theta_1}{\eta\cos\theta_2 + \eta_0\cos\theta_1}$$

Check: If $\beta = 0$, then $\theta_1 = \theta_2 = 0$, and $A = B = \dfrac{-J_0\eta\eta_0}{\eta + \eta_0}$,

which agrees with Problem 1.1 ✓

(1.13) This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1, k_2, η_1, and η_2. Thus,

$$k_0 \sin\theta_i = k_0 \sin\theta_r = k \sin\theta_t \quad ; \quad k = k_0\sqrt{\mu_r}$$

$$\Gamma = \frac{\eta\cos\theta_t - \eta_0\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i} \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

$$\eta = \eta_0\sqrt{\mu_r}$$

There will be a Brewster angle if $\Gamma = 0$. This requires that,

$$\eta\cos\theta_t = \eta_0\cos\theta_i$$

$$\sqrt{\mu_r}\sqrt{1-\left(\frac{k_0}{k}\right)^2\sin^2\theta_i} = \cos\theta_i = \sqrt{1-\sin^2\theta_i}$$

$$\mu_r\left(1-\frac{1}{\mu_r}\cancel{\sin^2\theta_i}\right) = 1-\cancel{\sin^2\theta_i}$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

(1.14) Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1, k_2, η_1, η_2. Thus,

$$\Gamma = \frac{\eta\cos\theta_i - \eta_0\cos\theta_t}{\eta\cos\theta_i + \eta_0\cos\theta_t}, \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

a Brewster angle exists if

$$\eta\cos\theta_i = \eta_0\cos\theta_t$$

$$\sqrt{\mu_r}\sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{1}{\mu_r}\sin^2\theta_i}$$

$$\mu_r^2 - \mu_r^2\sin^2\theta_i = \mu_r - \sin^2\theta_i$$

$$\mu_r = (\mu_r+1)\sin^2\theta_i$$

$$\sin\theta_i = \sin\theta_b = \sqrt{\frac{\mu_r}{1+\mu_r}} < 1 \quad ✓$$

Thus a Brewster angle does exist for this case.

(1.15)

$$D_x = \epsilon_0(\epsilon_r E_x + jK E_y)$$
$$D_y = \epsilon_0(-jKE_x + \epsilon_r E_y)$$
$$D_z = \epsilon_0 E_z$$

Then,

$$D_+ = D_x - jD_y = \epsilon_0(\epsilon_r - K)E_x - j\epsilon_0(\epsilon_r - K)E_y = \epsilon_0(\epsilon_r - K)E_+$$
$$D_- = D_x + jD_y = \epsilon_0(\epsilon_r + K)E_x + j\epsilon_0(\epsilon_r + K)E_y = \epsilon_0(\epsilon_r + K)E_-$$

or,

$$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - K) & 0 & 0 \\ 0 & (\epsilon_r + K) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,

$$\left.\begin{aligned} \nabla\times\bar{E} &= -j\omega\mu\bar{H} \\ \nabla\times\bar{H} &= j\omega[\epsilon]\bar{E} \end{aligned}\right\} \qquad \nabla\times\nabla\times\bar{E} = -j\omega\mu\nabla\times\bar{H} = \omega^2\mu[\epsilon]\bar{E}$$

$$\nabla^2\bar{E} + \omega^2\mu[\epsilon]\bar{E} = 0 \qquad \text{(CARTESIAN)}$$

Expanding this wave equation gives,

$$\nabla^2 E_x + \omega^2\mu\epsilon_0[\epsilon_r E_x + jKE_y] = 0 \qquad (1)$$
$$\nabla^2 E_y + \omega^2\mu\epsilon_0[-jKE_x + \epsilon_r E_y] = 0 \qquad (2)$$
$$\nabla^2 E_z + k_0^2 E_z = 0 \qquad (3)$$

adding (1) + j(2) gives $\quad \nabla^2(E_x + jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r + K)E_x + j(\epsilon_r + K)E_y] = 0$

$$\nabla^2 E^+ + \omega^2\mu\epsilon_0(\epsilon_r + K)E^+ = 0$$
$$\therefore \beta_+ = k_0\sqrt{\epsilon_r + K} \quad ✓$$

adding (1) − j(2) gives $\quad \nabla^2(E_x - jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r - K)E_x - j(\epsilon_r - K)E_y] = 0$

$$\nabla^2 E^- + \omega^2\mu\epsilon_0(\epsilon_r - K)E^- = 0$$
$$\therefore \beta_- = k_0\sqrt{\epsilon_r - K} \quad ✓$$

Note: The wave equations for E^+, E^- must be satisfied simultaneously. Thus, for E^+, we must have $E^- \equiv 0$. This implies that $E_y = jE_x = jE_o$. The actual electric field is then,

$$\bar{E}^+ = \hat{x}E_x + \hat{y}E_y = E_o(\hat{x} + j\hat{y})e^{-j\beta_+ z} \qquad (LHCP)$$

This is a LHCP wave. Similarly for E^-, we must have $E^+ \equiv 0$:

$$\bar{E}^- = \hat{x}E_x + \hat{y}E_y = E_o(\hat{x} - j\hat{y})e^{-j\beta_- z} \qquad (RHCP)$$

1.16 Comparing (1.123) and (1.131) indicates that $\bar{E}_t = R_s \bar{J}_s = R_s \hat{n} \times \bar{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S,

$$(\bar{E}_1 \times \bar{H}_2 - \bar{E}_2 \times \bar{H}_1)\cdot\hat{n} = R_s[(\hat{n}\times\bar{H}_{1t})\times\bar{H}_{2t} - (\hat{n}\times\bar{H}_{2t})\times\bar{H}_{1t}]$$

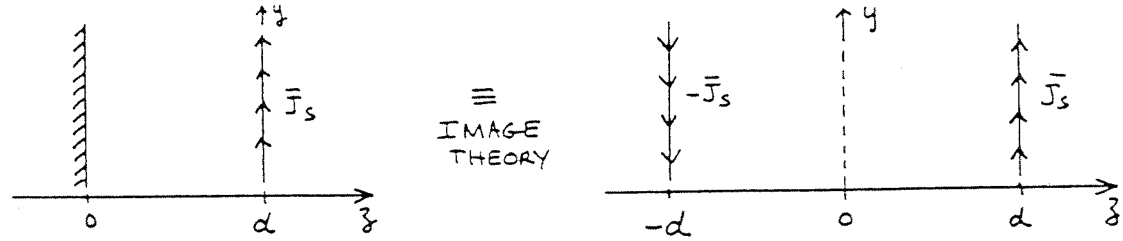
USING (B.5)

$$= R_s[(\underbrace{\bar{H}_{2t}\cdot\hat{n}}_{0})\bar{H}_{1t} - \cancel{(\bar{H}_{2t}\cdot\bar{H}_{1t})\hat{n}} - (\underbrace{\bar{H}_{1t}\cdot\hat{n}}_{0})\bar{H}_{2t} + \cancel{(\bar{H}_{1t}\cdot\bar{H}_{2t})\hat{n}}]$$

$$= 0$$

so (1.157) is obtained

(1.17)



First find the fields due to the source at $z=d$. From (1.139)–(1.140),

For $z<d$,
$$\bar{E}_1 = A\hat{y}\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$
$$\bar{H}_1 = \frac{A}{\eta_o}(\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$

For $z>d$,
$$\bar{E}_2 = B\hat{y}\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$
$$\bar{H}_2 = \frac{B}{\eta_o}(-\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$

apply boundary conditions at $z=d$:

$$\hat{z}\times[\bar{E}(d^+) - \bar{E}(d^-)] = 0 \Rightarrow A e^{jk_o d\cos\theta} = B e^{-jk_o d\cos\theta}$$
$$\hat{z}\times[\bar{H}(d^+) - \bar{H}(d^-)] = \bar{J}_s \Rightarrow [-B\cos\theta\, e^{-jk_o d\cos\theta} - A\cos\theta\, e^{jk_o d\cos\theta}]\cdot e^{-jk_o x\sin\theta} = \eta_o J_o e^{-j\beta x}$$

For phase matching, $k_o\sin\theta = \beta$

Then, $A = \dfrac{-\eta_o J_o}{2\cos\theta} e^{jk_o d\cos\theta}$ $\qquad B = \dfrac{-\eta_o J_o}{2\cos\theta} e^{jk_o d\cos\theta}$

$$\bar{E} = \frac{-\eta_o J_o\hat{y}}{2\cos\theta}\begin{cases} e^{-jk_o[x\sin\theta - (z-d)\cos\theta]} & z<d \\ e^{-jk_o[x\sin\theta + (z-d)\cos\theta]} & z>d \end{cases}$$

The fields due to the source at $z=-d$ can then be found by replacing d with $-d$, and J_o with $-J_o$:

$$\bar{E} = \frac{\eta_o J_o \hat{y}}{2\cos\theta} \begin{cases} e^{-jk_o[x\sin\theta - (z+d)\cos\theta]} & z < -d \\ e^{-jk_o[x\sin\theta + (z+d)\cos\theta]} & z > -d \end{cases}$$

Combining these results gives the total fields:

$$\bar{E} = \frac{-j\eta_o J_o \hat{y}}{\cos\theta} \begin{cases} e^{-jk_o x\sin\theta}\, e^{-jk_o d} \sin(k_o z\cos\theta) & 0<z<d \\ e^{-jk_o x\sin\theta}\, e^{-jk_o z} \sin(k_o d\cos\theta) & z>d \end{cases}$$

CHECK: If $\beta=0$, then $\theta=0$ and we have,

$$\bar{E} = -j\eta_o J_o \hat{y} \begin{cases} e^{-jk_o d} \sin k_o z & \text{for } 0<z<d \\ e^{-jk_o z} \sin k_o d & \text{for } z>d \end{cases}$$

This agrees with the results in (1.161)–(1.162).

Chapter 2

(2.1) $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(5+j628)(.01+j.94)} = \sqrt{(628\angle 89.54^\circ)(.94\angle 89.39^\circ)}$

$= 24.3\angle 89.465^\circ = 0.23 + j24.3 = \alpha + j\beta$ np/m, rad/m ✓

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{628\angle 89.54^\circ}{.94\angle 89.39^\circ}} = 25.8\angle .08 = 25.8 + j.03\,\Omega$$ ✓

If $R = G = 0$, $\alpha = 0$

$\beta = \omega\sqrt{LC} = 24.3$ ✓

$Z_0 = \sqrt{L/C} = 25.8\,\Omega$ ✓

Note that β and Z_0 for the lossless case are very close to the corresponding results for the lossy (low-loss) case.

(2.2) Using KVL:

$$-v(z) + R\frac{\Delta z}{2} i(z) + L\frac{\Delta z}{2}\frac{\partial i(z)}{\partial t} + R\frac{\Delta z}{2} i(z+\Delta z) + L\frac{\Delta z}{2}\frac{\partial i(z+\Delta z)}{\partial t} + v(z+\Delta z) = 0$$

divide by Δz and let $\Delta z \to 0$:

$$\frac{\partial v(z)}{\partial z} = -R\,i(z) - L\frac{\partial i(z)}{\partial t}$$ ✓

Using KCL:

$$i(z) - \Delta z\left[G + C\frac{\partial}{\partial t}\right]\left[v(z) - \frac{\Delta z}{2}\left(R + L\frac{\partial}{\partial t}\right)i(z)\right] - i(z+\Delta z) = 0$$

divide by Δz and let $\Delta z \to 0$:

$$\frac{\partial i(z)}{\partial z} = -G\,v(z) - C\frac{\partial v(z)}{\partial t}$$ ✓

These results agree with (2.2a,b).

2.3 Ignoring fringing fields, $\bar{E}$ and $\bar{H}$ can be assumed as,

$$E_y = \frac{-V_o}{d} \text{ V/m}, \quad H_x = \frac{V_o}{d\eta} = \frac{I_o}{W} \text{ A/m}, \quad \eta = \sqrt{\mu/\epsilon}.$$

Then $\bar{E} \times \bar{H}^* = \hat{z}|S|$ ✓ and $I_o = V_o\left(\frac{W}{\eta d}\right)$.

From (2.17) - (2.20),

$$L = \frac{\mu_o}{I_o^2}\int_S |\bar{H}|^2 ds = \frac{\mu_o}{I_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{I_o}{W}\right)^2 dx\,dy = \frac{\mu_o d}{W} \quad \text{H/m} \quad ✓$$

$$C = \frac{\epsilon}{V_o^2}\int_S |\bar{E}|^2 ds = \frac{\epsilon}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\epsilon W}{d} \quad \text{Fd/m} \quad ✓$$

$$R = \frac{R_s}{I_o^2}\int_{C_1+C_2} |\bar{H}|^2 dl = \frac{2R_s}{I_o^2}\int_{x=0}^{W}\left(\frac{I_o}{W}\right)^2 dx = \frac{2R_s}{W} \quad \Omega/\text{m} \quad ✓$$

$$G = \frac{\omega\epsilon''}{V_o^2}\int_S |\bar{E}|^2 ds = \frac{\omega\epsilon''}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\omega\epsilon'' W}{d} \quad \text{S/m} \quad ✓$$

These results agree with those in Table 2.1

2.4 Assume $E_z = H_z = 0$, $\partial/\partial x = \partial/\partial y = 0$.

Then Maxwell's curl equations reduce to,

$$\frac{-\partial E_y}{\partial z} = -j\omega\mu H_x \quad (1) \qquad \frac{-\partial H_y}{\partial z} = j\omega\epsilon E_x \quad (3)$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \quad (2) \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \quad (4)$$

Since $E_x = 0$ at $y=0$ and $y=d$, and $\partial/\partial y = 0$, we must have $E_x \equiv 0$. Then (3) implies $H_y \equiv 0$. So we have,

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y$$

Now let $E_y = \frac{1}{d} V(z)$ and $H_x = \frac{-1}{W} I(z)$.

Then the voltage and current are,

$$V(z) = \int_{y=0}^{d} E_y \, dy \qquad I(z) = \int_{x=0}^{W} (\hat{y} \times \bar{H}) \cdot \hat{z} \, dx = -\int_{x=0}^{W} H_x \, dx$$

Then,

$$\left.\begin{aligned} \frac{\partial V(z)}{\partial z} &= -j \frac{\omega \mu d}{W} I(z) \quad \Longrightarrow \quad L = \frac{\mu d}{W} \\ \frac{\partial I(z)}{\partial z} &= -j \frac{\omega \epsilon W}{d} V(z) \quad \Longrightarrow \quad C = \frac{\epsilon W}{d} \end{aligned}\right\} \text{ agrees with Table 2.1}$$

2.5 From Table 2.1 $(a = 0.5\,mm,\ b = 1.5\,mm,\ \epsilon_r = 2.8,\ \tan\delta = .005)$

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = 0.219 \ \mu H/m \ ✓$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln b/a} = 142. \ pF/m \ ✓$$

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = 6.07 \ \Omega/m \ ✓$$

$$G = \frac{2\pi \omega \epsilon_0 \epsilon_r \tan\delta}{\ln b/a} = 0.0134 \ S/m \ ✓$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{4128 \angle 89.916°}{2.677 \angle 89.713°}} = 39.3 \angle 0.10°$$

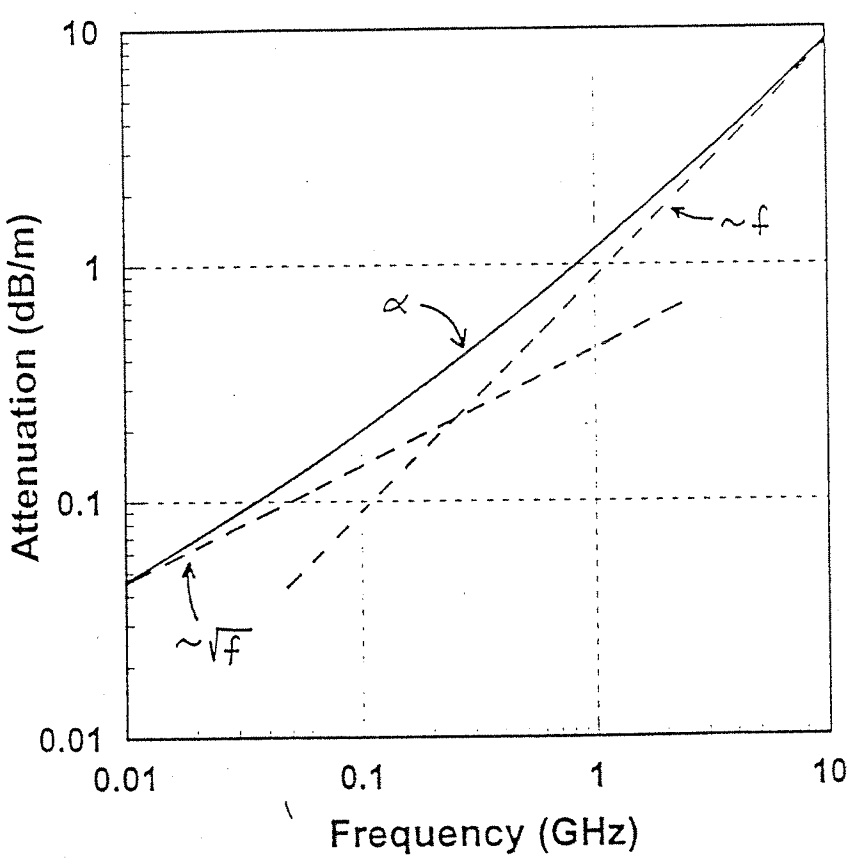
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = 105.1 \angle 89.81° \ ✓$$

$$= 0.34 + j\,105.1 = \alpha + j\beta \quad (np/m,\ rad/m)$$

(2.6) Using the results from P. 2.5, with $\alpha \simeq \frac{1}{2}(R/Z_o + GZ_o)$, gives:

f	R_s	$R(\Omega)$	$G(S)$	α (Np/m)	α (dB/m)
10 MHz	8.24×10^{-4}	.350	4.47×10^{-5}	.0053 ✓	.046
100 MHz	2.6×10^{-3}	1.10	4.47×10^{-4}	.0228 ✓	.198 ✓
3 GHz	1.43×10^{-2}	6.07	1.34×10^{-2}	.341 ✓	2.96
10 GHz	2.61×10^{-2}	11.08	4.47×10^{-2}	1.02	8.85

(The above approximation (2.85a) is much simpler than the exact method as used in Problem 2.5, and is accurate for small loss, as shown by comparing the results at 3 GHz.) Results are plotted below:



Note that the frequency dependence is between $\sqrt{f}$ $(R \sim \sqrt{f})$ and f $(G \sim f)$, at low and high frequencies.

2.7 $z_L = Z_L/Z_0 = 0.53 + j0.266$

From Smith chart, $z_{in} = 0.93 - j0.7$

So $Z_{in} = Z_0 z_{in} = 69.8 - j52.5\,\Omega$ ✓

SWR = 2.05 , $\Gamma = 0.34\angle 140°$ ✓

2.8 $|\Gamma| = \frac{S-1}{S+1} = \frac{0.5}{2.5} = 0.2$

$|\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{100 - Z_0}{100 + Z_0}\right|$ Z_0 real.

So either, $\frac{100 - Z_0}{100 + Z_0} = 0.2 \Rightarrow Z_0 = Z_L \frac{1-\Gamma}{1+\Gamma} = 100\left(\frac{.8}{1.2}\right) = 66.7\,\Omega$ ✓

or, $\frac{100 - Z_0}{100 + Z_0} = -0.2 \rightarrow Z_0 = Z_L \frac{1-\Gamma}{1+\Gamma} = 100\left(\frac{1.2}{.8}\right) = 150\,\Omega$ ✓

2.9 $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j40}{130 + j40} = \frac{50\angle 53°}{136\angle 17°} = 0.367\angle 36°$ ✓

$P_{LOAD} = P_{INC} - P_{REF} = P_{INC}(1 - |\Gamma|^2) = 30[1 - (.367)^2] = 25.9$ W ✓

2.10 $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{300}{3000\sqrt{2.56}} = 6.25$ cm

$\ell = \frac{2.0\text{ cm}}{6.25\text{ cm}/\lambda_g} = 0.320\lambda_g$ $\beta\ell = \frac{2\pi}{\lambda_g}(.320\lambda_g) = 115.2°$

SMITH CHART SOLUTION:

$Z_{in} = 18.98 - j20.55\,\Omega$

$\Gamma_{in} = 0.62\angle 212°$

$\Gamma_L = 0.62\angle 83°$

SWR = 4.27

ANALYTICAL SOLUTION: $\tan\beta\ell = -2.125$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 75 \frac{(37.5+j75)+j75(-2.125)}{75+j(37.5+j75)(-2.125)} = 18.99 - j\,20.55\,\Omega \checkmark$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.62\,\angle 82.9° \checkmark$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0.62\,\angle -147.5° \checkmark$$

$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 4.26 \checkmark$$

2.11

$$RL = -20 \log |\Gamma|$$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = 10^{-RL/20}$$

$$|\Gamma| = \frac{SWR-1}{SWR+1}$$

SWR	$\|\Gamma\|$	RL(dB)
1.00	0.0	∞
1.01	.005	46.0
1.02	.01	40.0
1.05	.024	32.3
1.07	.0316	30.
1.10	.0476	26.4
1.20	.091	20.8
1.22	.100	20.
1.50	.200	14.
1.92	.316	10.
2.00	.333	9.5
2.50	.429	7.4

2.12 $V_g = 15\text{ V RMS}$, $Z_g = 75\,\Omega$, $Z_0 = 75\,\Omega$, $Z_L = 60 - j40\,\Omega$, $\ell = 0.7\lambda$.

a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-15 - j40}{135 - j40} = \frac{42.7\angle -110.6^\circ}{140.8\angle -16.5^\circ} = 0.303\angle -94^\circ = -0.021 - j0.302$$

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}\left(1 - |\Gamma|^2\right) = 0.681\text{ W} \checkmark$$

This method is actually based on $P_L = P_{inc}(1 - |\Gamma|^2)$. It is the simplest method, but only applies to lossless lines.

b) $$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 75\,\frac{60 + j190.8}{198.1 + j184.7} = 75\,\frac{200\angle 72.5^\circ}{270.8\angle 43^\circ}$$

$$= 55.4\angle 29.5^\circ = 48.2 + j27.3\,\Omega$$

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}(Z_{in}) = \left|\frac{15}{123.2 + j27.3}\right|^2 (48.2) = 0.681\text{ W} \checkmark$$

This method computes $P_L = P_{in} = |I_{in}|^2 R_{in}$, and also applies only to lossless lines.

c) $$V(z) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$V_L = V(0) = V^+(1 + \Gamma) \qquad V^+ = \frac{V_g}{2} = 7.5\text{ V}$$

$$= 7.5(1 - .021 - j.302)$$

$$= 7.68\angle -17^\circ$$

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 \mathrm{Re}(Z_L) = \left(\frac{7.68}{72.1}\right)^2 (60) = 0.681\text{ W} \checkmark$$

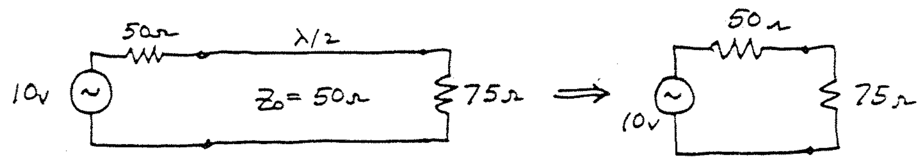
This method computes $P_L = |I_L|^2 R_L$, and applies to lossy as well as lossless lines. Note the concept that $V^+ = V_g/2$ requires a good understanding of the transmission line equations, and only applies here because $Z_g = Z_0$.

(2.13) $Z_L = jX$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$|\Gamma|^2 = \Gamma\Gamma^* = \frac{(jX - Z_0)}{(jX + Z_0)}\frac{(-jX - Z_0)}{(-jX + Z_0)} = \frac{X^2 - \cancel{jZ_0X} + \cancel{jZ_0X} + Z_0^2}{X^2 + Z_0^2} = 1 \checkmark$$

(2.14)



POWER DELIVERED BY SOURCE $= \frac{1}{2}\frac{(10)^2}{50+75} = 0.400\text{ W}$ ✓

POWER DISSIPATED IN 50Ω LOAD $= \frac{1}{2}(50)\left(\frac{10}{50+75}\right)^2 = 0.160\text{ W}$ ✓

POWER TRANSMITTED DOWN LINE $= \frac{1}{2}(75)\left(\frac{10}{50+75}\right)^2 = 0.240\text{ W}$ ✓

INCIDENT POWER $= \frac{1}{2}(50)\left(\frac{10}{50+50}\right)^2 = 0.250\text{ W}$ ✓

REFLECTED POWER $= P_{INC}|\Gamma|^2 = .250\left|\frac{75-50}{75+50}\right|^2 = 0.010\text{ W}$ ✓

$P_{INC} - P_{REF} = .250 - .010 = 0.240 = P_{TRANS}$ ✓

$P_{DISS} + P_{TRANS} = .160 + .240 = 0.400 = P_{SOURCE}$ ✓

(2.15)

$$\Gamma = \frac{-20 - j40}{180 - j40} = \frac{44.7\angle -116.6^\circ}{184.4\angle -12.5^\circ} = 0.24\angle -104^\circ = -0.058 - j0.233 \checkmark$$

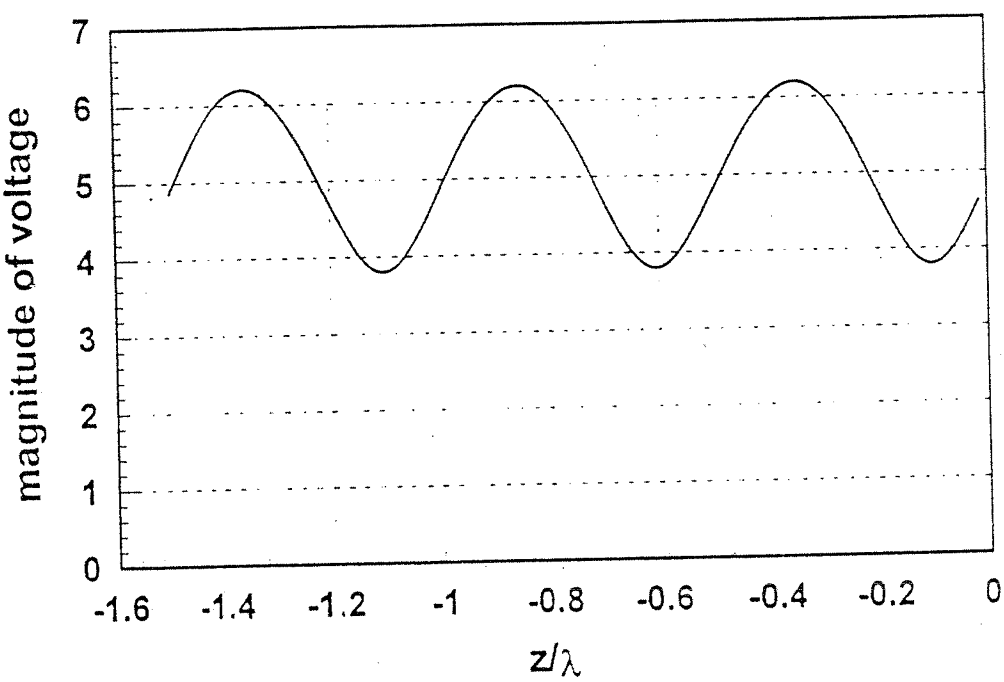
$$V_L = 10\frac{80 - j40}{180 - j40} = 10\frac{89.4\angle -26^\circ}{184\angle -12.5^\circ} = 4.86\angle -13.5^\circ$$

$V(z) = V^+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$ $\quad V^+ = 10\frac{100}{100+100} = 5\text{v}$ ✓

So $V(z) = 5\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$

$$\left.\begin{aligned} V_{MAX} &= 5(1+|\Gamma|) = 5(1.24) = 6.2 \quad \text{at } z = -0.355\lambda, \\ V_{MIN} &= 5(1-|\Gamma|) = 5(.76) = 3.8 \quad \text{at } z = -0.105\lambda \end{aligned}\right\} \text{REPEATS EVERY } \lambda/2$$

$|V(z)|$ is plotted below.



(2.16) The real characteristic impedance Z_1 must satisfy,

$$100 = Z_1 \frac{(80+j20)+jZ_1 t}{Z_1 + j(80+j20)t}, \quad \text{where } t = \tan\beta l$$

multiplying out and separating real and imaginary parts gives,

$$Re: \quad 100Z_1 - 2000t = 80Z_1 \quad \Rightarrow Z_1 = 100t$$

$$Im: \quad 8000t = Z_1(20 + Z_1 t)$$

Solving for Z_1, t: $\quad 6 = 10t^2 \Rightarrow t = 0.775 \Rightarrow \beta l = 37.8°$, $\underline{\underline{l = 0.105\lambda}}$ ✓

$Z_1 = 100t = \underline{\underline{77.5\,\Omega}}$ ✓ $\qquad$ (or, $l = 0.605\lambda$, etc)

2.17 $Z_0 = 50\,\Omega$, $Z_L = 60 + j50\,\Omega$, $\ell = 0.4\lambda$

From Smith chart, ($z_L = 1.2 + j1.0$)

a) $SWR = 2.46$ ✓

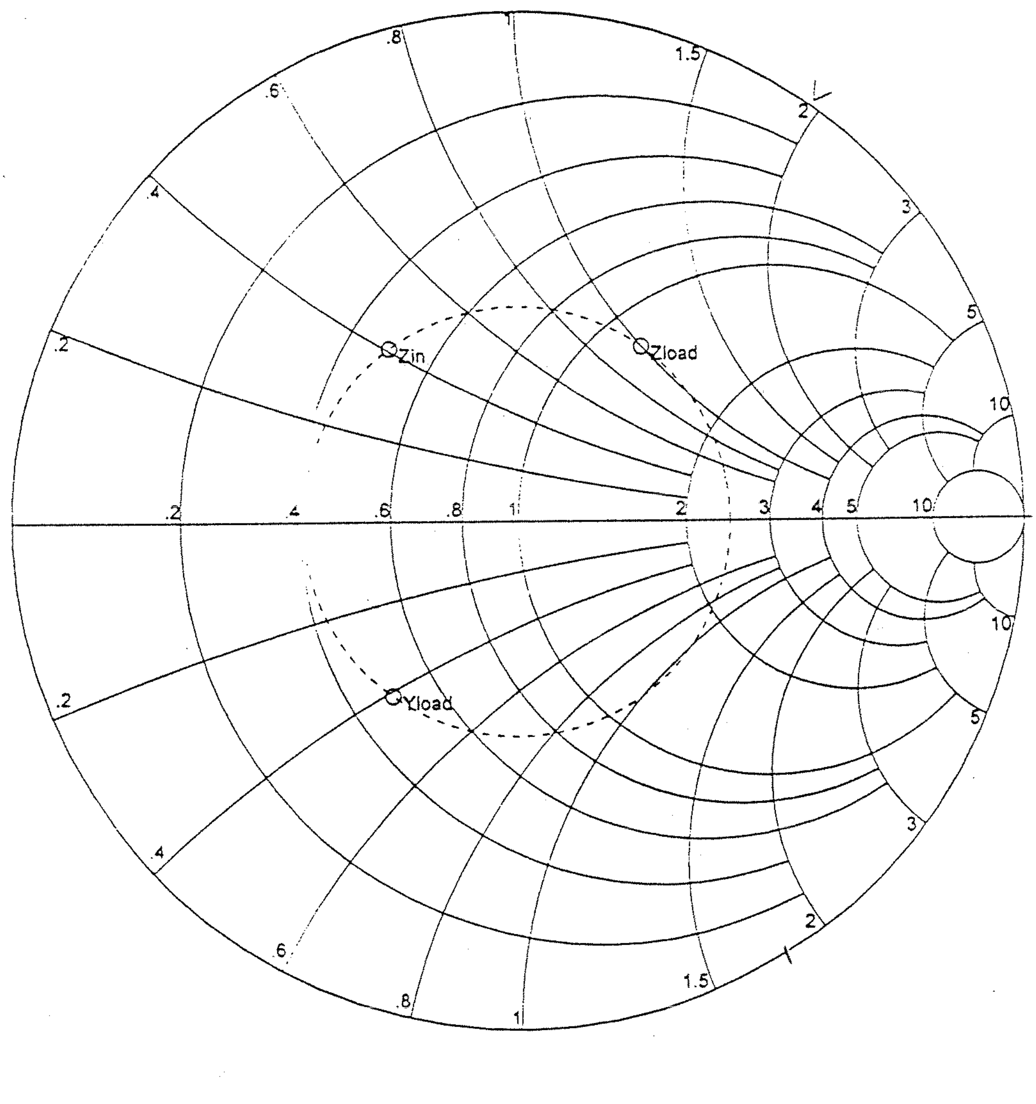
b) $\Gamma = 0.422 \angle 54°$ ✓

c) $Y_L = (.492 - j.410)/50 = 9.84 - j8.2$ mS ✓

d) $Z_{in} = 24.5 + j20.3\ \Omega$

e) $\ell_{MIN} = 0.325\lambda$

f) $\ell_{MAX} = 0.075\lambda$



Smith Chart for Problem 2.17

2.18 $Z_o = 50\,\Omega$, $Z_L = 40 - j30\,\Omega$, $l = 0.4\lambda$

From Smith chart, $(z_L = 0.80 - j0.60)$

a) SWR = 2.00

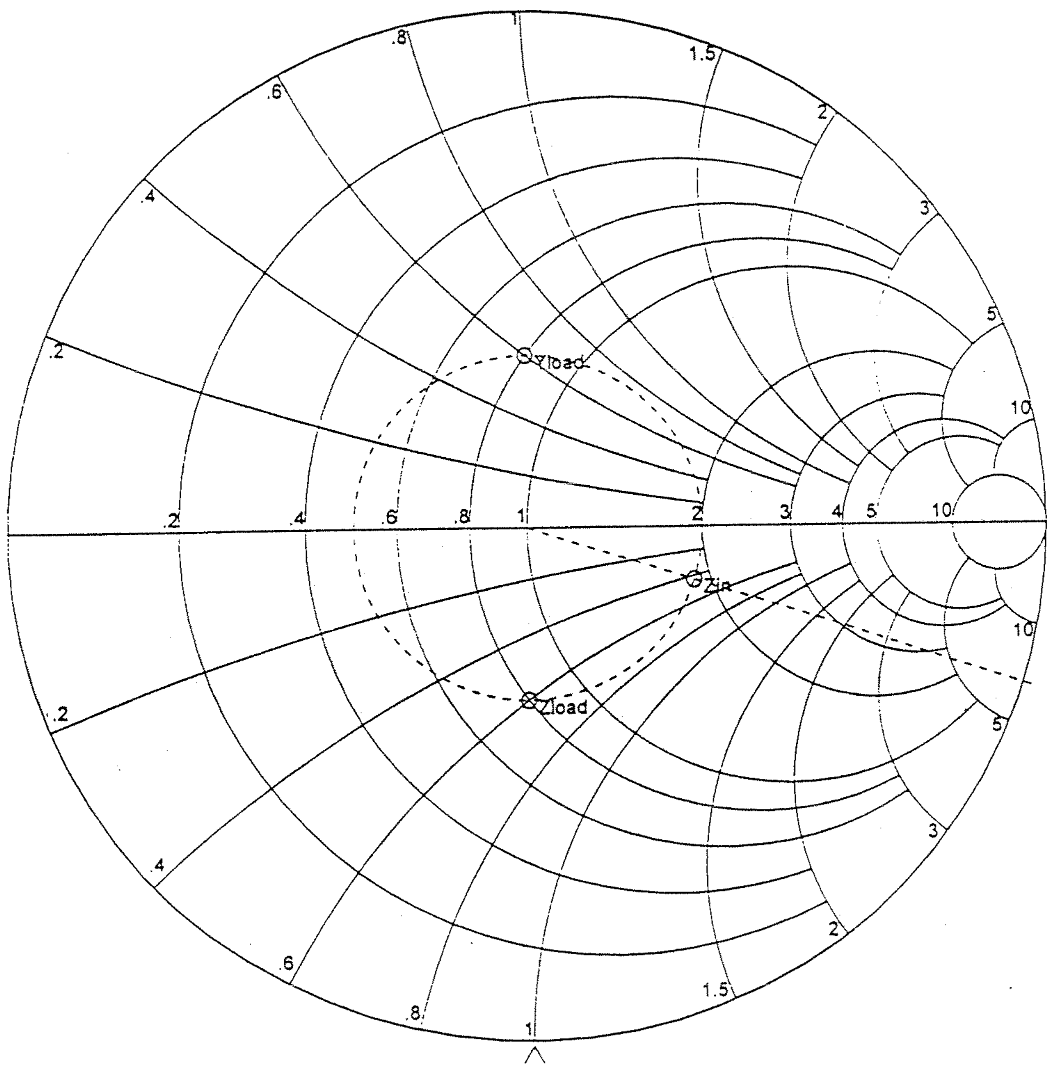
b) $\Gamma = 0.333\angle 270°$

c) $Y_L = (.800 + j.600)/50 = 16.0 + j12.0$ mS ✓

d) $Z_{in} = 93.2 - j21.6\,\Omega$

e) $l_{MIN} = 0.125\lambda$

f) $l_{MAX} = 0.375\lambda$



Smith Chart for Problem 2.18

2.19 $Z_0 = 50\Omega$, $Z_L = 60 + j50\Omega$, $\ell = 1.8\lambda$

From Smith chart, $(z_L = 1.2 + j1.0)$

a) $SWR = 2.46$

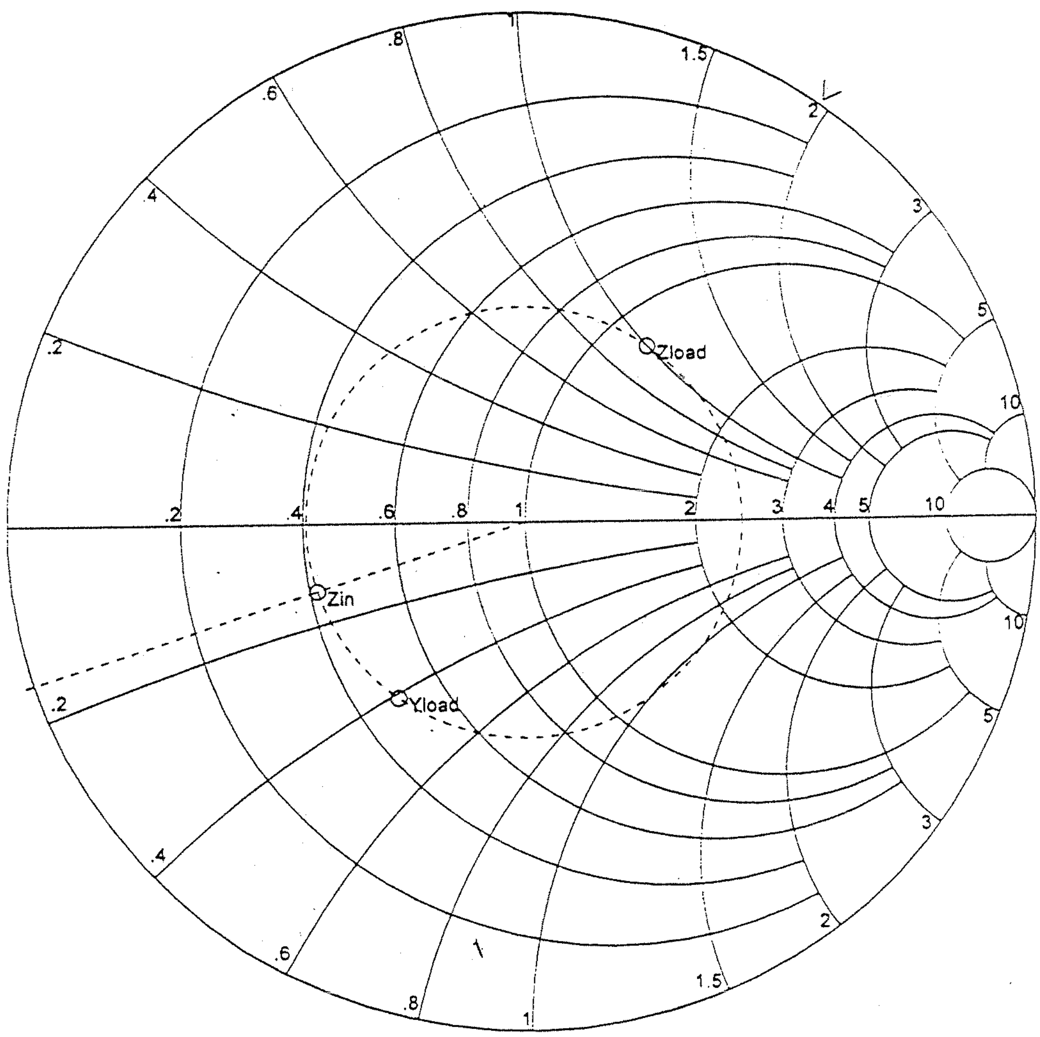
b) $\Gamma = 0.422\angle 54^\circ$

c) $Y_L = (.492 - j.410)/50 = 9.84 - j8.2$ mS ✓

d) $Z_{in} = 20.8 - j6.7\,\Omega$

e) $\ell_{MIN} = 0.325\lambda$

f) $\ell_{MAX} = 0.075\lambda$



Smith Chart for Problem 2.19

2.20

a) $\ell = 0$ or $\ell = 0.5\lambda$ ✓

b) $\ell = 0.25\lambda$ ✓

c) $\ell = 0.125\lambda$ ✓

d) $\ell = 0.406\lambda$ ✓

e) $\ell = 0.021\lambda$ ✓

These results check with $Z_{in} = jZ_0 \tan\beta\ell$.

2.21

a) $\ell = 0.25\lambda$ ✓

b) $\ell = 0\lambda$ or 0.5λ ✓

c) $\ell = 0.375\lambda$ ✓

d) $\ell = 0.656\lambda - 0.5\lambda = 0.156\lambda$ ✓

e) $\ell = 0.271\lambda$ ✓

(add $\lambda/4$ to results of P.2.20)

(also check with $Z_{in} = -jZ_0 \cot\beta\ell$)

2.22

$\lambda = 4.2$ cm. From the Smith chart, $\ell_{MIN} = .9/4.2 = 0.214\lambda$ from the load, so $z_L = 2 - j.9 \Rightarrow \underline{\underline{Z_L = 100 - j45\ \Omega}}$ ✓

Analytically, using (2.58)–(2.60),

$$\Gamma = |\Gamma| e^{j\theta}, \quad |\Gamma| = \frac{2.5-1}{2.5+1} = 0.428$$

$$\theta = \pi + 2\beta\ell_{MIN} = 180 + 2(360)(.214) = -26^\circ \checkmark$$

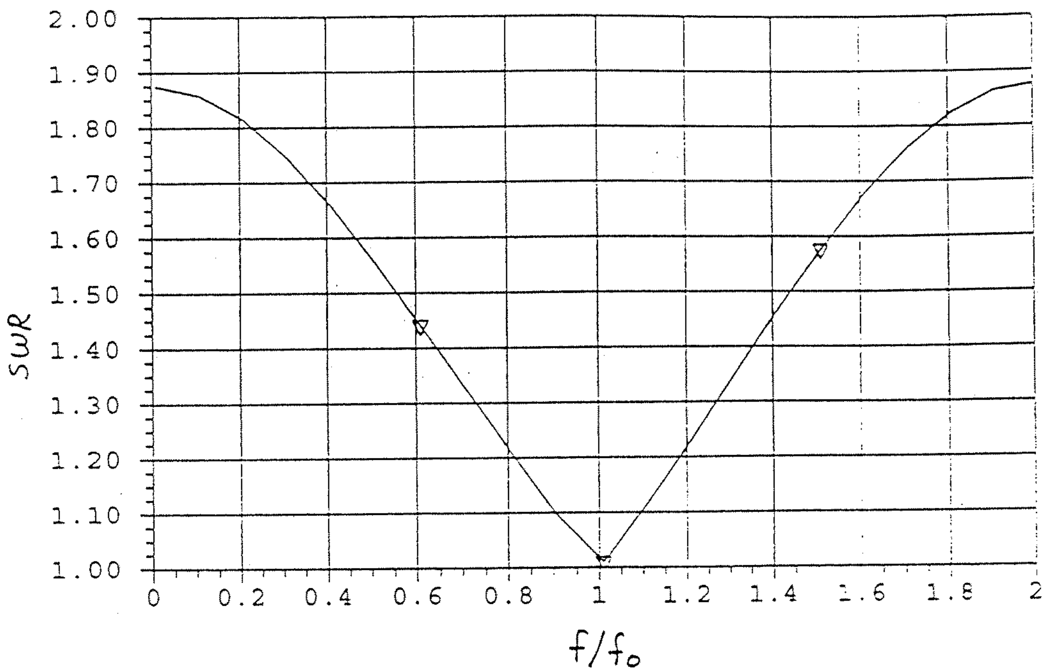
Then,

$$Z_L = \frac{1 + .428\angle{-26^\circ}}{1 - .428\angle{-26^\circ}}(50) = 50\,\frac{1.4\angle{-7.7^\circ}}{.643\angle{17^\circ}} = 109\angle{-25^\circ}$$

$$= \underline{\underline{99 - j46\ \Omega}}$$

2.23 $Z_t = \sqrt{40(75)} = 54.77\ \Omega$

The VSWR is plotted vs f/f_0 below:



2.24 On the $\lambda/4$ transformer, the voltage can be expressed as,

$$V(z) = V^+ e^{-j\beta z} + \Gamma V^+ e^{j\beta z} \quad , \quad \Gamma = \frac{R_L - \sqrt{Z_0 R_L}}{R_L + \sqrt{Z_0 R_L}}$$

at $z=-\ell$, $V(-\ell) = V^i = V^+\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]$

$$V^+ = \frac{V^i}{\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]} \quad , \quad V^- = \Gamma V^+$$

(assuming V^i with a phase reference at $z=-\ell$.)

2.25 From (2.70),

$$V_0^+ = V_g \frac{Z_{in}}{Z_{in}+Z_g} \frac{1}{(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})}$$

From (2.67),

$$Z_{in} = Z_0 \frac{1+\Gamma_\ell e^{-2j\beta\ell}}{1-\Gamma_\ell e^{-2j\beta\ell}}$$

Then,

$$\frac{Z_{in}}{Z_{in}+Z_g} = \frac{Z_0(1+\Gamma_\ell e^{-2j\beta\ell})}{Z_0(1+\Gamma_\ell e^{-2j\beta\ell})+Z_g(1-\Gamma_\ell e^{-2j\beta\ell})}$$

$$= \frac{Z_0(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})\, e^{-j\beta\ell}}{(Z_0+Z_g)+\Gamma_\ell(Z_0-Z_g)\, e^{-2j\beta\ell}}$$

$$= \frac{Z_0(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})\, e^{-j\beta\ell}}{(Z_0+Z_g)\left[1+\Gamma_\ell \frac{Z_0-Z_g}{Z_0+Z_g} e^{-2j\beta\ell}\right]}$$

Thus,

$$V_0^+ = V_g \frac{Z_0 e^{-j\beta\ell}}{(Z_0+Z_g)(1-\Gamma_\ell\Gamma_g e^{-2j\beta\ell})}, \quad \text{since } \Gamma_g = \frac{Z_g-Z_0}{Z_g+Z_0}.$$

2.26

$$\frac{\partial \alpha_c}{\partial a} = \frac{R_s}{2\eta}\left[\frac{1}{a}\left(\frac{1}{\ln b/a}\right)^2\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{1}{\ln b/a}\left(\frac{-1}{a^2}\right)\right] = 0$$

$$a\left(\frac{1}{a}+\frac{1}{b}\right) = \ln b/a$$

$$(1+b/a) = b/a \ \ln b/a$$

If $x = b/a$, then $1+x = x \ln x$.

(If $\frac{\partial \alpha_c}{\partial b}$ is taken, the same result is obtained if $x = a/b$)

Now solve this equation for x:

Using interval-halving method:

x	$x \ln x - x - 1$
1	-2.0
2	-1.6
3	-.704
4	.545
3.5	-.115
3.6	.011
3.55	-.052
→ 3.59	-.01

For $x = \frac{b}{a} = 3.59$,

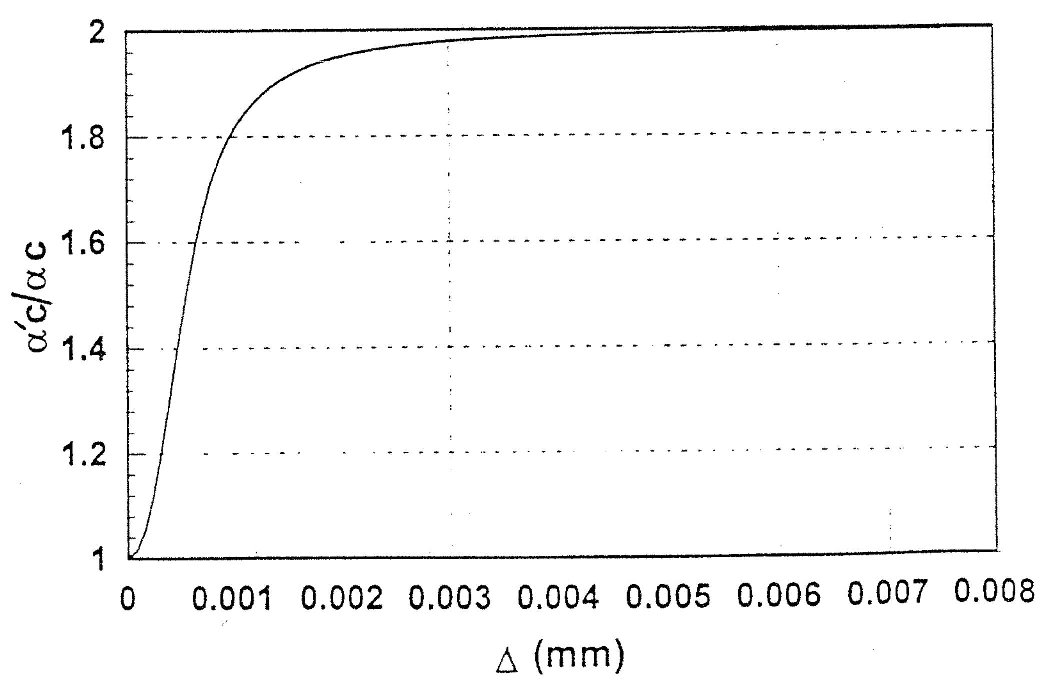
$$Z_o = \frac{\eta}{2\pi} \ln \frac{b}{a} = \frac{377}{\sqrt{\epsilon_r}} \ln(3.59) = \frac{76.7}{\sqrt{\epsilon_r}} \simeq 77\Omega \text{ for } \epsilon_r = 1.$$

Thus, for an air dielectric, minimum attenuation occurs for a characteristic impedance near 77Ω.

2.27 The skin depth of copper at 10 GHz is $\delta_s = 6.60 \times 10^{-7}$ m. Then, compute

$$\frac{\alpha_c'}{\alpha_c} = 1 + \frac{2}{\pi} \tan^{-1} 1.4 \left(\frac{\Delta}{\delta_s}\right)^2 \qquad (2.107)$$

The results are plotted below.



2.28 Since the generator is matched to the line,

$$V_o^+ = \frac{V_g}{2} e^{-\gamma \ell} \quad \text{(phase reference at } z=0)$$

$$\alpha = 0.5\, dB/\lambda = 0.0576 \text{ neper}/\lambda$$

$$\gamma \ell = (\alpha + j\beta)\ell = 0.1325 + j\,108° \checkmark$$

Thus $|V_o^+| = \frac{10}{2} e^{-\alpha \ell} = 4.38\,v.$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100-50}{100+50} = 0.333 \quad , \; \Gamma(\ell) = \Gamma e^{-2\gamma \ell}$$

From (2.92)-(2.94) we then have,

$$P_{in} = \frac{|V_o^+|^2}{2Z_o}\left[1 - |\Gamma(\ell)|^2\right] e^{2\alpha \ell} = \frac{(4.38)^2}{100}\left[e^{2(.1325)} - (.333)^2 e^{-2(.1325)}\right]$$

$$= \underline{0.2337\ w} \quad \text{(power delivered to line)}$$

$$P_L = \frac{|V_o^+|^2}{2Z_o}\left(1 - |\Gamma|^2\right) = \frac{(4.38)^2}{100}\left[1 - (.333)^2\right] = \underline{0.1706\,w} \quad \text{(power to load)}$$

$$P_{Loss} = P_{in} - P_L = 0.2337 - 0.1706 = \underline{0.0631\,w}$$

The input impedance is,

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} = 50\,\frac{100 + 50(.845 + j2.19)}{50 + 100(.845 + j2.19)} = 32.5 - j\,12.4\ \Omega$$

The input current is,

$$I_{in} = \frac{V_g}{R_g + Z_{in}} = \frac{10}{82.5 - j12.4} = 0.1199\angle 8.5°\ A$$

The generator power is,

$$P_s = \tfrac{1}{2} V_g |I_{in}| = 5(.1199) = \underline{0.600}\ w$$

Power lost in R_g is,

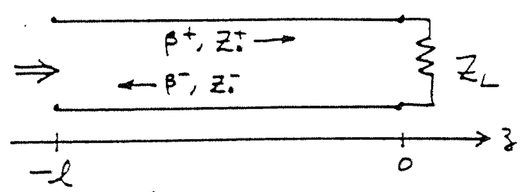
$$P_{R_g} = \tfrac{1}{2}|I_{in}|^2 R_g = \tfrac{1}{2}(.1199)^2(50) = \underline{0.3594\ w}$$

CHECK:

$$P_L + P_{Loss} + P_{R_g} = .1706 + .0631 + .3594 = 0.5931\ w \simeq P_s \checkmark$$

$$P_{in} + P_{R_g} = .2337 + .3594 = 0.5931\ w \simeq P_s \checkmark$$

2.29



$$V(z) = V_0^+ e^{-j\beta^+ z} + V_0^- e^{j\beta^- z}$$

$$I(z) = \frac{V_0^+}{Z_0^+} e^{-j\beta^+ z} - \frac{V_0^-}{Z_0^-} e^{j\beta^- z}$$

at $z=0$ (load), $\quad V(0) = V_0^+ + V_0^-$

$$I(0) = \frac{V_0^+}{Z_0^+} - \frac{V_0^-}{Z_0^-}$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+/Z_0^+ - V_0^-/Z_0^-} = \frac{1 + V_0^-/V_0^+}{\frac{1}{Z_0^+} - \frac{V_0^-}{V_0^+}\frac{1}{Z_0^-}}$$

as usual, let $\Gamma(0) = V_0^-/V_0^+$. Then,

$$Z_L\left(\frac{1}{Z_0^+} - \Gamma\frac{1}{Z_0^-}\right) = 1 + \Gamma$$

$$\frac{Z_L}{Z_0^+} - 1 = \Gamma\left(1 + \frac{Z_L}{Z_0^-}\right)$$

$$\Gamma = \Gamma(0) = \frac{Z_L - Z_0^-}{Z_L + Z_0^+} \qquad \text{(at load)}$$

The input impedance is,

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = \frac{\cancel{V_0^+}\left[e^{j\beta^+\ell} + \Gamma e^{-j\beta^-\ell}\right]}{\cancel{V_0^+}\left[\frac{1}{Z_0^+}e^{j\beta^+\ell} - \Gamma\frac{1}{Z_0^-}e^{-j\beta^-\ell}\right]}$$

$$= \frac{(Z_L+Z_0^+)e^{j\beta^+\ell} + (Z_L - Z_0^-)e^{-j\beta^-\ell}}{\frac{1}{Z_0^+}(Z_L+Z_0^+)e^{j\beta^+\ell} - \frac{1}{Z_0^-}(Z_L-Z_0^-)e^{-j\beta^-\ell}}$$

This result does not simplify much further. From (2.42),

$\Gamma(-\ell) = \Gamma(0)\, e^{-j(\beta^- + \beta^+)\ell}$ (reflection coefficient at the input)

Chapter 3

3.1 Let $k_c^2 = k^2 - \beta^2$

H_x: multiply (3.3a) by $\omega\epsilon$, multiply (3.4b) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial y} - j\beta^2 H_x - \beta \frac{\partial H_z}{\partial x} = -j\omega^2\mu\epsilon H_x$$

$$H_x = \frac{j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x}\right] \checkmark$$

H_y: multiply (3.3b) by $-\omega\epsilon$, multiply (3.4a) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} + j\beta^2 H_y = j\omega^2\mu\epsilon H_y$$

$$H_y = \frac{-j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_x: multiply (3.3b) by $-\beta$, multiply (3.4a) by $\omega\mu$, and add:

$$j\beta^2 E_x + \beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} = j\omega^2\mu\epsilon E_x$$

$$E_x = \frac{-j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_y: multiply (3.3a) by β, multiply (3.4b) by $\omega\mu$, and add:

$$\beta \frac{\partial E_z}{\partial y} + j\beta^2 E_y - \omega\mu \frac{\partial H_z}{\partial x} = j\omega^2\mu\epsilon E_y$$

$$E_y = \frac{j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial y} - \omega\mu \frac{\partial H_z}{\partial x}\right] \checkmark$$

3.2 From (3.66)-(3.67),

$$H_z = B_n \cos\frac{n\pi y}{d} e^{-j\beta z}$$

$$H_y = \frac{j\beta}{k_c} B_n \sin\frac{n\pi y}{d} e^{-j\beta z}$$

From (3.71), $P_o = \dfrac{\omega\mu d W \beta}{4k_c^2} |B_n|^2$ for $n>0$, β real.

From (2.97), the power lost in both plates is,

$$P_\ell = 2\left(\frac{R_s}{2}\right)\int_s |\bar{H}_t|^2 ds = R_s \int_{z=0}^{\ell}\int_{x=0}^{w}\left[|H_y(y=0)|^2 + |H_z(y=0)|^2\right] dx\,dz$$

$$= R_s W |B_n|^2$$

Then, $\alpha_c = \dfrac{P_\ell}{2P_o} = \dfrac{2R_s k_c^2}{k d \eta \beta}$. (agrees with (3.72)) ✓

(3.3) From Appendix I, $a = 1.07$ cm, $b = 0.43$ cm.

$$\left.\begin{aligned} f_{c_{10}} &= \frac{c}{2a} = 14.02 \text{ GHz} ✓ \\ f_{c_{20}} &= \frac{c}{a} = 28.04 \text{ GHz} ✓ \end{aligned}\right\} \text{LOWEST ORDER MODES}$$

$$f_{c_{01}} = \frac{c}{2b} = 34.88 \text{ GHz} ✓$$

The bandwidth covered by $f_{c_{10}}$ and $f_{c_{20}}$ is 100%.
The recommended operating range for K-band guide is 18.0 - 26.5 GHz, or a bandwidth of 47%. The reduction in bandwidth is 53%.

(3.4) $\sigma = 2.56 \times 10^7$ S/m for brass, so $R_s = \sqrt{\dfrac{\omega\mu}{2\sigma}} = 0.0555\,\Omega$

$$k = \sqrt{\epsilon_r}\,k_o = \sqrt{2.6}\,\frac{2\pi(20{,}000)}{300} = 675.\ m^{-1}$$

$$\beta = \sqrt{k^2 - k_c^2} = 608.3\ m^{-1} ✓$$

$$\eta = \frac{377}{\sqrt{\epsilon_r}} = 234\,\Omega ✓$$

From (3.29) the attenuation due to dielectric loss is,

$$\alpha_d = \frac{k^2 \tan\delta}{2\beta} = \frac{(675)^2(.001)}{2(608.3)} = 0.375 \text{ neper/m}$$

From (3.96) the attenuation due to conductor loss is,

$$\alpha_c = \frac{R_s}{a^3 b \beta k \eta}\left(2b\pi^2 + a^3k^2\right) = \frac{.0555\left[2(.0043)\pi^2 + (.0107)^3(675)^2\right]}{(.0107)^3(.0043)(608.3)(675)(234)} = 0.0705 \text{ neper/m} ✓$$

Note that the attenuation due to dielectric loss is much greater than the conductor loss, even for a relatively low-loss dielectric material.

The total attenuation is

$$\alpha = \alpha_c + \alpha_d = 0.446 \text{ neper/m}$$

In dB/m, $\alpha = 0.446(8.686) = 3.87$ dB/m ✓ (agrees with PCAAD)

(3.5) In the section of guide of width $a/2$, the TE_{10} mode is below cutoff (evanescent), with an attenuation constant α:

$$k = \frac{2\pi(12{,}000)}{300} = 251.3 \text{ m}^{-1} \checkmark$$

$$\alpha = \sqrt{\left(\frac{\pi}{a/2}\right)^2 - k^2} = \sqrt{\left(\frac{2\pi}{.02286}\right)^2 - (251.3)^2} = 111.3 \text{ neper/m} \checkmark$$

To obtain 100 dB attenuation (ignoring reflections),

$$-100 \text{ dB} = 20 \log e^{-\alpha l}$$

$$10^{-5} = e^{-\alpha l}$$

$$l = \frac{11.5}{111.3} = 0.103 \text{ m} = \underline{10.3 \text{ cm}} \checkmark$$

(3.6) The TE_{10} H-fields from (3.89) are :

$$H_x = \frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta x}$$

$$H_y = 0$$

$$H_z = A \cos\frac{\pi x}{a} e^{-j\beta x}$$

$\bar{J}_s = \hat{n} \times \bar{H}$, so the surface currents are,

ON BOTTOM WALL: $\hat{n} = \hat{y}$; $\bar{J}_s = -\hat{z} \frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} + \hat{x} A \cos\frac{\pi x}{a} e^{-j\beta z}$ ✓

ON TOP WALL : $\hat{n} = -\hat{y}$; $\bar{J}_s = \hat{z} \frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} - \hat{x} A \cos\frac{\pi x}{a} e^{-j\beta z}$ ✓

ON LEFT SIDE WALL: $\hat{n} = \hat{x}$, $x=0$; $\bar{J}_s = -\hat{y} A e^{-j\beta z}$

ON RIGHT SIDE WALL: $\hat{n} = -\hat{x}$, $x=a$; $\bar{J}_s = \hat{y} A e^{-j\beta z}$

Note that the top and bottom currents are the negative of each other, as are the left and right side wall currents.

Along the centerline of the top or bottom (broad) walls, $x = a/2$, so the surface currents can be reduced to,

$$\bar{J}_s = \pm\hat{z} \frac{j\beta a A}{\pi} e^{-j\beta z},$$

which shows that current flow is only in the longitudinal direction. Thus a narrow longitudinal slot will not break any current lines, and will have a negligible effect on the operation of the waveguide.

3.7 From (3.101),

$$\bar{E} \times \bar{H}^* \cdot \hat{z} = E_x H_y^* - E_y H_x^*$$

$$= \frac{\omega\epsilon\beta m^2\pi^2}{a^2 k_c^4} |B|^2 \cos^2\frac{m\pi x}{a} \sin^2\frac{n\pi y}{b}$$

$$+ \frac{\omega\epsilon\beta n^2\pi^2}{b^2 k_c^4} |B|^2 \sin^2\frac{m\pi x}{a} \cos^2\frac{n\pi y}{b}$$

So the power flow down the guide is,

$$P_o = \frac{1}{2}\int_{x=0}^{a}\int_{y=0}^{b} \bar{E}\times\bar{H}^*\cdot\hat{z}\,dx\,dy = \frac{\omega\epsilon\beta\pi^2|B|^2}{2k_c^4}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\frac{ab}{4} = \frac{\omega\epsilon\beta ab}{8k_c^2}|B|^2$$

The power loss in the walls is,

$$P_\ell = \frac{R_s}{2}\int_c |\bar{H}_t|^2\,ds = R_s\left\{\int_{x=0}^{a}|H_x(y=0)|^2\,dx + \int_{y=0}^{b}|H_y(x=0)|^2\,dy\right\}$$

$$= R_s\left\{\frac{\omega^2\epsilon^2 n^2\pi^2}{b^2 k_c^4}|B|^2\frac{a}{2} + \frac{\omega^2\epsilon^2 m^2\pi^2}{a^2 k_c^4}|B|^2\frac{b}{2}\right\}$$

$$= R_s\frac{\omega^2\epsilon^2\pi^2}{2k_c^4}|B|^2\left(\frac{n^2 a}{b^2}+\frac{m^2 b}{a^2}\right)$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{R_s\omega^2\epsilon^2\pi^2 4k_c^2}{2k_c^4\omega\epsilon\beta ab}\left(\frac{n^2 a}{b^2}+\frac{m^2 b}{a^2}\right)$$

$$= \frac{2R_s k\pi^2}{k_c^2\beta\eta}\left(\frac{n^2}{b^3}+\frac{m^2}{a^3}\right) \quad \text{neper/m} \checkmark$$

3.8 From (3.109), the propagation constant is a solution of,

$$k_a \tan k_d t + k_d \tan k_a (a-t) = 0,$$

where

$$\beta = \sqrt{k_0^2 - k_a^2} = \sqrt{\epsilon_r k_0^2 - k_d^2} \qquad (3.106)$$

Since $\beta = 0$ at cutoff, we have that $k_a = k_0$, and $k_d = \sqrt{\epsilon_r}\, k_0$. Thus we must find the root of the following equation:

$$f(k_0) = k_0 \tan \sqrt{\epsilon_r}\, k_0 t + \sqrt{\epsilon_r}\, k_0 \tan k_0 t = 0 \quad (\text{since } t = a/2)$$

We know that $k_c = k_0$ must be between k_c of the empty guide, and k_c for the completely filled guide:

$k_c(\text{EMPTY}) = \frac{\pi}{a} = 137.\ m^{-1}$

$k_c(\text{FILLED}) = \frac{\pi}{\sqrt{\epsilon_r}\, a} = 92.\ m^{-1}$

k_0	$f(k_0)$
95	−1366
100	−362
105	−44
110	171
106	2.3
105.9	−2.25
✓ → 105.95	.017

This result is accurate to at least four figures, and agrees with a result given in the Waveguide Handbook[*].

The cutoff frequency is,

$$f_c = \frac{k_c c}{2\pi} = 5.06 \text{ GHz}$$

(3.9) The lowest order mode will have an H_z component which is even in x, and no variation in y. Thus, h_z can be written as,

$$h_z(x,y) = \begin{cases} A \cos k_d x & \text{for } |x| < W/2 \quad (k_c = k_d) \\ B e^{-k_a |x|} & \text{for } |x| > W/2 \quad (k_c = j k_a) \end{cases}$$

where k_d and k_a are the cutoff wavenumbers in the dielectric and air regions, respectively, satisfying

$$\beta = \sqrt{\epsilon_r k_0^2 - k_d^2} = \sqrt{k_0^2 - k_a^2} \qquad \text{(phase matching)}$$

Next, we need e_y, from (3.19d):

$$e_y(x,y) = \frac{j\omega\mu}{k_c^2} \frac{\partial h_z}{\partial x} = \begin{cases} -j\frac{\omega\mu A}{k_d} \sin k_d x & \text{for } |x| < W/2 \\ \frac{j\omega\mu B}{k_a} e^{-k_a x} & \text{for } x > W/2 \end{cases}$$

Matching h_z and e_y at $x = W/2$ gives,

$$A \cos k_d W/2 = B e^{-k_a W/2}$$

$$-\frac{A}{k_d} \sin k_d W/2 = \frac{B}{k_a} e^{-k_a W/2}$$

Setting the determinant of these equations to zero gives,

$$k_a \tan k_d W/2 + k_d = 0.$$

A TEM mode cannot exist by itself because of the impossibility of phase matching at $x = W/2$. (For a TEM mode, $\beta = k$ in both regions, which is not possible.)

3.10

$$h_z(y) = \begin{cases} A \cos k_d y & \text{for } 0<y<t \quad (k_c = k_d) \\ B \cos k_a (d-y) & \text{for } t<y<d \quad (k_c = k_a) \end{cases}$$

where k_d and k_c are the cutoff wavenumbers in the dielectric and air regions, satisfying,

$$\beta = \sqrt{\epsilon_r k_o^2 - k_d^2} = \sqrt{k_o^2 - k_a^2}$$

Also, h_z has been chosen to satisfy $E_x = 0$ at $y = 0, d$. (since $E_x \sim \partial h_z/\partial y$). From (3.19),

$$e_x(y) = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y} = \begin{cases} \frac{j\omega\mu A}{k_d} \sin k_d y & \text{for } 0<y<t \\ \frac{-j\omega\mu B}{k_a} \sin k_a (d-y) & \text{for } t<y<d \end{cases}$$

Enforcing continuity of h_z and e_x at $y=t$ yields,

$$A \cos k_d t - B \cos k_a (d-t) = 0$$

$$\frac{A}{k_d} \sin k_d t + \frac{B}{k_a} \sin k_a (d-t) = 0$$

Setting the determinant of this set of equations to zero gives,

$$k_d \tan k_a (d-t) + k_a \tan k_d t = 0 \quad \checkmark$$

(This result agrees with a result on p. 160 of Harrington)

The TEM mode cannot exist on this structure.

3.11 Maxwell's curl equations are,

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} \quad , \quad \nabla \times \bar{H} = j\omega\epsilon \bar{E}$$

The ρ and ϕ components in cylindrical form are,

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_\rho \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_\rho$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \qquad \frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi$$

Now assume $\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi)e^{-j\beta z}$

$\bar{H}(\rho,\phi,z) = \bar{h}(\rho,\phi)e^{-j\beta z}$

Then $\partial/\partial z \rightarrow -j\beta$, and the above equations reduce to:

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta E_\phi = -j\omega\mu H_\rho \quad (1) \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta H_\phi = j\omega\epsilon E_\rho \quad (3)$$

$$-j\beta E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \quad (2) \qquad -j\beta H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi \quad (4)$$

Multiply (2) by $-\beta$, multiply (3) by $\omega\mu$, and add:

$$j\beta^2 E_\rho + \beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega^2\mu\epsilon E_\rho$$

$$E_\rho = \frac{-j}{k_c^2}\left[\beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

multiply (1) by β, multiply (4) by $\omega\mu$, and add:

$$\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta^2 E_\phi - \omega\mu\frac{\partial H_z}{\partial \rho} = j\omega^2\mu\epsilon E_\phi$$

$$E_\phi = \frac{-j}{k_c^2}\left[\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} - \omega\mu\frac{\partial H_z}{\partial \rho}\right]$$

multiply (1) by $\omega\epsilon$, multiply (4) by β, and add:

$$\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - j\beta^2 H_\rho - \beta\frac{\partial H_z}{\partial \rho} = -j\omega^2\mu\epsilon H_\rho$$

$$H_\rho = \frac{j}{k_c^2}\left[\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \beta\frac{\partial H_z}{\partial \rho}\right]$$

$$\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta^2 H_\phi = j\omega^2\mu\epsilon H_\phi$$

$$H_\phi = \frac{-j}{k_c^2}\left[\omega\epsilon\frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

with $k_c^2 = k^2 - \beta^2$.

These results agree with those of (3.110). ✓

3.12 Let $A=1$, $B=0$ in (3.141). Then the transverse fields are,

$$E_\rho = \frac{-j\beta}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$$E_\phi = \frac{-j\beta n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$\bar{E}\times\bar{H}^*\cdot\hat{z} = E_\rho H_\phi^* - E_\phi H_\rho^*$.

The power flow down the guide is, for $n>0$,

$$P_o = \frac{1}{2}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[\frac{\beta\omega\epsilon}{k_c^2}\sin^2 n\phi\, J_n'^2(k_c\rho) + \frac{\beta\omega\epsilon n^2}{k_c^4\rho^2}\cos^2 n\phi\, J_n^2(k_c\rho)\right]\rho\, d\phi\, d\rho$$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^2}\int_{\rho=0}^{a}\left[J_n'^2(k_c\rho) + \frac{n^2}{k_c^2\rho^2}J_n^2(k_c\rho)\right]\rho\, d\rho \qquad \text{Let } x = k_c\rho,\; dx = k_c\, d\rho,\; k_c a = p_{nm}$$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^4}\int_{x=0}^{p_{nm}}\left[J_n'^2(x) + \frac{n^2}{x^2}J_n^2(x)\right]x\, dx = \frac{\beta\omega\epsilon\pi}{4k_c^4}\, p_{nm}^2\, J_n'^2(p_{nm}) \quad \text{(SEE C.16)}$$

The power lost in the conducting wall is,

$$P_\ell = \frac{R_s}{2}\int_{z=0}^{1}\int_{\phi=0}^{2\pi}|H_\phi(\rho=a)|^2 a\, d\phi\, dz = \frac{aR_s}{2}\frac{\omega^2\epsilon^2}{k_c^2}J_n'^2(p_{nm})\int_{\phi=0}^{2\pi}\sin^2 n\phi\, d\phi$$

$$= \frac{aR_s\omega^2\epsilon^2\pi}{2k_c^2}J_n'^2(p_{nm})$$

The attenuation is then,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{aR_s\omega^2\epsilon^2\pi\, 4k_c^4}{4k_c^2\beta\omega\epsilon\pi p_{nm}^2} = \frac{aR_s\omega\epsilon k_c^2}{\beta p_{nm}^2} = \frac{kR_s}{\beta\eta a} \text{ neper/m} ✓$$

(3.13) From Figure 3.13, the first four modes to propagate are the TE_{11}, TM_{01}, TE_{21}, and TE_{01} (TM_{11} has same f_c as TE_{01}).

Using (3.127) and (3.140):

$$TE_{11}: \quad f_c = \frac{p'_{11} c}{2\pi a} = \frac{1.841(300)}{2\pi(.008)} = 10,988 \text{ MHz} \checkmark$$

$$TM_{01}: \quad f_c = \frac{p_{01} c}{2\pi a} = \frac{2.405(300)}{2\pi(.008)} = 14,354 \text{ MHz} \checkmark$$

$$TE_{21}: \quad f_c = \frac{p'_{21} c}{2\pi a} = \frac{3.054(300)}{2\pi(.008)} = 18,227 \text{ MHz} \checkmark$$

$$TE_{01}: \quad f_c = \frac{p'_{01} c}{2\pi a} = \frac{3.832(300)}{2\pi(.008)} = 22,871 \text{ MHz} \checkmark$$

(3.14) From (3.153), $\Phi(\rho,\phi) = \dfrac{V_o \ln b/\rho}{\ln b/a}$

From (3.13) and Appendix,

$$\bar{e}(\rho,\phi) = -\nabla_t \Phi(\rho,\phi) = -\left(\hat{\rho}\frac{\partial \Phi}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\right) = \frac{V_o \hat{\rho}}{\rho \ln b/a}$$

Then,

$$\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi) e^{-j\beta z} = \frac{V_o \hat{\rho} e^{-j\beta z}}{\rho \ln b/a} \checkmark \qquad (4.155) \text{ of 1st Ed.}$$

From (3.18),

$$\bar{h}(\rho,\phi) = \frac{1}{\eta} \hat{z} \times \bar{e}(\rho,\phi) = \frac{V_o \hat{\phi}}{\eta \rho \ln b/a}$$

Then,

$$\bar{H}(\rho,\phi,z) = \frac{V_o \hat{\phi} e^{-j\beta z}}{\eta \rho \ln b/a} \checkmark \qquad (4.157) \text{ of 1st Ed.}$$

The potential between the two conductors is,

$$V_{ab} = \int_{\rho=a}^{b} E_\rho(\rho,\phi,z)\, d\rho = V_o e^{-j\beta z} \checkmark \qquad (4.158) \text{ of 1st Ed.}$$

The current on the inner conductor is,

$$I_a = \int_{\phi=0}^{2\pi} H_\phi(a,\phi,z)\, a\, d\phi = \frac{2\pi V_o e^{-j\beta z}}{\eta \ln b/a} \checkmark \qquad (4.159) \text{ of 1st Ed.}$$

The characteristic impedance is,

$$Z_o = \frac{V_{ab}}{I_o} = \frac{\eta \ln b/a}{2\pi} \checkmark \qquad (4.162) \text{ of 1st Ed.}$$

3.15 The solution is similar to the TE mode case for the coax, but with e_z in place of h_z:

$$e_z(\rho,\phi) = (A \sin n\phi + B \cos n\phi)\left[C J_n(k_c\rho) + D Y_n(k_c\rho)\right]$$

Then the boundary condition that $e_z = 0$ at $\rho = a$ and at $\rho = b$ yields two equations:

$$C J_n(k_c a) + D Y_n(k_c a) = 0$$
$$C J_n(k_c b) + D Y_n(k_c b) = 0$$

or,

$$J_n(k_c a) Y_n(k_c b) = J_n(k_c b) Y_n(k_c a)$$

For the TM_{01} mode, $n = 0$. Let $x = k_c a$. Then for $b = 2a$, we have that $k_c b = 2x$, and so the above equation can be written as,

$$f(x) = J_0(x) Y_0(2x) - J_0(2x) Y_0(x) = 0$$

We know that k_c should be greater than k_c for a circular waveguide of radius b, for which $k_{c_{01}} = p_{01}/b = 2.405/2a$, which implies that $x = 1.2$. So we can begin the root search at $x = 1.2$. Using a table of Bessel functions gives the following results in only a few minutes:

x	$J_0(x)$	$Y_0(x)$	$J_0(2x)$	$Y_0(2x)$	$f(x)$
1.2	.671	.228	.003	.510	.342
1.5	.512	.382	−.260	.377	.292
2.0	.224	.510	−.397	−.017	.198
3.1	−.292	.343	.202	−.248	.003
3.2	−.320	.307	.243	−.200	−.011

Linear interpolation between $x = 3.1$ and 3.2 gives a more accurate value for the root:

$$f(x) \simeq .003 + \frac{.003 - (-.011)}{3.1 - 3.2}(x - 3.1)$$

$$\simeq .437 - .14x = 0$$

$$x = \frac{.437}{.14} = \underline{\underline{3.12 = k_c a}}$$

3.16 From (3.175),

$$\bar{E}\times\bar{H}^*\cdot\hat{z} = -E_y H_x^* = \begin{cases} \dfrac{\omega\mu_0\beta|B|^2}{k_c^2}\sin^2 k_c x & \text{for } 0\le x\le d \\ \dfrac{\omega\mu_0\beta|B|^2}{h^2}\cos^2 k_c d\, e^{-2h(x-d)} & \text{for } d\le x<\infty \end{cases}$$

The power flow is,

$$P_0 = \frac{1}{2}\int_{x=0}^{\infty}\int_{y=0}^{1}\bar{E}\times\bar{H}^*\cdot\hat{z}\,dy\,dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2k_c^2}\int_{x=0}^{d}\sin^2 k_c x\,dx + \frac{\omega\mu_0\beta|B|^2}{2h^2}\cos^2 k_c d\int_{x=d}^{\infty}e^{-2h(x-d)}dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{x}{2}-\frac{\sin 2k_c x}{4k_c}\right)\Big|_0^d + \frac{\cos^2 k_c d}{h^2}\left(\frac{e^{-2h(x-d)}}{-2h}\right)\Big|_d^\infty\right]$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right)+\frac{\cos^2 k_c d}{2h^3}\right]$$

The power loss is,

$$P_\ell = \frac{R_s}{2}\int_S|\bar{H}_t|^2 ds = \frac{R_s}{2}\int_{y=0}^{1}\int_{z=0}^{1}\left[|\overset{\to 0}{H_x(x=0)}|^2+|H_z(x=0)|^2\right]dz\,dy$$

$$= \frac{R_s}{2}|B|^2$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_0} = \frac{2R_s}{4\omega\mu_0\beta\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right)+\frac{\cos^2 k_c d}{2h^3}\right]}$$

$$= \frac{R_s}{k_0\eta_0\beta\left[\frac{d}{k_c^2}-\frac{\sin 2k_c d}{2k_c^3}+\frac{\cos^2 k_c d}{h^3}\right]}$$ ✓

(3.17) Following the derivation in Section 3.6 for the TM surface waves of a dielectric slab:

$$k_c^2 = \mu_r k_0^2 - \beta^2 \qquad \text{for } 0 \le y \le d$$

$$h^2 = \beta^2 - k_0^2 \qquad \text{for } y \ge d$$

Then,

$$e_z(x,y) = \begin{cases} A \sin k_c y & \text{for } 0 \le y \le d \\ B e^{-hy} & \text{for } y > d \end{cases}$$

This form of e_z is selected to satisfy $e_z = 0$ at $y=0$, and to have exponential decay for $y \to \infty$ (radiation condition). next, we need H_x ($H_y = E_x = H_z = 0$); From (3.23a),

$$H_x = \frac{j\omega\epsilon_0}{k_c^2} \frac{\partial E_z}{\partial y} = \begin{cases} \frac{j\omega\epsilon_0}{k_c} A \cos k_c y & \text{for } 0 \le y \le d \\ \frac{j\omega\epsilon_0}{h} B e^{-hy} & \text{for } y > d \end{cases}$$

at $y=d$:

E_z continuous $\Rightarrow$ $A \sin k_c d = B e^{-hd}$

H_x continuous $\Rightarrow$ $\frac{A}{k_c} \cos k_c d = \frac{B}{h} e^{-hd}$

or,

$$h \cos k_c d = k_c \sin k_c d$$

$$\underline{h = k_c \tan k_c d} \quad ✓$$

and,

$$\underline{h^2 + k_c^2 = (\mu_r - 1) k_0^2} \quad ✓$$

These two equations must be solved simultaneously to find h and k_c.

(3.18) TM_{om} mode. $H_z = 0$ $\quad E_z(\rho,\phi,z) = e_z(\rho,\phi)\, e^{-j\beta z}$

(No TEM mode can be supported by this line because of the impossibility of phase matching at $\rho = b$)

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k_c^2\right) e_z(\rho,\phi) = 0$$

$\partial/\partial\phi = 0$ for $n=0$ modes $\Rightarrow E_\phi = H_\rho = 0$.

Thus,

$$e_z(\rho,\phi) = \begin{cases} A\,J_0(k_d\rho) + B\,Y_0(k_d\rho) & \text{for } a \le \rho \le b \\ C\,J_0(k_a\rho) + D\,Y_0(k_a\rho) & \text{for } b \le \rho \le c \end{cases}$$

where $\quad \beta^2 = \epsilon_r k_0^2 - k_d^2 = k_0^2 - k_a^2$.

The boundary conditions are:

$e_z = 0$ at $\rho = a$ and $\rho = c$.

e_z and H_ϕ are continuous at $\rho = b$.

From (3.110d),

$$H_\phi = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial\rho},$$

So we get the following four equations:

$$A\,J_0(k_d a) + B\,Y_0(k_d a) = 0$$

$$C\,J_0(k_a c) + D\,Y_0(k_a c) = 0$$

$$A\,J_0(k_d b) + B\,Y_0(k_d b) = C\,J_0(k_a b) + D\,Y_0(k_a b)$$

$$\epsilon_r k_d\left[A\,J_0'(k_d b) + B\,Y_0'(k_d b)\right] = k_a\left[C\,J_0'(k_a b) + D\,Y_0'(k_a b)\right]$$

k_a and k_d can be expressed in terms of β, and β can be found so that the determinant of the above system of equations vanishes. This is as far as we can go without actual values for $a, b, c,$ and ϵ_r.

3.19 To use (3.180) we compute $\sqrt{\epsilon_r}\, Z_0 = \sqrt{2.2}\,(100) = 148.3 > 120.$

and, $x = \dfrac{30\pi}{\sqrt{\epsilon_r}\, Z_0} - .441 = 0.195$

Then $W/b = .85 - \sqrt{.6 - x} = 0.2136$

$$W = .2136(.316\text{cm}) = \underline{0.0675 \text{ cm}} \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_r}\, f} = \frac{300}{\sqrt{2.2}\,(4000)} = \underline{5.06 \text{ cm}} \checkmark$$

3.20 First try $W/d < 2$:

From (3.197), $A = \dfrac{Z_0}{60}\sqrt{\dfrac{\epsilon_r+1}{2}} + \dfrac{\epsilon_r-1}{\epsilon_r+1}(.23 + .11/\epsilon_r) = 2.21$ ✓

Then, $W/d = \dfrac{8e^A}{e^{2A} - 2} = 0.90 < 2$ OK ✓

$$W = .9(.158 \text{ cm}) = \underline{0.142 \text{ cm}} \checkmark$$

From (3.195) the effective permittivity is,

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\,\frac{1}{\sqrt{1+12d/W}} = \underline{1.758} \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_e}\, f} = \frac{300}{\sqrt{1.758}\,(4000)} = \underline{5.656 \text{ cm}} \checkmark$$

3.21 $k_0 = \frac{2\pi f}{c} = 104.7 \text{ m}^{-1}$ ✓ ; $R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi(5\times10^9)(4\pi\times10^{-7})}{2(5.813\times10^7)}} = 0.018\,\Omega$ ✓

MICROSTRIP CASE:

First try $W/d > 2$: From (3.197), $B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}} = 8.0$

$$W/d = \frac{2}{\pi}\left[B-1-\ln(2B-1)+\frac{\epsilon_r-1}{2\epsilon_r}\left\{\ln(B-1)+.39-\frac{.61}{\epsilon_r}\right\}\right] = 3.09 > 2 \checkmark$$

$W = 3.09(.16\text{ cm}) = \underline{0.494\text{ cm}}$

From (3.195), $\epsilon_e = \frac{\epsilon_r+1}{2}+\frac{\epsilon_r-1}{2}\frac{1}{\sqrt{1+12d/W}} = 1.87$ ✓ $\Rightarrow \lambda_g = \frac{c}{\sqrt{\epsilon_e} f} = \underline{4.38\text{ cm}}$ ✓

From (3.198), $\alpha_d = \frac{k_0\epsilon_r(\epsilon_e-1)}{2\sqrt{\epsilon_e}(\epsilon_r-1)}\tan\delta = \underline{0.061}$ neper/m ✓

From (3.199), $\alpha_c = \frac{R_s}{Z_0 W} = \underline{0.073}$ neper/m ✓

Total MS attenuation: $\alpha_{MS} = (.061+.073)\left(\frac{np}{m}\right)(16\lambda_g)\left(.0438\frac{m}{\lambda_g}\right)\left(\frac{8.686\text{ dB}}{\text{neper}}\right)$

$= 0.82\text{ dB}$ ✓

STRIPLINE CASE:

From (3.180), $\sqrt{\epsilon_r}Z_0 = \sqrt{2.2}(50) = 74 < 120$. $x = \frac{30\pi}{\sqrt{\epsilon_r}Z_0} - .441 = 0.833$

$W/b = x = 0.833 \Rightarrow W = .833(.32\text{ cm}) = \underline{0.267\text{ cm}}$ ✓

$\lambda_g = \frac{c}{\sqrt{\epsilon_r}f} = \underline{4.045\text{ cm}}$ ✓ $A = 1+\frac{2W}{b-t}+\frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right) = 4.73$

From (3.181), $\alpha_c = \frac{2.7\times10^{-3}R_s\epsilon_r Z_0}{30\pi b}A = \underline{0.084}$ neper/m ✓

From (3.30), $\alpha_d = \frac{k\tan\delta}{2} = \frac{\sqrt{2.2}(104.7)(.001)}{2} = \underline{0.078}$ neper/m ✓

Total S.L. attenuation:

$\alpha_{SL} = (.084+.078)\left(\frac{np}{m}\right)(16\lambda_g)\left(.04045\frac{m}{\lambda_g}\right)\left(\frac{8.686\text{ dB}}{\text{neper}}\right)$

$= \underline{0.91\text{ dB}}$ ✓

Thus the microstrip line should be used.

(3.22) $H_z(x,y,z) = h_z(x,y)\, e^{-j\beta z}$; h_z real, β real.

From (3.19),

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial x} e^{-j\beta z} \qquad H_y = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial y} e^{-j\beta z}$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y} e^{-j\beta z} \qquad E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x} e^{-j\beta z}$$

$$\bar{E}\times\bar{H}^* = (E_x H_y^* - E_y H_x^*)\hat{z} - E_x H_z^* \hat{y} + E_y H_z^* \hat{x}$$

$$= \frac{\omega\mu\beta}{k_c^4}\left[\left(\frac{\partial h_z}{\partial y}\right)^2 + \left(\frac{\partial h_z}{\partial x}\right)^2\right]\hat{z} + \frac{j\omega\mu}{k_c^2}\left(\frac{\partial h_z}{\partial y}\hat{y} + \frac{\partial h_z}{\partial x}\hat{x}\right) h_z$$

So if h_z is real (or a real function times a complex constant), there is real power flow only in the z-direction.

(3.23) Write the incident, reflected, and transmitted TE_{10} fields as follows:

$$E_y^i = E_0 \sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad E_y^r = E_0\Gamma \sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$H_x^i = \frac{-E_0}{Z_a}\sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad H_x^r = \frac{E_0\Gamma}{Z_a}\sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$E_y^t = E_0 T \sin\frac{\pi x}{a} e^{-j\beta_d z}$$

$$H_x^t = \frac{-E_0 T}{Z_d}\sin\frac{\pi x}{a} e^{-j\beta_d z}$$

where $\beta_a = \sqrt{k_0^2 - (\pi/a)^2}$ $\qquad Z_a = k_0\eta_0/\beta_a$

$\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$ $\qquad Z_d = k_0\eta_0/\beta_d$

Match fields at $z=0$ to obtain:

$$1+\Gamma = T \qquad (E_y \text{ continuous})$$

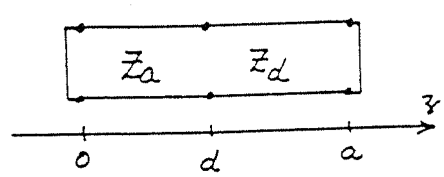
$$\frac{1}{Z_a}(-1+\Gamma) = \frac{-T}{Z_d} \qquad (H_x \text{ continuous})$$

Solving for Γ gives,

$$\Gamma = \frac{Z_d - Z_a}{Z_d + Z_a}$$

which agrees with the transmission line theory result if Z_{TE} is used as Z_0 in each region.

3.24 $Z_{TM} = \eta\beta/k = \eta_0\beta/k_0\sqrt{\epsilon_r}$



for $0 < x < d$, $Z_a = \eta_0 k_{xa}/k_0$

for $d < x < a$, $Z_d = \eta_0 k_{xd}/\sqrt{\epsilon_r} k_0$

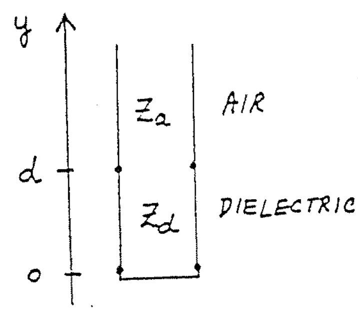
$$\beta = \sqrt{\epsilon_r k_0^2 - k_{xd}^2 - (n\pi/b)^2} = \sqrt{k_0^2 - k_{xa}^2 - (n\pi/b)^2}$$

Applying (3.215):

$$Z_a \tan \beta_a d + Z_d \tan \beta_d (a-b) = 0$$

The m-th root of this equation applies to the TM_{mn} mode.

3.25



$$Z_a = k_{ya}\eta_0/k_0 = -jh\eta_0/k_0$$

$$Z_d = k_{yd}\eta/k = k_{yd}\eta_0/k_0$$

$$\beta = \sqrt{\mu_r k_0^2 - k_{yd}^2} = \sqrt{k_0^2 - k_{ya}^2} = \sqrt{k_0^2 + h^2}$$

Applying (3.215):

$$Z_a + j Z_d \tan k_{yd} d = 0$$

$$h = k_{yd} \tan k_{yd} = 0$$

This agrees with the solution to Problem 3.17, with $k_c = k_{yd}$. ✓

3.26 For X-band guide, $a = 2.286$ cm.

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = \frac{2\pi(9500)\sqrt{2.08}}{300} = 287.\ m^{-1} \checkmark$$

$$\beta = \sqrt{k^2 - (\pi/a)^2} = 252.\ m^{-1} \checkmark$$

speed of light in Teflon $= \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.08}} = \underline{2.08\times10^8}$ m/sec ✓

phase velocity $= v_p = \frac{\omega}{\beta} = \frac{2\pi(9.5\times10^9)}{252.} = \underline{2.37\times10^8}$ m/sec ✓

From (3.234),

$$\text{group velocity} = v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\frac{d\beta}{dk}\frac{dk}{d\omega}\right)^{-1} = \left(\frac{k}{\beta}\sqrt{\mu\epsilon}\right)^{-1}$$

$$= \frac{\beta}{k\sqrt{\mu\epsilon}} = \frac{252(2.08\times10^8)}{287.} = 1.83\times10^8\ m/sec$$

Note that $v_g < \frac{c}{\sqrt{\epsilon_r}} < v_p$.

3.27

$$P_{MAX} = c\,a^2 \ln\frac{b}{a}$$

$$\frac{dP_{MAX}}{da} = 2a\ln\frac{b}{a} - \frac{a^2}{a} = 0$$

$$2\ln\frac{b}{a} - 1 = 0$$

$$2\ln x = 1$$

$$\ln x = 0.5$$

$$x = 1.65$$

$$Z_0 = \frac{377}{2\pi}\ln\frac{b}{a} = \frac{120\pi}{2\pi}\left(\frac{1}{2}\right) = \underline{30\,\Omega}$$

(4.1) (This problem is essentially the same as P. 3.23)
Write the fields of incident, reflected, and transmitted TE_{10} modes in each region:

$$E_y^i = E_0 \sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad E_y^r = E_0 \Gamma \sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$H_x^i = \frac{-E_0}{Z_a} \sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad H_x^r = \frac{E_0 \Gamma}{Z_0} \sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$E_y^t = E_0 T \sin\frac{\pi x}{a} e^{-j\beta_d z}$$

$$H_x^t = \frac{-E_0 T}{Z_d} \sin\frac{\pi x}{a} e^{-j\beta_d z}$$

where $\beta_a = \sqrt{k_0^2 - (\pi/a)^2}$, $\quad Z_a = k_0 \eta_0 / \beta_a$

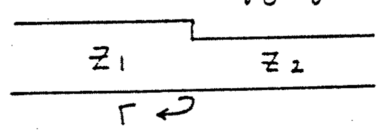
$\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$, $\quad Z_d = k_0 \eta_0 / \beta_d$

Match fields at $z=0$:
$$1 + \Gamma = T$$
$$\frac{1}{Z_a}(-1+\Gamma) = \frac{-T}{Z_d}$$

Solving gives, $\Gamma = \dfrac{Z_d - Z_a}{Z_d + Z_a}$. As in Example 4.2, $\Gamma = -0.629$ ✓

(4.2) Using a transmission line analogy gives,

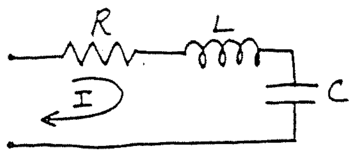
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



where $Z_1 = k_0 \eta_0 / \beta_1$, $Z_2 = k_0 \eta_0 / \beta_2$.

But $\beta_1 = \beta_2 = \sqrt{k_0^2 - (\pi/a)^2}$ in both regions, since only the height (b) of the guide changes. Thus, $\Gamma = 0$ from above. This is obviously not correct, as E_y should be zero for $b/2 < y < b$. Higher order TE_{1n} modes must be considered, in a mode matching procedure. This will result in a solution where $\Gamma \neq 0$. Consideration of only the dominant mode is not adequate.

(4.3)



$$P_\ell = \tfrac{1}{2}|I|^2 R \implies R = \frac{P_\ell}{\frac{1}{2}|I|^2}$$

$$W_m = \tfrac{1}{4} L |I|^2 \implies L = \frac{2W_m}{\frac{1}{2}|I|^2}$$

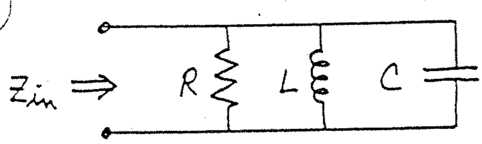
$$W_e = \tfrac{1}{4} C|V_c|^2 = \frac{1}{4\omega^2 C}|I|^2 \implies \frac{1}{\omega^2 C} = \frac{2W_e}{\frac{1}{2}|I|^2}$$

The input impedance is,

$$Z_{in} = R + j\left(\omega L + \frac{1}{\omega C}\right) = \frac{P_\ell + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2} \quad ✓$$

In agreement with (4.17)

(4.4)



$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\omega\left(C - \frac{1}{\omega^2 L}\right)}$$

$$Z(-\omega) = \frac{1}{\frac{1}{R} - j\omega\left(C - \frac{1}{\omega^2 L}\right)} = Z^*(\omega) \quad ✓$$

4.5 $$P_{in} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}[V]^t[Y]^*[V]^*$$
$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} V_m Y_{mn}^* V_n^*$$

If lossless, $Re\{P_{in}\}=0$. Since the V_n's are independent, we first let all $V_m=0$, except for V_n. Then,

$$P_{in} = \frac{1}{2} V_n Y_{nn}^* V_n^* = \frac{1}{2}|V_n|^2 Y_{nn}^*$$
$$\therefore \quad Re\{Y_{nn}^*\} = Re\{Y_{nn}\} = 0 \quad ✓$$

Now let all port voltages be zero except for V_m and V_n. Then,

$$P_{in} = \frac{1}{2} V_m Y_{mn}^* V_n^* + \frac{1}{2} V_n Y_{nm}^* V_m^*$$

So,
$$Re\{V_m Y_{mn}^* V_n^* + V_n Y_{nm}^* V_m^*\} = 0$$

If $Y_{mn} = Y_{nm}$ (reciprocal), then

$$Re\{Y_{mn}^*(V_m V_n^* + V_n V_m^*)\} = Re\{Y_{mn}^*[(V_m V_n^* + (V_m V_n^*)^*]\} = 0$$

Since $A + A^*$ is real, we must have $Re\{Y_{mn}\} = 0$ ✓

4.6 Let $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, and show that Z_{ij}'s can be found such that $P_{in}=0$, but not all Z_{ij}'s are pure imaginary.

$$P_{in} = \frac{1}{2}[I]^t[Z]^t[I]^* = \frac{1}{2}(I_1 Z_{11} I_1^* + I_1 Z_{21} I_2^* + I_2 Z_{12} I_1^* + I_2 Z_{22} I_1^*)$$
$$= \frac{1}{2}(Z_{11}|I_1|^2 + Z_{22}|I_2|^2 + Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*)$$

To be lossless, we must have $Re\{Z_{11}\} = Re\{Z_{22}\} = 0$.

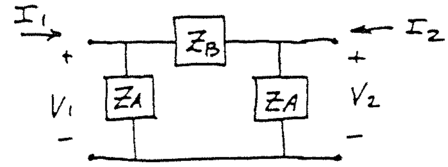
Also, $Re\{Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*\} = 0$.

This will occur if $Z_{12} = -Z_{21}^*$ (since $Re\{A - A^*\} = 0$).

For example, if $Z_{12} = a + jb$, then $Z_{21} = -a + jb$.

Thus, $[Z]$ is not symmetric, and the answer is NO.

4.7



From (4.28),

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{V_1}{V_1\left(\frac{2Z_A+Z_B}{Z_A(Z_A+Z_B)}\right)} = \frac{Z_A(Z_A+Z_B)}{2Z_A+Z_B} \checkmark = Z_{22} \quad \text{(BY SYMMETRY)}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{I_1 Z_{11}\left(\frac{Z_A}{Z_A+Z_B}\right)}{I_1} = \frac{Z_A^2}{2Z_A+Z_B} \checkmark = Z_{12} \quad \text{(BY RECIPROCITY)}$$

From (4.29),

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{I_1}{I_1\left(\frac{Z_A Z_B}{Z_A+Z_B}\right)} = \frac{Z_A+Z_B}{Z_A Z_B} \checkmark = Y_{22} \quad \text{(BY SYMMETRY)}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{-V_1/Z_B}{V_1} = \frac{-1}{Z_B} \checkmark = Y_{12} \quad \text{(BY RECIPROCITY)}$$

$$I_2 = \frac{V_2}{Z_A} + \frac{V_2 - V_1}{Z_B}$$

CHECK: $[Z][Y] = [U]$?

$$Z_{11}Y_{11} + Z_{12}Y_{21} = \frac{(Z_A+Z_B)^2}{Z_B(2Z_A+Z_B)} - \frac{Z_A^2}{Z_B(2Z_A+Z_B)} = \frac{2Z_AZ_B + Z_B^2}{Z_B(2Z_A+Z_B)} = 1 \checkmark$$

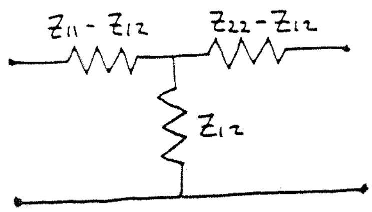
$$Z_{11}Y_{12} + Z_{12}Y_{22} = \frac{-Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} + \frac{Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} = 0 \checkmark$$

Similarly for the T-network. The results are,

$$Z_{11} = Z_{22} = \frac{Y_A+Y_B}{Y_A Y_B} \checkmark \qquad Z_{12} = Z_{21} = \frac{-1}{Y_B} \checkmark$$

$$Y_{11} = Y_{22} = \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} \checkmark \qquad Y_{12} = Y_{21} = \frac{Y_A^2}{2Y_A+Y_B} \checkmark$$

(4.8) Model the two-port as below:



Then, $$Z_{sc}^{(1)} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22} - Z_{12})}{Z_{22}} = Z_{11} - Z_{12}^2/Z_{22}$$

$$Z_{sc}^{(2)} = Z_{22} - Z_{12}^2/Z_{11}$$

$$Z_{oc}^{(1)} = Z_{11} \checkmark$$

$$Z_{oc}^{(2)} = Z_{22} \checkmark$$

From the first equation,
$$Z_{12}^2 = -(Z_{sc}^{(1)} - Z_{11})Z_{22} = (Z_{oc}^{(1)} - Z_{sc}^{(1)})\, Z_{oc}^{(2)}.$$

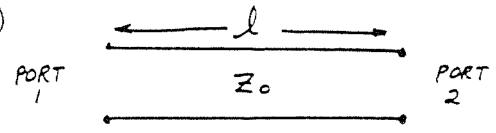
(4.9) From (4.58), $$V_n = V_n^+ + V_n^-$$
$$Z_0 I_n = V_n^+ - V_n^-$$

Solving for V_n^+, V_n^-: $$V_n^+ = (V_n + Z_0 I_n)/2$$
$$V_n^- = (V_n - Z_0 I_n)/2$$

at port 1: $$V_1^+ = \tfrac{1}{2}\left[10\angle 0^\circ + 50(.1\angle 30^\circ)\right] = 7.27\angle 9.9^\circ$$
$$V_1^- = \tfrac{1}{2}\left[10\angle 0^\circ - 50(.1\angle 30^\circ)\right] = 3.10\angle -23.8^\circ$$

at port 2: $$V_2^+ = \tfrac{1}{2}\left[12\angle 90^\circ + 50(.15\angle 120^\circ)\right] = 9.44\angle 101.5^\circ \checkmark$$
$$V_2^- = \tfrac{1}{2}\left[12\angle 90^\circ - 50(.15\angle 120^\circ)\right] = 3.33\angle 55.7^\circ \checkmark$$

(4.10) a)

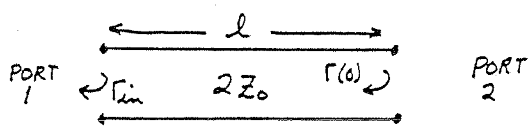


$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = 0 \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = 0$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0} = e^{-j\beta \ell} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = e^{-j\beta \ell}$$

$$[S] = \begin{bmatrix} 0 & e^{-j\beta \ell} \\ e^{-j\beta \ell} & 0 \end{bmatrix} \quad \text{unitary} \checkmark$$

b)



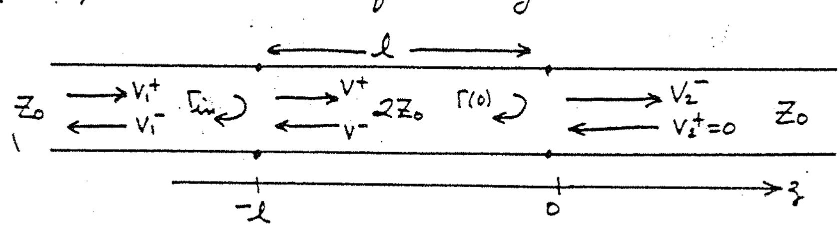
$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma_{in} = S_{22} \qquad \Gamma(0) = \frac{Z_0 - 2Z_0}{Z_0 + 2Z_0} = \frac{-1}{3}$$

$$Z_{in} = 2Z_0 \frac{1 + \Gamma(0)\, e^{-2j\beta \ell}}{1 - \Gamma(0)\, e^{-2j\beta \ell}} = 2Z_0 \frac{1 - \frac{1}{3} e^{-2j\beta \ell}}{1 + \frac{1}{3} e^{-2j\beta \ell}}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{2\left(1 - \frac{1}{3} e^{-2j\beta \ell}\right) - \left(1 + \frac{1}{3} e^{-2j\beta \ell}\right)}{2\left(1 - \frac{1}{3} e^{-2j\beta \ell}\right) + \left(1 + \frac{1}{3} e^{-2j\beta \ell}\right)} = \frac{1 - e^{-2j\beta \ell}}{3 - \frac{1}{3} e^{-2j\beta \ell}}$$

CHECK: if $\ell = \lambda/2$, $\Gamma_{in} = 0$ ✓

For $S_{21} = S_{12}$, consider the following circuit:



On the $2Z_0$ line we have,

$$V_1 = V_1^+ + V_1^- = V_1^+(1 + \Gamma_{in}) = V^+ e^{j\beta \ell} - \frac{1}{3} V^+ e^{-j\beta \ell}$$

$$V_2 = V_2^+ + V_2^- = V_2^- = V^+\left(1 - \frac{1}{3}\right)$$

Thus, $$V_2^- = \frac{2}{3}V^+ = \frac{\frac{2}{3}V_1^+(1+\Gamma_{in})}{e^{j\beta\ell} - \frac{1}{3}e^{-j\beta\ell}}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{\frac{2}{3}(1+\Gamma_{in})}{e^{j\beta\ell} - \frac{1}{3}e^{-j\beta\ell}} = \frac{8}{3}\,\frac{e^{-j\beta\ell}}{3 - \frac{1}{3}e^{-2j\beta\ell}}$$

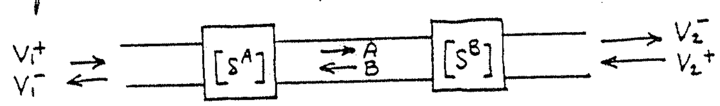
CHECK: if $\ell = \lambda$, $Z_{in} = Z_0$, $\Gamma_{in} = 0$, $S_{11} = 0$, $S_{21} = 1$ ✓

if $\ell = \lambda/2$, $\Gamma_{in} = 0$, $S_{11} = 0$, $S_{21} = -1$ ✓

$$|S_{11}|^2 + |S_{21}|^2 = \left|\frac{1 - e^{-2j\beta\ell}}{3 - \frac{1}{3}e^{-2j\beta\ell}}\right|^2 + \frac{64}{9}\,\frac{1}{|3 - \frac{1}{3}e^{-2j\beta\ell}|^2}$$

$$= \frac{|1 - e^{-2j\beta\ell}|^2 + \frac{64}{9}}{|3 - \frac{1}{3}e^{-2j\beta\ell}|^2} = \frac{1 - e^{-2j\beta\ell} - e^{2j\beta\ell} + 1 + \frac{64}{9}}{9 - e^{2j\beta\ell} - e^{-2j\beta\ell} + \frac{1}{9}} = 1 \checkmark \quad \text{(UNITARY)}$$

4.11 Define wave amplitudes as shown:



Then, $$\begin{bmatrix} V_1^- \\ A \end{bmatrix} = [S^A]\begin{bmatrix} V_1^+ \\ B \end{bmatrix} \qquad \begin{bmatrix} B \\ V_2^- \end{bmatrix} = [S^B]\begin{bmatrix} A \\ V_2^+ \end{bmatrix} \qquad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = [S]\begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$. When $V_2^+ = 0$, we have $B = S_{11}^B A$, $V_2^- = S_{21}^B A$.

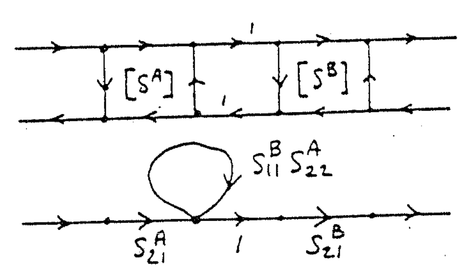
Then, $$A = S_{21}^A V_1^+ + S_{22}^A B = S_{21}^A V_1^+ + S_{22}^A S_{11}^B A$$

$$\frac{V_2^-}{S_{21}^B} = S_{21}^A V_1^+ + S_{22}^A S_{11}^B \frac{V_2^-}{S_{21}^B}$$

$$V_2^-\left(\frac{1 - S_{22}^A S_{11}^B}{S_{21}^B}\right) = S_{21}^A V_1^+$$

So, $$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B} \checkmark$$

SIGNAL FLOWGRAPH SOLUTION:



$$\therefore S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{11}^B S_{22}^A} \checkmark$$

4.12

a) $[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix}$ $S_{12}=S_{21}$ since reciprocal

$[S]$ is unitary if lossless, so

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$ (or 1st col.)

$|S_{21}|^2 = 1 - |S_{11}|^2$ ✓

b) $[S] = \begin{bmatrix} S_{11} & S_{21} \\ 0 & S_{22} \end{bmatrix}$ $S_{12} \neq S_{21}$ since nonreciprocal

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$

1st Col.: $|S_{11}|^2 = 1$

$\therefore |S_{21}| = 0$

4.13 a matched, reciprocal, 3-port network has an $[S]$ matrix of the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is lossless, then $[S]$ must be unitary:

$|S_{12}|^2 + |S_{13}|^2 = 1$ (1) $S_{13} S_{23}^* = 0$ (4)

$|S_{12}|^2 + |S_{23}|^2 = 1$ (2) $S_{12} S_{13}^* = 0$ (5)

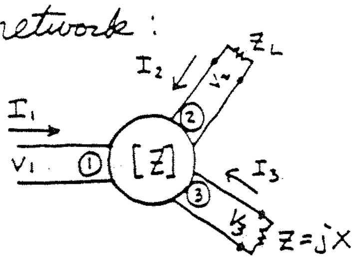
$|S_{13}|^2 + |S_{23}|^2 = 1$ (3) $S_{12} S_{23}^* = 0$ (6)

To show that a contradiction exists, assume that $S_{12}=0$, in order to satisfy (5) and (6). Then from (1), $|S_{13}|^2=1$, and from (3), $|S_{23}|=0$. But then (2) will be contradicted. Similarly, a contradiction will follow if we let $S_{13}=0$, or $S_{23}=0$.

a circulator is an example of a nonreciprocal, lossless, matched 3-port network.

(4.14) For this problem it is easiest to use the z-matrix for a lossless reciprocal 3-port network:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} jX_{11} & jX_{12} & jX_{13} \\ jX_{12} & jX_{22} & jX_{23} \\ jX_{13} & jX_{23} & jX_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



If we terminate port 3 in a reactance jX, then $V_3 = -jX I_3$. Then we must find jX so that $V_2 = 0$ for $V_1 \neq 0$. If $V_2 = 0$, then $I_2 = 0$:

$$V_3 = jX_{13} I_1 + jX_{33} I_3 = -jX I_3$$

$$I_3 = \frac{-X_{13} I_1}{X_{33} + X}$$

$$V_2 = jX_{12} I_1 + jX_{23} I_3 = \left(jX_{12} - \frac{jX_{23}X_{13}}{X_{33}+X}\right) I_1 = 0$$

So,

$$X_{12}X_{33} + XX_{12} - X_{13}X_{23} = 0$$

$$X = \frac{X_{13}X_{23} - X_{12}X_{33}}{X_{12}} \checkmark$$

CHECK: The input impedance at Port 1 is,

$$Z_{in}^{(1)} = \frac{V_1}{I_1} = \frac{jX_{11}I_1 + jX_{13}I_3}{I_1} = jX_{11} + jX_{13}\left(\frac{-X_{13}}{X_{33}+X}\right)$$

$$= j\left(X_{11} - \frac{X_{13}^2}{X_{33}+X}\right)$$ which is pure imaginary ✓

4.15 $$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

Assume the network is fed at port 1, so $V_1^+ \neq 0$. Port 2 is terminated in a matched load, so $V_2^+ = 0$. Port 3 is terminated in a reactive load, so $V_3^+ = e^{j\phi} V_3^-$. We must find $e^{j\phi}$ so that $V_1^-/V_1^+ = 0$.

$$V_3^- = S_{13} V_1^+ + S_{33} V_3^+ = e^{-j\phi} V_3^+$$

$$V_3^+ = \frac{S_{13} V_1^+}{e^{-j\phi} - S_{33}}$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+ = S_{11} V_1^+ + \frac{S_{13}^2 V_1^+}{e^{-j\phi} - S_{33}}$$

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{13}^2}{e^{-j\phi} - S_{33}} = 0 \Rightarrow e^{-j\phi} = S_{33} - \frac{S_{13}^2}{S_{11}} . \checkmark$$

We should also verify that this quantity has unit magnitude:

$$\left| S_{33} - S_{13}^2/S_{11} \right|^2 = \frac{|S_{11}|^2 |S_{33}|^2 - |S_{13}|^4 - S_{11}^* S_{33}^* S_{13}^2 - S_{11} S_{33} S_{13}^{*2}}{|S_{11}|^2}$$

$= (\quad)(\quad)^*$

The unitary properties of [S] lead to four equations:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{11} S_{12}^* + S_{12} S_{11}^* + |S_{13}|^2 = 0$$

$$2|S_{13}|^2 + |S_{33}|^2 = 1 \qquad S_{12} S_{13}^* + S_{11} S_{13}^* + S_{13} S_{33}^* = 0$$

Eliminating S_{12} from the two equations on the right yields,

$$-2|S_{11}|^2 - \frac{S_{11} S_{13}^* S_{33}}{S_{13}} - \frac{S_{11}^* S_{13} S_{33}^*}{S_{13}^*} + |S_{13}|^2 = 0$$

or,

$$-S_{11} S_{13}^{*2} S_{33} - S_{11}^* S_{13}^2 S_{33}^* = 2|S_{11}|^2 |S_{13}|^2 - |S_{13}|^4$$

Then,

$$|S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 + \cancel{|S_{13}|^4} + 2|S_{11}|^2 |S_{13}|^2 - \cancel{|S_{13}|^4}}{|S_{11}|^2}$$

$$= |S_{33}|^2 + 2|S_{13}|^2 = 1 \checkmark$$

4.16

a) To be lossless, [S] must be unitary:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = (.1)^2 + (.6)^2 + (.6)^2 = 0.73 \neq 1$$

Thus the network is <u>not lossless</u>

b) To be reciprocal, [S] must be symmetric:

Since $S_{13} \neq S_{31}$, the network is <u>not reciprocal</u>.

c) When all other ports are matched, $\Gamma = S_{11}$

Thus, $RL = -20 \log |\Gamma| = -20 \log(.1) = \underline{20\ dB}$ at port 1. ✓

d) When all other ports are matched, the insertion loss from port 2 to port 4 is,

$IL = -20 \log |S_{42}| = -20 \log(.6) = \underline{4.4\ dB}$. Phase delay $= \underline{45^\circ}$ ✓

e) For a short circuit at port 3, and matched loads at other ports, we have,

$$V_2^+ = V_4^+ = 0 \ , \quad V_3^+ = -V_3^-$$
$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+ = S_{11}V_1^+ - S_{13}V_3^-$$
$$V_3^- = S_{31}V_1^+ + S_{33}V_3^+ = S_{31}V_1^+ - S_{33}V_3^-$$

Solving the second equation for V_3^-:

$$V_3^- = \frac{S_{31}V_1^+}{1 + S_{33}}$$

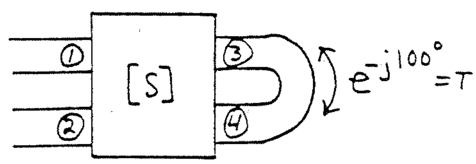
Then,

$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{13}S_{31}}{1 + S_{33}} \checkmark$$

$$= .1j - \frac{(.6\angle 45^\circ)(.6\angle -45^\circ)}{1+0} = .1j - .36 = \underline{0.374\angle 164^\circ} \checkmark$$

(verified with supercompact)

4.17



$V_4^+ = T V_3^-$
$V_3^+ = T V_4^-$

Assume feed at port 1, port 2 matched. Then $V_2^+ = 0$. Also,
$S_{12} = S_{13} = S_{21} = S_{24} = S_{31} = S_{34} = S_{42} = S_{43} = 0$.

DIRECT SOLUTION:

$$V_2^- = S_{23} V_3^+ = S_{23} T V_4^-$$
$$V_3^- = S_{33} V_3^+$$
$$V_4^- = S_{41} V_1^+ + S_{44} V_4^+ = S_{41} V_1^+ + S_{44} T V_3^-$$
$$= S_{41} V_1^+ + S_{44} S_{33} T V_3^+$$
$$= S_{41} V_1^+ + S_{44} S_{33} T^2 V_4^-$$

Solve for V_4^-:

$$V_4^- = \frac{S_{41} V_1^+}{1 - S_{44} S_{33} T^2}$$

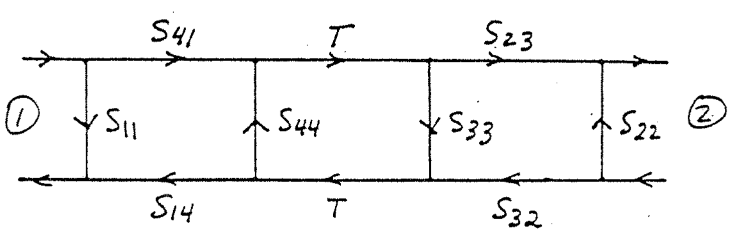
Then,

$$T_{21} = \frac{V_2^-}{V_1^+} = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^2} = \frac{(.7\angle{-45^\circ})(.8)(1\angle{-100^\circ})}{1 - (.7\angle{45^\circ})(.6\angle{90^\circ})(1\angle{-200^\circ})}$$

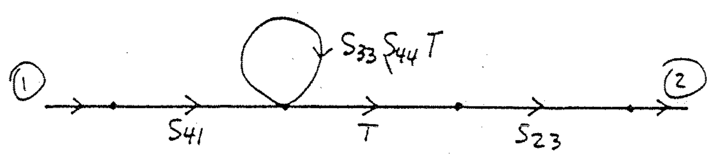
$$= 0.453\angle{0.4^\circ}$$

$$IL = -20 \log(.453) = 6.88 \text{ dB}$$

SIGNAL FLOWGRAPH SOLUTION:

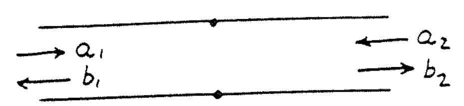


(SPLITTING RULE AND SELF-LOOP RULE)



$$T_{21} = \frac{S_{41} S_{23} T}{1 - S_{33} S_{44} T^2}$$

4.18



From (4.61),

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}} \checkmark$$ (reflection coefficient at port 1 when port 2 is matched)

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{2Z_{01}}{Z_{01}+Z_{02}} \checkmark$$ (transmission coefficient from port 2 to port 1)

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{2Z_{02}}{Z_{01}+Z_{02}} \checkmark$$ (transmission coefficient from port 1 to port 2)

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}} \checkmark$$ (reflection coefficient at port 2 when port 1 is matched)

4.19 Using Table 4.2 for S to Z transformation:

$(1+S_{11}) = 1.4765\angle 28.3°$

$(1-S_{11}) = .9899\angle -45°$

$S_{12}S_{21} = -0.36$

$(1+S_{22}) = 1.4765\angle -28.3°$

$(1-S_{22}) = .9899\angle 45°$

$(1-S_{11})(1-S_{22}) - S_{12}S_{21} = 1.340$

Then,

$$Z_{11} = \frac{50(1.4765\angle 28.3°)(.9899\angle 45°) - .36}{1.340} = 2.24 + j52.2\,\Omega \checkmark$$

$$Z_{12} = Z_{21} = 50\,\frac{2(.6\angle 90°)}{1.340} = j44.8\,\Omega \checkmark$$

$$Z_{22} = \frac{50(.9899\angle -45°)(1.4765\angle -28.3°) - .36}{1.340} = 2.24 - j52.2\,\Omega \checkmark$$

(verified with supercompact)

4.20 From (4.62),

$$S'_{ij} = \frac{\sqrt{Z_{0j}}}{\sqrt{Z_{0i}}} S_{ij}$$

So,

$$S'_{11} = S_{11} \checkmark$$

$$S'_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}}\, S_{12} \checkmark$$

$$S'_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}}\, S_{21} \checkmark$$

$$S'_{22} = S_{22} \checkmark$$

4.21 From Table 4.1 the ABCD parameters for a transmission line section are,

$A = D = \cos\beta\ell$, $B = jZ_0 \sin\beta\ell$, $C = jY_0 \sin\beta\ell$.

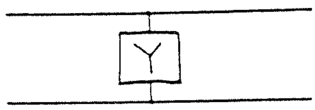
Now use Table 4.2 to convert to Z-parameters:

$$Z_{11} = \frac{A}{C} = \frac{\cos\beta\ell}{jY_0 \sin\beta\ell} = -jZ_0 \cot\beta\ell \checkmark$$

$$Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc\beta\ell \checkmark$$

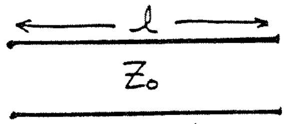
$$Z_{22} = \frac{D}{C} = \frac{\cos\beta\ell}{jY_0 \sin\beta\ell} = -jZ_0 \cot\beta\ell \checkmark$$

4.22



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$ ✓ $\qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = Y$ ✓

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0$ ✓ $\qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$ ✓



for $I_2 = 0$, $\quad V_1 = V^+(e^{j\beta l} + e^{-j\beta l}) = V^+ 2\cos\beta l$

$V_2 = 2V^+ = V_1/\cos\beta l$

So, $\quad A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \cos\beta l$ ✓

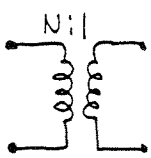
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{V_1}{Z_{in} V_2} = \frac{\cos\beta l}{-jZ_0 \cot\beta l} = jY_0 \sin\beta l \checkmark$$

for $V_2 = 0$, $\quad V_1 = V^+(e^{j\beta l} - e^{-j\beta l}) = V^+ 2j \sin\beta l$

$I_2 = \frac{2V^+}{Z_0}$

So, $\quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = jZ_0 \sin\beta l$ ✓

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_1}{Z_{in} I_2} = \frac{B}{Z_{in}} = \frac{jZ_0 \sin\beta l}{jZ_0 \tan\beta l} = \cos\beta l \checkmark$$



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{NV_2}{V_2} = N$ ✓

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0$ ✓

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$ ✓

$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{1}{N}$ ✓

(4.23) NOTE: Difference in signs for Z and ABCD.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{V_2}\frac{V_2}{I_1}\right|_{I_2=0} = A/C \checkmark$$

for $I_1 = 0$, $\quad V_1 = AV_2 - BI_2$

$$0 = CV_2 - DI_2 \Rightarrow \quad V_2 = DI_2/C$$

$$V_1 = \left(\frac{AD}{C} - B\right) I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{AD-BC}{C} \checkmark$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1/C \checkmark$$

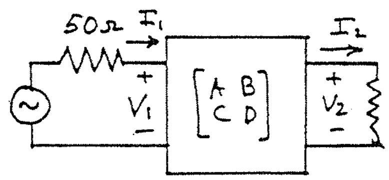
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = D/C \checkmark$$

(4.24) Using Table 4.1, the ABCD matrix of the cascade of four components is,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 40+j30 \\ 0 & 1 \end{bmatrix}}_{R+jX} \underbrace{\begin{bmatrix} 3 & 0 \\ 0 & 1/3 \end{bmatrix}}_{\text{TRANSF.}} \underbrace{\begin{bmatrix} 0 & j75 \\ j/75 & 0 \end{bmatrix}}_{\text{T-LINE}} = \begin{bmatrix} (-.1333+j.1778) & j225 \\ j/225 & 0 \end{bmatrix}$$

CHECK: $AD - BC = 1 \checkmark$

Then we have,



$I_2 = V_2/60$.

$$V_1 = AV_2 + BI_2 = (A + B/60)\, V_2$$

$$I_1 = CV_2 + DI_2 = (C + D/60)\, V_2$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{(A+B/60)}{(C+D/60)} = \frac{(-.1333+j.1778+j225/60)}{j/225} = 883.8 + j30.0\,\Omega \checkmark$$

$$V_L = V_2 = \frac{I_1}{(C+D/60)} = \frac{V_g}{(C+D/60)(Z_{in}+50)} = \frac{10(-j225)}{933.8+j30} = 2.41\underline{/-92^\circ}$$

4.25

DIRECT CALCULATION:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_1 \frac{1/Y}{Z+1/Y}} = 1+YZ$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1/Y} = Y$$

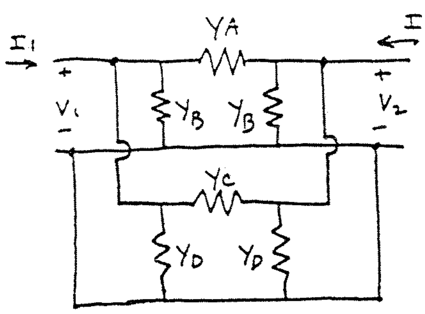
$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

CHECK: $AD - BC = 1 + YZ - ZY = 1$ ✓

CALCULATION USING CASCADE: (From Table 4.1)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}$$ ✓

4.26



$$[Y_1] = \begin{bmatrix} Y_A+Y_B & -Y_A \\ -Y_A & Y_A+Y_B \end{bmatrix}$$

$$[Y_2] = \begin{bmatrix} Y_C+Y_D & -Y_C \\ -Y_C & Y_C+Y_D \end{bmatrix}$$

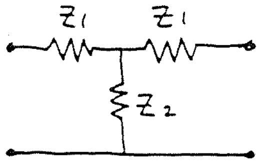
adding [Y] matrices gives:

$$[Y] = [Y_1] + [Y_2] = \begin{bmatrix} Y_A+Y_B+Y_C+Y_D & -Y_A-Y_C \\ -Y_A-Y_C & Y_A+Y_B+Y_C+Y_D \end{bmatrix}$$

By direct calculation, we obtain similar results:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A+Y_B+Y_C+Y_D$$ ✓ $$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -(Y_A+Y_C)$$ ✓

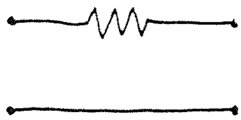
Now apply to bridged-T network. (Example 5.7 of 1st edition)



$$[Z_A] = \begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_1+Z_2 \end{bmatrix}$$

$$[Y_A] = \frac{1}{D}\begin{bmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & Z_1+Z_2 \end{bmatrix} \checkmark$$

$$D = (Z_1+Z_2)^2 - Z_2^2 = Z_1^2 + 2Z_1Z_2 \checkmark$$



$$[Y_B] = \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix} \checkmark$$

$$[Y_{TOT}] = [Y_A] + [Y_B] = \begin{bmatrix} \frac{1}{Z_3} + \frac{Z_1+Z_2}{D} & -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) \\ -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) & \frac{1}{Z_3} + \frac{Z_1+Z_2}{D} \end{bmatrix} \checkmark$$

4.27

$$V_1 = AV_2 - BI_2 \qquad V_n = V_n^+ + V_n^-$$
$$I_1 = CV_2 - DI_2 \qquad I_n = (V_n^+ - V_n^-)/Z_0$$

So,

$$V_1^+ + V_1^- = A(V_2^+ + V_2^-) - B(V_2^+ - V_2^-)/Z_0$$
$$V_1^+ - V_1^- = C(V_2^+ + V_2^-)Z_0 - D(V_2^+ - V_2^-)$$

For $V_2^+ = 0$,

$$V_1^+ + V_1^- = (A + B/Z_0)V_2^- \quad -①$$
$$V_1^+ - V_1^- = (CZ_0 + D)V_2^- \quad -②$$

eliminate V_2^+:

$$V_1^+ + V_1^- = \frac{A + B/Z_0}{CZ_0 + D}(V_1^+ - V_1^-)$$

$$V_1^-(CZ_0 + D + A + B/Z_0) = V_1^+(A + B/Z_0 - CZ_0 - D)$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \checkmark$$

eliminate V_1^-:

$$2V_1^+ = (A + B/Z_0 + CZ_0 + D)V_2^-$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{2}{A + B/Z_0 + CZ_0 + D} \checkmark$$

for $V_1^+ = 0$ the above set reduces to,

$$V_1^- = (A - B/Z_0)V_2^+ + (A + B/Z_0)V_2^-$$
$$-V_1^- = (CZ_0 - D)V_2^+ + (CZ_0 + D)V_2^-$$

eliminate V_1^-:

$$(A - B/Z_0 + CZ_0 - D)V_2^+ + (A + B/Z_0 + CZ_0 + D)V_2^- = 0$$

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \checkmark$$

eliminate V_2^-:

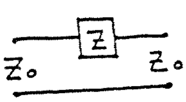
$$\frac{V_1^-}{A + B/Z_0} - \frac{A - B/Z_0}{A + B/Z_0}V_2^+ = \frac{-V_1^-}{CZ_0 + D} - \frac{CZ_0 - D}{CZ_0 + D}V_2^+$$

$$V_1^-\left(\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}\right) = V_2^+\left(\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}\right)$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0} = \frac{\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}}{\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \checkmark$$

These results agree with Table 4.2.

4.28

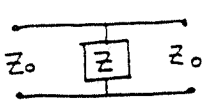


From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

convert to [S] using Table 4.2:

$$S_{11} = \frac{1 + Z/Z_0 - 1}{1 + Z/Z_0 + 1} = \frac{Z}{2Z_0 + Z} \quad ; \quad S_{12} = \frac{2}{1 + Z/Z_0 + 1} = \frac{2Z_0}{2Z_0 + Z}$$

$$1 - S_{11} = \frac{2Z_0}{2Z_0 + Z} = S_{12} \checkmark$$



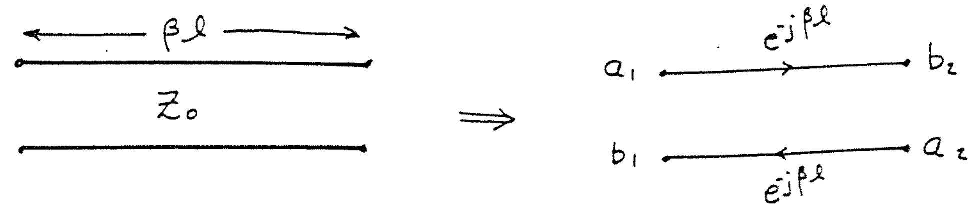
From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$

convert to [S]:

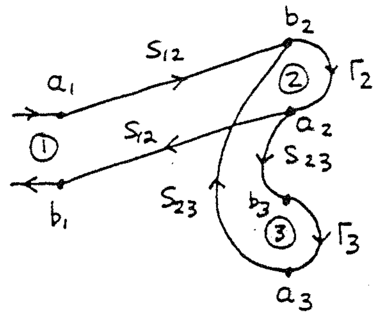
$$S_{11} = \frac{1 - Z_0/Z - 1}{1 + Z_0/Z + 1} = \frac{-Z_0}{2Z + Z_0} \quad ; \quad S_{12} = \frac{2}{1 + Z_0/Z + 1} = \frac{2Z}{2Z + Z_0}$$

$$1 + S_{11} = \frac{2Z}{2Z + Z_0} = S_{12} \checkmark$$

4.29

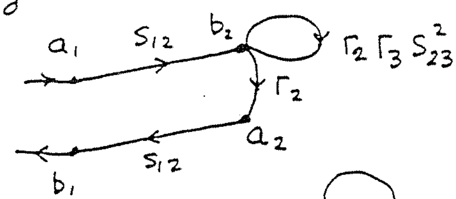


4.30 The signal flowgraph is as follows:



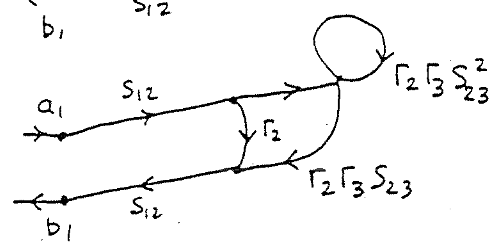
Let $\Gamma_{in} = \dfrac{b_1}{a_1}$

Using the reduction rules:



$$\therefore\ b_2 = a_1 \frac{S_{12}}{1-\Gamma_2\Gamma_3 S_{23}^2} \checkmark$$

$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1-\Gamma_2\Gamma_3 S_{23}^2} \checkmark$$



$$\therefore\ b_3 = b_2 \frac{\Gamma_2 S_{23}}{1-\Gamma_2\Gamma_3 S_{23}^2}$$

Then,

$$\frac{P_2}{P_1} = \frac{b_2^2 - a_2^2}{a_1^2 - b_1^2} = \frac{b_2^2(1-|\Gamma_2|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|\left(1-\dfrac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2} \checkmark$$

$$\frac{P_3}{P_1} = \frac{b_3^2 - a_3^2}{a_1^2 - b_1^2} = \frac{b_3^2(1-|\Gamma_3|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2|\Gamma_2 S_{23}|^2(1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^4\left(1-\dfrac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2|\Gamma_2|^2|S_{23}|^2(1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2\left(|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2\right)} \checkmark$$

(4.31) The complex reflected power can be computed using (4.88):

$$P_r = \int_s \bar{E}^r \times \bar{H}^{r*} \cdot \hat{z}\, ds = -\int_{x=0}^{a}\int_{y=0}^{b} E_y^r H_x^{r*}\, dx\, dy$$

$$= -b\int_{x=0}^{a}\left[\sum_n A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z}\right]\left[\sum_m \frac{A_m^*}{Z_m^{a*}} \sin\frac{m\pi x}{a} e^{-j\beta_m^{a*} z}\right] dx$$

$$= \frac{-ab}{2}\sum_{n=1}^{\infty}\frac{|A_n|^2}{Z_n^{a*}} e^{j(\beta_n^a - \beta_n^{a*})z}$$

The only propagating mode is the $n=1$ (TE_{10}) mode, so β_1^a is real, and β_n^a is imaginary for $n>1$. Let $\alpha_n = j\beta_n = \sqrt{(n\pi/a)^2 - k_0^2}$ for $n>1$. Then $Z_1^a = k_0\eta_0/\beta_1^a$, and

$Z_n^a = k_0\eta_0/\beta_n^a = j k_0\eta_0/\alpha_n$ for $n>1$.

Then $$P_r = \frac{-ab}{2}\left[\frac{|A_1|^2\beta_1^a}{k_0\eta_0} - j\sum_{n=2}^{\infty}\frac{|A_n|^2\alpha_n}{k_0\eta_0} e^{2\alpha_n z}\right] \quad \text{for } z<0.$$

So we see that $\text{Im}\{P_r\} > 0$, indicating an inductive load.

(4.32) This problem can be solved by setting up two equations for A_1 and A_2 using (4.97). But a general computer program had been written for Section 4.6, so this was used to obtain the results that $A_1 = 0.071\angle 50^\circ$, and $A_2 = 0.29\angle -13^\circ$. The computer program listing is shown on the following page.

```
C MODAL ANALYSIS OF ASYMMETRIC H-PLANE STEP
      COMPLEX AM(100),QM(100,100),PM(100),ZKC,ZNA
      COMPLEX DET,XOZO,XN,BTNA,BTKC
      DIMENSION LL(100),MM(100)
      PI=3.14159265
      Z0=377.
100   write(6,*) ' Enter lambda/a, c/a, N:'
      READ(6,*) A,C,NE
      XLAM=1.
      a=1./a
      c=c*a
      XKO=2.*PI/XLAM
C FILL ARRAYS
      BT1A=SQRT(XKO*XKO-(PI/A)**2)
      Z1A=XKO*Z0/BT1A
      DO 40 M=1,NE
      DO 10 N=1,NE
      BTNA=CSQRT((1.,0.)*XKO*XKO-(N*PI/A)**2)
      IF(AIMAG(BTNA).GT.0.) BTNA=-BTNA
      ZNA=XKO*Z0/BTNA
      QM(M,N)=(0.,0.)
      IF(M.EQ.N) QM(M,N)=A/2.
      DO 50 K=1,NE
      BTKC=CSQRT((1.,0.)*XKO*XKO-(K*PI/C)**2)
      IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
      ZKC=XKO*Z0/BTKC
      QM(M,N)=QM(M,N)-ZKC*2./C*XI(K,M,A,C)*XI(K,N,A,C)/ZNA
50    CONTINUE
10    CONTINUE
      PM(M)=(0.,0.)
      IF(M.EQ.1) PM(M)=-A/2.
      DO 60 K=1,NE
      BTKC=CSQRT((1.,0.)*XKO*XKO-(K*PI/C)**2)
      IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
      ZKC=XKO*Z0/BTKC
      PM(M)=PM(M)-ZKC*2./C*XI(K,M,A,C)*XI(K,1,A,C)/Z1A
60    CONTINUE
40    CONTINUE
C INVERT MATRIX AND COMPUTE A VECTOR (standard matrix inversion)
      CALL CMINV(QM,DET,LL,MM,100,NE)
      DO 20 M=1,NE
      AM(M)=(0.,0.)
      DO 20 N=1,NE
20    AM(M)=AM(M)-QM(M,N)*PM(N)
      write(6,*) ' A vector (mag, phase):'
      DO 30 N=1,NE
      AMM=CABS(AM(N))
      PHS=180.*BTAN2(AIMAG(AM(N)),REAL(AM(N)))/PI
30    WRITE(6,*) N,AMM,PHS
C     COMPUTE REACTANCE
      XOZO=(0.,-1.)*(1.+AM(1))/(1.-AM(1))
      XLAMG=2.*PI/BT1A
      XN=XOZO*XLAMG/(2.*A)
      WRITE(6,*) ' c/a=',C/A,'lambda/a=',XLAM/A,'Xn=',XN
      GOTO 100
200   CALL EXIT
      END
      FUNCTION XI(M,N,A,C)
      PI=3.14159265
      ARGM=PI*(M/C-N/A)
      ARGP=PI*(M/C+N/A)
      XIP=SIN(ARGP*C)/(2.*ARGP)
      XIM=C/2.
      IF(ABS(ARGM)*A.GT.1.E-6) XIM=SIN(ARGM*C)/(2.*ARGM)
      XI=XIM-XIP
      RETURN
      END
```

4.33 This solution is essentially the same as the analysis in Section 4.6. Let $d = (a-c)/2$

$$E_y^i = \sin\frac{\pi x}{a} e^{-j\beta_1^a z} \qquad H_x^i = \frac{-1}{Z_1^a}\sin\frac{\pi x}{a} e^{-j\beta_1^a z}$$

$$E_y^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z} \qquad H_x^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{A_n}{Z_n^a}\sin\frac{n\pi x}{a} e^{j\beta_n^a z}$$

$$E_y^t = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} B_n \sin\frac{n\pi}{c}(x-d) e^{-j\beta_n^c z} \qquad H_x^t = -\sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{B_n}{Z_n^c}\sin\frac{n\pi}{c}(x-d) e^{-j\beta_n^c z}$$

where $\beta_n^a = \sqrt{k_0^2 - (n\pi/a)^2}$, $\beta_n^c = \sqrt{k_0^2 - (n\pi/c)^2}$

The solution has the same form as (4.97):

$$\frac{a}{2}A_m + \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2 Z_k^c I_{km} I_{kn}}{c Z_n^a} A_n = \sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2 Z_k^c I_{km} I_{k1}}{c Z_1^a} - \frac{a}{2}\delta_{m1}$$

for $m = 1, 3, 5 \ldots$,

and,
$$I_{mn} = \int_{x=d}^{d+c} \sin\frac{m\pi}{c}(x-d) \sin\frac{n\pi x}{a}\, dx$$

$$\delta_{mn} = \begin{cases} 1 & \text{for } m=n \\ 0 & \text{for } m\neq n \end{cases}$$

4.34 From (4.110) the source current is,

$$\bar{J}_s = \hat{x}\,\frac{2B^+ m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + \hat{y}\,\frac{2B^+ n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

From Table 3.2, the transverse fields for ± traveling TM_{mn} modes are,

$$E_x = \frac{\mp j\beta m\pi}{k_c^2 a} C^{\pm}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, e^{\mp j\beta z}$$

$$E_y = \frac{\mp j\beta n\pi}{k_c^2 b} C^{\pm}\sin\frac{m\pi x}{a}\cos\frac{m\pi y}{b}\, e^{\mp j\beta z}$$

$$H_x = \frac{j\omega\epsilon n\pi}{k_c^2 b} C^{\pm}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\, e^{\mp j\beta z}$$

$$H_y = \frac{-j\omega\epsilon m\pi}{k_c^2 a} C^{\pm}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\, e^{\mp j\beta z}$$

where $C^{\pm}$ are the unknown amplitudes. At $z=0$, E_t is continuous, so $C^+ = -C^-$. Also, $\hat{z}\times(\bar{H}^+ - \bar{H}^-) = \bar{J}_s$, or $H_y^+ - H_y^- = J_{sx}$ and $-H_x^+ + H_x^- = J_{sy}$. So,

J_{sx}: $$\frac{-j\omega\epsilon m\pi}{k_c^2 a}(C^+ - C^-) = 2B^+\frac{m\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon}$$

J_{sy}: $$\frac{j\omega\epsilon n\pi}{k_c^2 b}(-C^+ + C^-) = 2B^+\frac{n\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon} \checkmark$$

Since these fields satisfy Maxwell's equations and the boundary conditions, they must form the unique solution.

4.35 From (4.114), the source current is,

$$\bar{M}_s = -\hat{x}\, 2B_{mn}^+ \frac{n\pi}{b} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} + \hat{y}\, 2B_{mn}^+ \frac{m\pi}{a} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

From Table 3.2, the transverse TM_{mn} fields for $\pm$ traveling waves can be written as,

$$E_x^\pm = \frac{-m\pi}{a} C^\pm \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$E_y^\pm = \frac{-n\pi}{b} C^\pm \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_x^\pm = \frac{\pm\omega\epsilon}{\beta} \frac{n\pi}{b} C^\pm \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_y^\pm = \mp\frac{\omega\epsilon}{\beta} \frac{m\pi}{a} C^\pm \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

where $C^\pm$ is the unknown amplitude. At $z=0$, $\bar{H}_t$ is continuous, so $C^+ = -C^-$. At $z=0$, $(\bar{E}^+ - \bar{E}^-)\times\hat{z} = \bar{M}_s$, so we have,

$$-E_x^+ + E_x^- = M_{sy} \Rightarrow C^+ - C^- = 2B_{mn}^+ \Rightarrow C^+ = -C^- = B_{mn}^+$$

$$E_y^+ - E_y^- = M_{sx} \Rightarrow -C^+ + C^- = -2B_{mn}^+ \Rightarrow C^+ = -C^- = B_{mn}^+ \checkmark$$

Since these fields satisfy Maxwell's equations and the relevant boundary conditions, they must form the unique solution.

4.36 Following Example 4.8:

$$\bar{J}(x,y,z) = I(y)\,\delta(x-a/2)\,\delta(z) \quad \text{for } 0<y<d.$$

$$\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}, \quad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}, \quad Z_1 = k_0\eta_0/\beta_1$$

From (4.119),
$$P_1 = \frac{ab}{Z_1}$$

From (4.118),

$$A_1^+ = \frac{-1}{P_1}\int_v \sin\frac{\pi x}{a}\, e^{j\beta_1 z}\, I(y)\,\delta(x-a/2)\,\delta(z)\,dx\,dy\,dz$$

$$= \frac{-I_0}{P_1}\int_{y=0}^{d}\frac{\sin k(d-y)}{\sin kd}\,dy \xrightarrow[\text{(let } w=d-y)]{} \frac{-I_0}{P_1\sin kd}\int_0^d \sin kw\,dw$$

$$= \frac{I_0 Z_1(\cos kd-1)}{kab\sin kd}$$

The total power flow in the TE_{10} mode is,

$$P = \frac{ab|A_1^+|^2}{2Z_1},$$

for both + and − traveling waves, since $|A_1^+| = |A_1^-|$.
Then the radiation resistance is,

$$R_{in} = \frac{2P}{I_0^2} = \frac{ab|A_1^+|^2}{I_0^2 Z_1} = \frac{Z_1}{ab}\,\frac{(1-\cos kd)^2}{k^2\sin^2 kd}.$$

$$= \frac{Z_1}{k^2ab}\,\frac{(2\sin^2\frac{kd}{2})^2}{4\sin^2\frac{kd}{2}\cos^2\frac{kd}{2}} = \frac{Z_1}{k^2ab}\tan^2\frac{kd}{2} \checkmark$$

4.37 Following Example 4.8:

$$\bar{J}(x,y,z) = I\,\delta(z)\left[\delta(x-a/4) - \delta(x-3a/4)\right]\hat{y} \quad \text{for } 0<y<b$$

From Table 3.2,

TE_{10}: $\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}$ $\quad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}$ $\quad Z_1 = k_0\eta_0/\beta_1$

TE_{20}: $\bar{e}_2 = \hat{y}\sin\frac{2\pi x}{a}$ $\quad \bar{h}_2 = \frac{-\hat{x}}{Z_2}\sin\frac{2\pi x}{a}$ $\quad Z_2 = k_0\eta_0/\beta_2$

$P_1 = ab/Z_1 \qquad \beta_1 = \sqrt{k_0^2-(\pi/a)^2}$

$P_2 = ab/Z_2 \qquad \beta_2 = \sqrt{k_0^2-(2\pi/a)^2}$

From (4.118):

$$A_1^+ = \frac{-1}{P_1}\int_v \bar{E}_1^- \cdot \bar{J}\,dv = -\frac{Ib}{P_1}\left(\sin\frac{\pi}{4} - \sin\frac{3\pi}{4}\right) = 0 \quad \checkmark$$

$$A_2^+ = \frac{-1}{P_2}\int_v \bar{E}_2^- \cdot \bar{J}\,dv = \frac{-Ib}{P_2}\left(\sin\frac{\pi}{2} - \sin\frac{3\pi}{2}\right) = \frac{-2Ib}{a}$$

Since the excitation has an odd symmetry about the center of the guide, it will only excite modes that have an electric field with an odd symmetry about $x=a/2$. This implies the TE_{m0} modes, for m even, will be excited. The TE_{10} mode is not excited.

4.38 As in Example 4.9,

$$\bar{P}_m = \hat{z}\, I_o \pi r_o^2\, \delta(x)\, \delta(y-b/2)\, \delta(z)$$

$$\bar{M} = j\omega\mu_o \bar{P}_m$$

$$= \hat{z}\, j\omega\mu_o I_o \pi r_o^2\, \delta(x)\, \delta(y-b/2)\, \delta(z) \qquad \text{V/m}^2$$

For the TE_{10} mode,

$$\bar{e}_1 = \hat{y} \sin\frac{\pi x}{a}$$

$$\bar{h}_1 = \frac{-\hat{x}}{Z_1} \sin\frac{\pi x}{a}$$

$$h_{z1} = \frac{j\pi}{k_o\eta_o a} \cos\frac{\pi x}{a}$$

and $Z_1 = k_o\eta_o/\beta_1$, $P_1 = ab/Z_1$.

From (4.122),

$$A_1^+ = \frac{1}{P_1}\int_v (-\bar{h}_1 + \hat{z}\, h_{z1}) \cdot \bar{M}\, e^{j\beta_1 z}\, dv$$

$$= \frac{Z_1}{ab}\int_v h_{z1} M\, dv = \frac{-\pi^2 Z_1 I_o r_o^2}{a^2 b} = A^-$$

Stronger coupling can usually be obtained by coupling to transverse field components – compare with the coefficient A^+ in Example 4.9, which is greater by a factor of $\beta_1 a/\pi$.

4.39

FIRST SOLUTION: (all fields and currents are TE_{10})

$$E_y = B \sin\frac{\pi x}{a}\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jB \sin\frac{\pi x}{a} \sin\beta z \qquad 0<z<d$$

$$H_x = \frac{B}{Z_1} \sin\frac{\pi x}{a}\left[-e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2B}{Z_1} \sin\frac{\pi x}{a} \cos\beta z \qquad 0<z<d$$

This satisfies $E_y = 0$ at $z=0$.

$$E_y = C \sin\frac{\pi x}{a} e^{-j\beta(z-d)} \qquad z>d$$

$$H_x = \frac{C}{Z_1} \sin\frac{\pi x}{a} e^{-j\beta(z-d)} \qquad z>d$$

at $z=d$, E_y is continuous, so

$$-2jB \sin\beta d = C$$

at $z=d$, $\hat{z}\times(\bar{H}^{+}-\bar{H}^{-}) = \bar{J}_s$, or

$$\frac{C}{Z_1} + \frac{2B}{Z_1}\cos\beta d = \frac{2\pi A}{a}$$

Solving for B, C:

$$B = \frac{\pi Z_1 A}{a} e^{-j\beta d}, \qquad C = \frac{\pi Z_1 A}{a}\left(e^{-2j\beta d} - 1\right)$$

SECOND SOLUTION: (Using (4.105) and (4.106b)):

E_y due to J_{sy} at $z=d$:

$$E_y^{\pm} = \frac{-\pi Z_1 A}{a} \sin\frac{\pi x}{a} e^{\mp j\beta(z-d)}$$

E_y due to $-J_{sy}$ at $z=-d$:

$$E_y^{\pm} = \frac{\pi Z_1 A}{a} \sin\frac{\pi x}{a} e^{\mp j\beta(z+d)}$$

For $0<z<d$,

$$E_y = \frac{\pi Z_1 A}{a} \sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a} e^{-j\beta d} \sin\frac{\pi x}{a} \sin\beta z \checkmark$$

For $z>d$,

$$E_y = \frac{\pi Z_1 A}{a}\cdot \sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{-j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a} \sin\beta d \sin\frac{\pi x}{a} e^{-j\beta z} \checkmark$$

These results agree with those from the first solution.

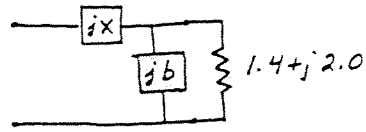
Chapter 5

5.1 (Smith chart solutions)

a) $z_L = 1.4 + j2.0$

inside $1+jx$ circle

SOL'N #1: $b_1 = -0.10$ ✓
$x_1 = -1.7$ ✓

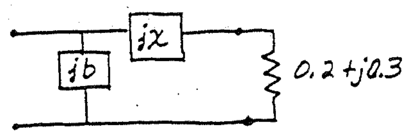


SOL'N #2: $b_2 = 0.78$ ✓
$x_2 = 1.7$ ✓

b) $z_L = 0.2 + j0.3$

outside $1+jx$ circle

SOL'N #1: $x_1 = 0.10$ ✓
$b_1 = 2.0$ ✓

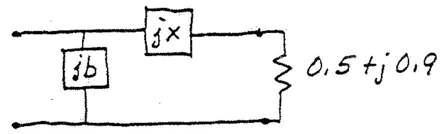


SOL'N #2: , $x_2 = -0.70$ ✓
$b_2 = -2.0$ ✓

c) $z_L = 0.5 + j0.9$

outside $1+jx$ circle

SOL'N #1: $x_1 = -0.40$ ✓
$b_1 = 0.96$ ✓

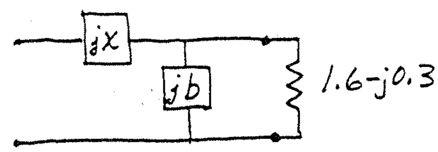


SOL'N #2: $x_2 = -1.4$ ✓
$b_2 = -0.96$ ✓

d) $z_L = 1.6 - j0.3$

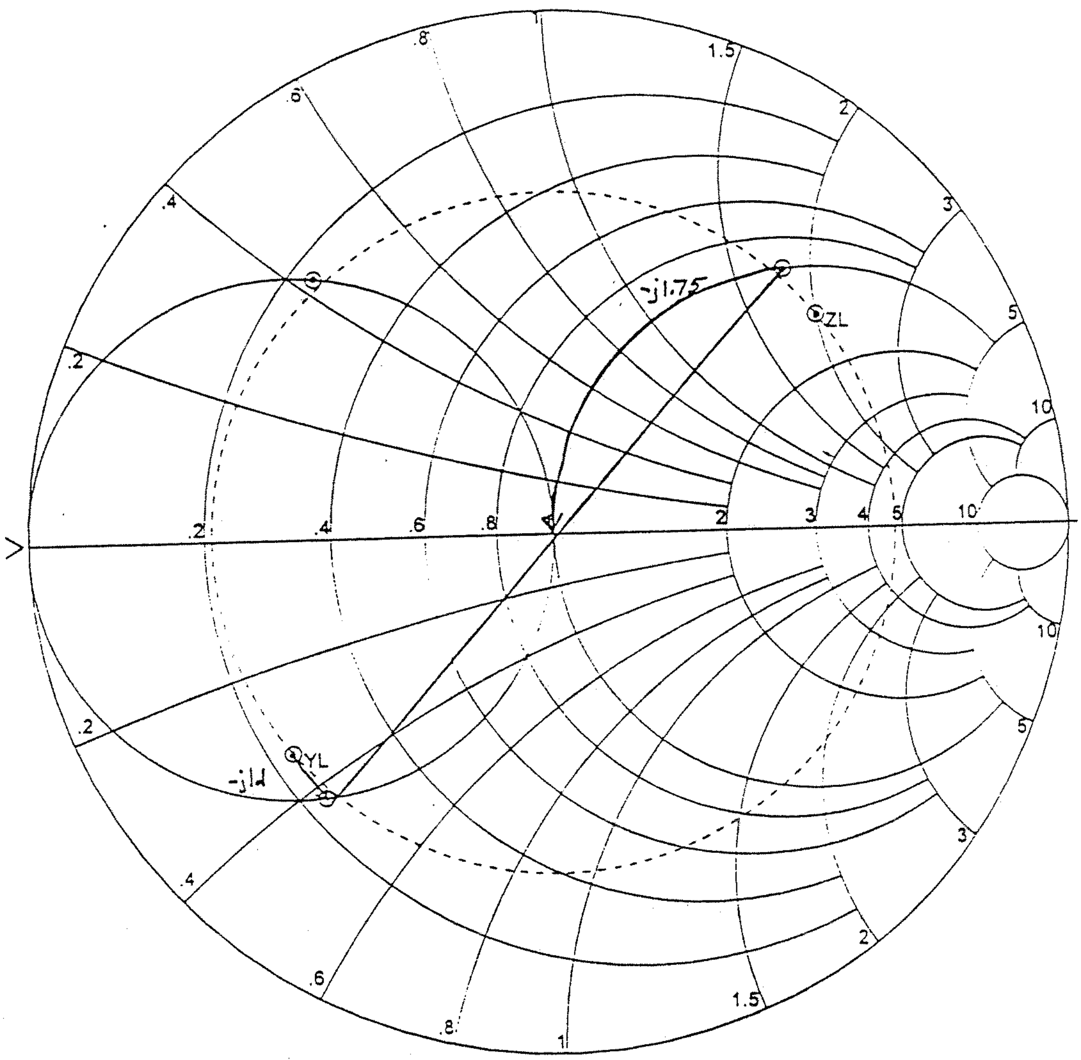
inside $1+jx$ circle

SOL'N #1: $b_1 = 0.38$ ✓
$x_1 = 0.80$ ✓



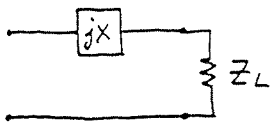
SOL'N #2: $b_2 = -0.62$ ✓
$x_2 = -0.80$ ✓

(The Smith chart for 5.1a is shown on the following page.)



Smith chart for Problem 5.1a

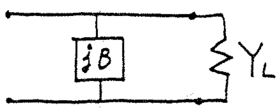
(5.2) a)



也就是說，當 $Re\{Z_L\} = Z_0$ 時，才可用單一元件達成匹配

matching possible if $Z_L = Z_0 - jX$ ✓

b)



matching possible if $Y_L = \frac{1}{Z_0} - jB$ ✓

(5.3) (analytical solutions)

From (5.9),

$$t = \frac{160 \pm \sqrt{200[(100-200)^2 + (160)^2]/100}}{200-100} = 1.6 \pm 2.67$$

So, $t_1 = 4.27$, $t_2 = -1.07$

Then from (5.10) the possible stub positions are,

$$d_1 = \frac{\lambda}{2\pi} \tan^{-1} t_1 = 0.213\lambda \checkmark$$

$$d_2 = \frac{\lambda}{2\pi} (\pi + \tan^{-1} t_2) = 0.370\lambda \checkmark$$

From (5.8b) the susceptances at the stub locations are,

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0 [R_L^2 + (X_L + Z_0 t)^2]}$$

So, $B_1 = 0.0133$, $B_2 = -0.0134$.

Then from (5.11a) the open-circuited stub lengths are,

$$\ell_1 = \frac{-\lambda}{2\pi} \tan^{-1}(B_1 Z_0) = 0.353\lambda \checkmark \quad (\lambda/2 \text{ added to get } \ell_1 > 0)$$

$$\ell_2 = \frac{-\lambda}{2\pi} \tan^{-1}(B_2 Z_0) = 0.148\lambda \checkmark$$

5.4 For short-circuited stubs, use B_1, B_2 from Problem 5.2 with (5.11b):

$$\ell_1 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_1} = 0.103\lambda$$

$$\ell_2 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_2} = 0.398\lambda$$

(These lengths are $\lambda/4$ shorter or longer than the open-circuited stub lengths.)

5.5 (Smith chart solutions)

The normalized load impedance is $z_L = 0.40 - j1.2$ To intersect the $1+jx$ circle, we must move back from the load either of the following distances:

$$d_1 = 0.19 + (0.5 - 0.355) = 0.335\lambda \checkmark$$

or,

$$d_2 = 0.31 + (0.5 - 0.355) = 0.455\lambda \checkmark$$

The reactances necessary for matching are then,

$$x_{s1} = -2.1$$

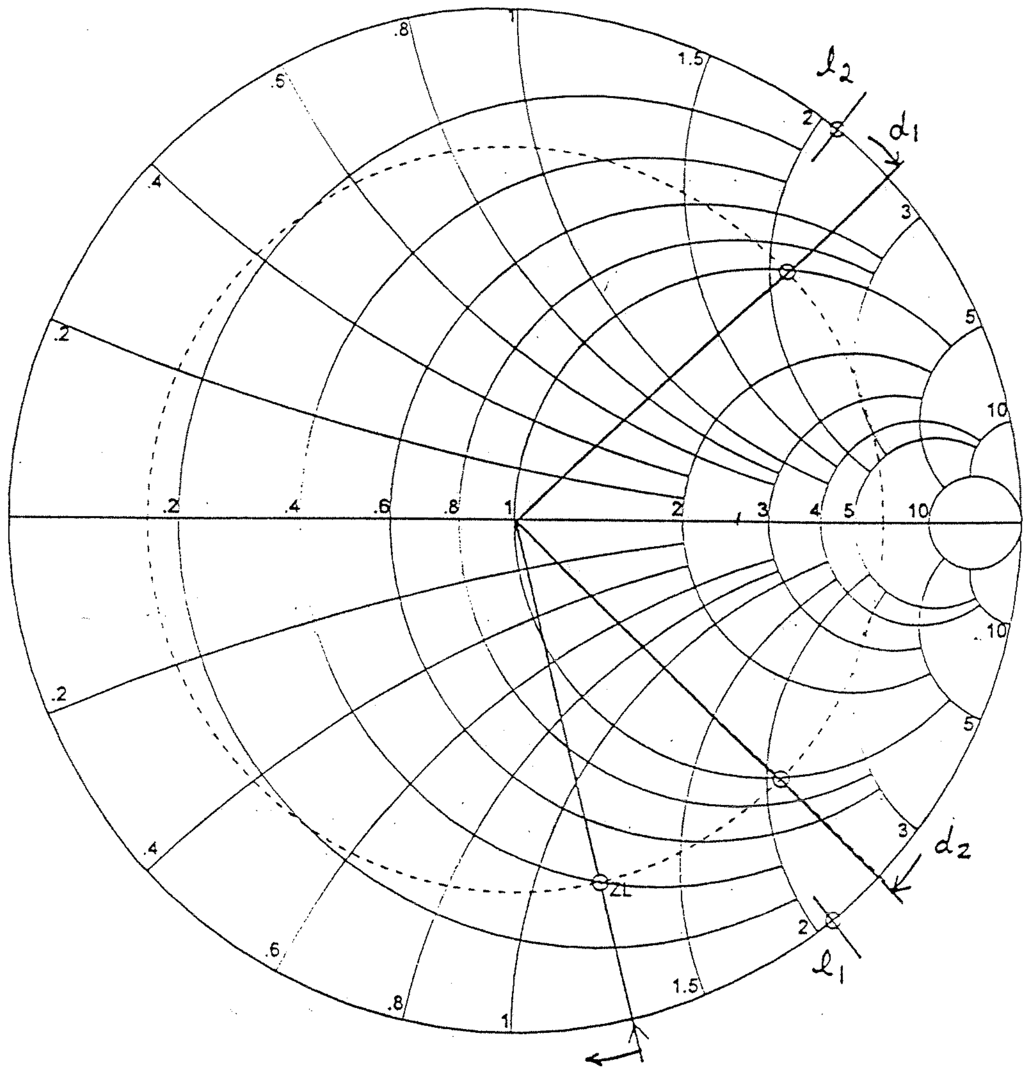
$$x_{s2} = 2.1$$

Then the open-circuited stub lengths are,

$$\ell_1 = 0.32 - 0.25 = 0.07\lambda \checkmark$$

$$\ell_2 = 0.25 + 0.18 = 0.43\lambda \checkmark$$

(The Smith chart for this problem is shown on the following page.)



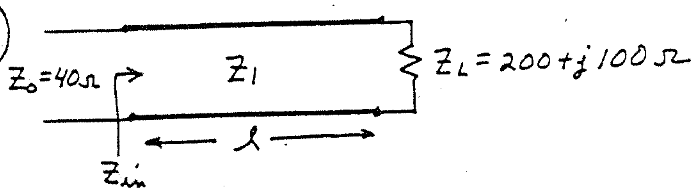
Smith chart for Problem 5.5

5.6 The stub lengths for short-circuited stubs are $\lambda/4$ longer or shorter than the above lengths:

$$\ell_1 = 0.07 + 0.25 = 0.32\lambda$$

$$\ell_2 = 0.43 - 0.25 = 0.18\lambda$$

5.7



To match this load, we must find Z_1 and ℓ so that $Z_{in} = Z_0 = 40\Omega$:

$$Z_{in} = 40 = Z_1 \frac{(200+j100) + jZ_1 t}{Z_1 + j(200+j100)t}, \quad \text{with } t = \tan\beta\ell.$$

$$(40Z_1 - 4000t) + j\,8000t = 200Z_1 + j(100 + Z_1 t)Z_1$$

Equating real and imaginary parts gives two equations for the two unknowns, Z_1 and t: (if they exist!)

$$\text{Re:} \quad 40Z_1 - 4000t = 200Z_1 \Rightarrow Z_1 = -25t$$

$$\text{Im:} \quad 8000t = Z_1(100 + Z_1 t)$$

$$8000t = -25t(100 - 25t^2)$$

$$t = \pm\sqrt{16.8} = \pm 4.10 \quad (\text{use } -4.10 \text{ so that } Z_1 > 0)$$

Then, $\beta\ell = \tan^{-1}(-4.10) = -76.3^\circ \equiv 104^\circ \Rightarrow \underline{\ell = 0.288\lambda}$

The characteristic impedance is then,

$$Z_1 = -25(-4.10) = 102.5\,\Omega$$

(Note: Not all load impedances can be matched in this way – a good exam problem to determine which impedances can be matched using this technique!)

5.8 From (2.91) the impedance of a terminated lossy line is,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \quad , \quad \gamma l = \alpha l + j\beta l$$

For $Z_L = \infty$ (o.c.), the normalized input admittance is,

$$y_{in} = \tanh \gamma l = \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l}$$

The normalized input susceptance is,

$$b_{in} = \frac{\tan\beta l\,(1-\tanh^2 \alpha l)}{1+\tanh^2 \alpha l \tan^2 \beta l}$$

(at this point, we could find max. b_{in} by calculating b_{in} vs. l)

Since maximum susceptance for a lossless line is obtained for $\beta l = \pi/2$, we expect βl to be close to $\pi/2$ for the lossy case. So let $\beta l = \pi/2 + \Delta$, where Δ is small. Also, αl is small, so we have $\tanh \alpha l \simeq \alpha l$, and $\tan\beta l = -\cot\Delta \simeq -1/\Delta$.

Then,

$$b_{in} \simeq \frac{\frac{-1}{\Delta}(1-\alpha^2 l^2)}{1+\alpha^2 l^2/\Delta^2} \simeq \frac{-1}{\Delta + \alpha^2 l^2/\Delta}$$

To maximize b_{in}, we can minimize $\Delta + \alpha^2 l^2/\Delta$ with respect to l:

$$\frac{d}{dl}\left(\Delta + \alpha^2 l^2/\Delta\right) = \frac{d\Delta}{dl} + \frac{2\alpha^2 l}{\Delta} + \alpha^2 l^2 \left(\frac{-1}{\Delta^2}\right)\frac{d\Delta}{dl} = 0$$

or, since $\dfrac{d\Delta}{dl} = \beta$, $\Delta = \beta l - \frac{\pi}{2}$

$$\beta + \frac{2\alpha^2 l}{\Delta} - \frac{\alpha^2 l^2}{\Delta^2}\beta = 0$$

since $\Delta = \beta l - \pi/2$, we have,

$$l^2 \beta(\alpha^2+\beta^2) - \pi l(\alpha^2+\beta^2) + \beta\frac{\pi^2}{4} = 0$$

Solve for l:

$$\ell = \frac{\pi(\alpha^2+\beta^2) \pm \sqrt{\pi^2(\alpha^2+\beta^2)^2 - \beta^2\pi^2(\alpha^2+\beta^2)}}{2\beta(\alpha^2+\beta^2)}$$

$$= \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta\sqrt{\alpha^2+\beta^2}} \simeq \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta^2} \qquad (\text{since } \alpha^2 << \beta^2)$$

Then, $\Delta = \beta\ell - \pi/2 \simeq \dfrac{\pi\alpha}{2\beta} \simeq \alpha\ell \qquad (\text{since } \beta \simeq \pi/2\ell)$

The corresponding value of b_{in} is,

$$b_{in}^{MAX} = \frac{\pm 1}{\alpha\ell + \alpha\ell} = \frac{\pm 1}{2\alpha\ell} = \frac{\pm 2}{\alpha\lambda} \qquad (\text{since } \ell \simeq \lambda/4)$$

For $\alpha = 0.01$ neper/λ, $b_{in}^{MAX} = \dfrac{\pm 2}{.01} = \underline{\pm 200}$

(This checks with direct calculation of y_{in} vs. ℓ.)

The reactance of a short-circuited line is the dual case of the above problem, so $x_{in}^{MAX} = \pm 200$.

5.9 Smith chart solution

1. plot $y_L = 1.4 + j2.0$ (ADMITTANCE CHART)
2. plot rotated $1+jb$ circle
3. add a stub susceptance of $-j0.1$ to y_L to move to rotated $1+jb$ circle
4. move $\lambda/8$ towards generator, to $1+jb$ circle
5. add a stub susceptance of $+j1.6$ to move to center of chart.
6. The open-circuited stub lengths are,

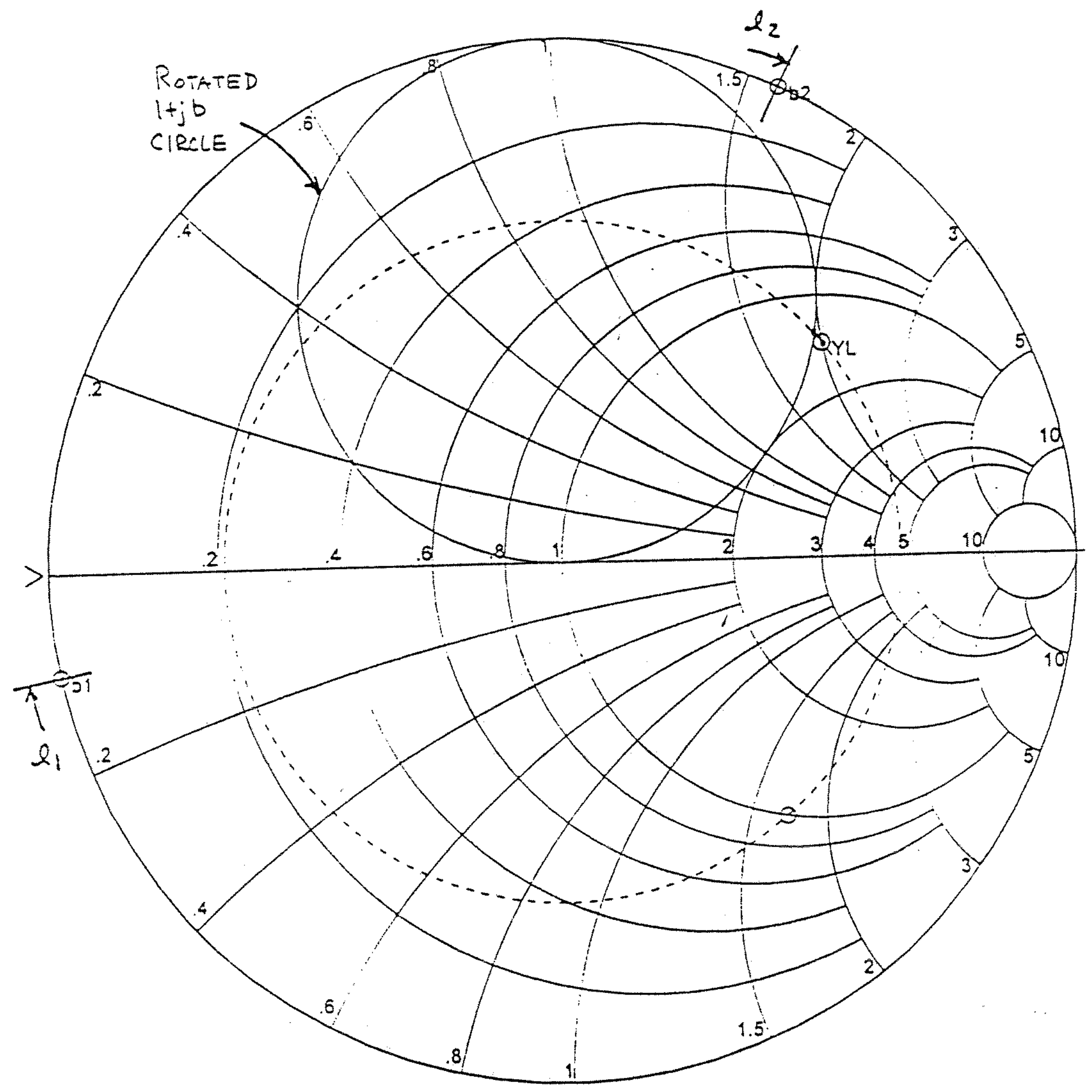
$$\ell_1 = 0.484\lambda \checkmark$$
$$\ell_2 = 0.161\lambda \checkmark$$

(Analytically, we obtain $\ell_1 = 0.487\lambda$, $\ell_2 = 0.163\lambda$)

The alternative matching solution has,

$$\ell_1' = 0.326\lambda \checkmark$$
$$\ell_2' = 0.053\lambda \checkmark$$

The Smith chart for the first solution is shown on the following page.



Smith chart for Problem 5.9

(5.10) analytic solution

let $t = \tan\beta d = \tan 135° = -1.0$

From (5.22) the first stub susceptance is,

$$b_1 = -b_L + \frac{1 \pm \sqrt{(1+t^2)g_L - g_L^2 t^2}}{t} = -3.92 \text{ or } -2.08 \checkmark$$

From (5.23) the second stub susceptance is,

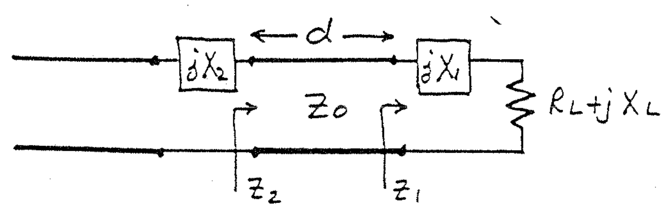
$$b_2 = \frac{\pm\sqrt{(1+t^2)g_L - g_L^2 t^2} + g_L}{g_L t} = -1.65 \text{ or } -3.45 \checkmark$$

From (5.24b) the S.C. stub lengths are,

$\ell_1 = 0.042\lambda$, or 0.071λ ✓

$\ell_2 = 0.087\lambda$, or 0.045λ ✓

(5.11)



$$Z_1 = R_L + j(X_L + X_1)$$

$$Z_2 = Z_0 \frac{R_L + j(X_L + X_1 + Z_0 t)}{Z_0 + jt(R_L + jX_L + jX_1)} = Z_0 \qquad t = \tan\beta d$$

Solving for R_L:

$$R_L = Z_0 \frac{1+t^2}{2t^2}\left[1 \pm \sqrt{1 - \frac{4t^2(Z_0 - X_L t - X_1 t)^2}{Z_0(1+t^2)^2}}\right]$$

So we must have,

$$0 \le R_L \le Z_0 \frac{1+t^2}{2t^2} = \frac{Z_0}{\sin^2\beta d}$$

The first stub reactance is,

$$X_1 = -X_L + \frac{Z_0 \pm \sqrt{(1+t^2)R_L Z_0 - R_L^2 t^2}}{t}$$

The second stub reactance is,

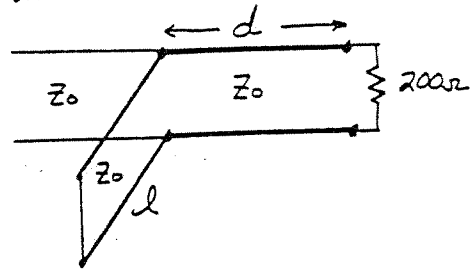
$$X_2 = \frac{\pm Z_0\sqrt{Z_0 R_L(1+t^2) - R_L^2 t^2} + R_L Z_0}{R_L t}$$

The stub lengths are given by,

$$\ell_{oc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right), \qquad \ell_{sc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right)$$

(5.12) Using the Smith chart ($Z_0 = 100\ \Omega$)

a) a single short-circuited shunt stub:



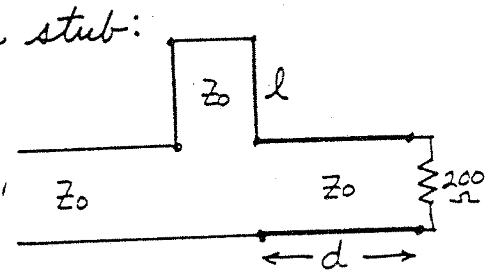
at f_0, $d_1 = 0.152\lambda$ $\quad d_2 = 0.348\lambda$

$b_1 = -0.7$ $\quad b_2 = +0.7$

$\ell_1 = 0.153\lambda$ $\quad \ell_2 = 0.347$

$|\Gamma_1| = 0$ $\quad |\Gamma_2| = 0$

b) a single short-circuited series stub:



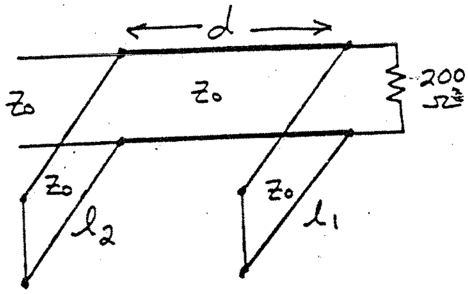
at f_0, $d_1 = 0.098\lambda$ $\quad d_2 = 0.402\lambda$

$X_1 = 0.7$ $\quad X_2 = -0.7$

$\ell_1 = 0.097\lambda$ $\quad \ell_2 = 0.403\lambda$

$|\Gamma_1| = 0$ $\quad |\Gamma_2| = 0$

c) a double short-circuited shunt stub: (let $d = \lambda/8$)



at f_0, $b_1 = 0.14$ $\quad b_1' = 1.85$

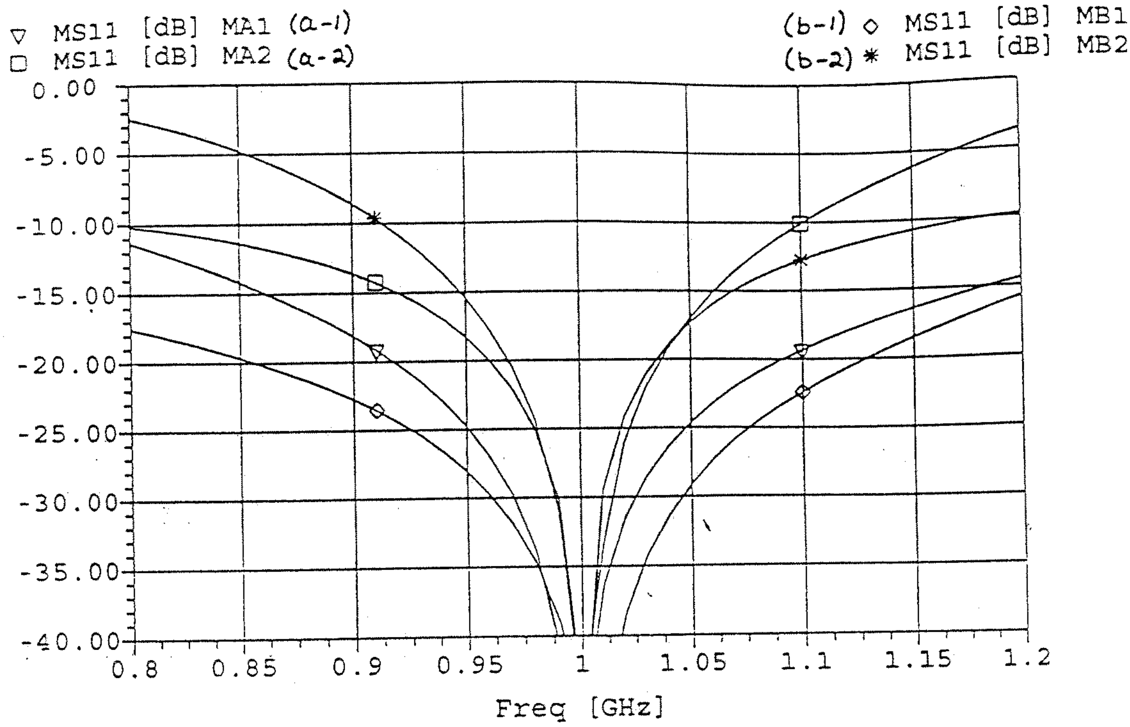
$\ell_1 = 0.272\lambda$ $\quad \ell_1' = 0.421\lambda$

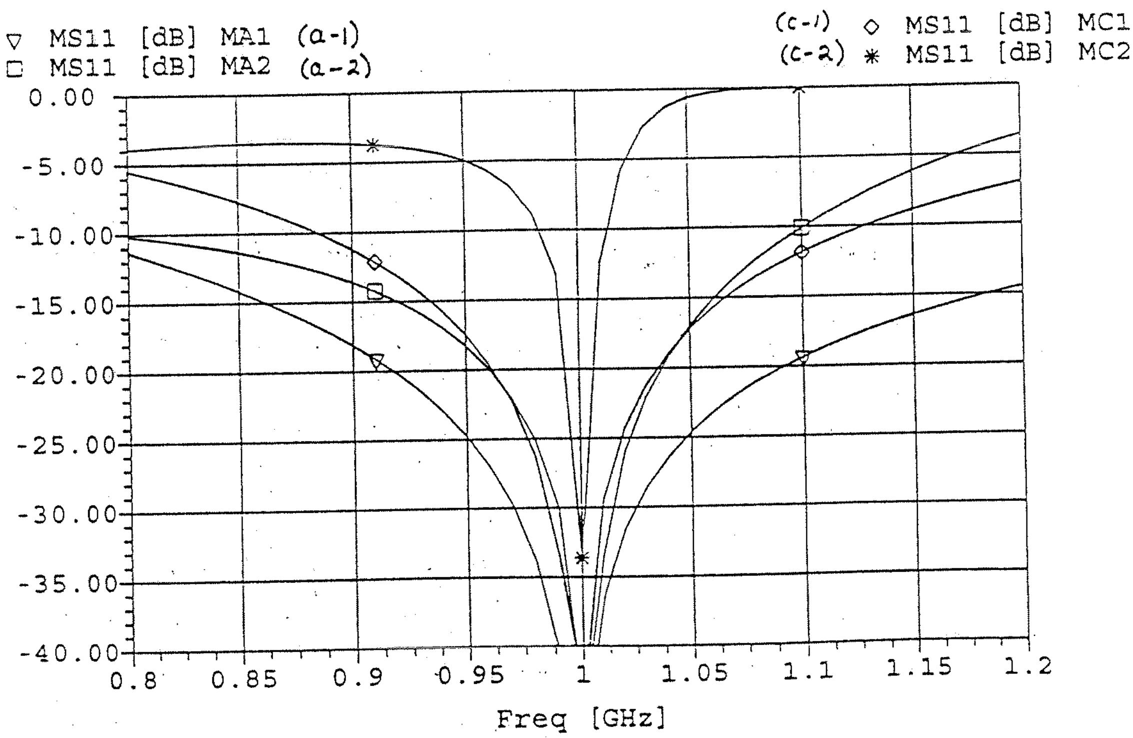
$b_2 = -0.73$ $\quad b_2' = 2.75$

$\ell_2 = 0.15\lambda$ $\quad \ell_2' = 0.444\lambda$

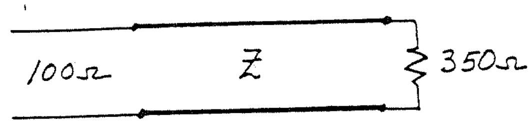
$|\Gamma| = 0$ $\quad |\Gamma'| = 0$

Plots of return loss vs. f/f for these six solutions are shown on the following page. (only 4 curves could be plotted per graph). These results show that the tuner of solution (b-1), the series stub tuner, gives the best bandwidth. This is probably because the stub length and line length are shortest for this case, giving the smallest frequency variation.


▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(b-1) ◇ MS11 [dB] MB1
(b-2) ✱ MS11 [dB] MB2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Freq [GHz]


▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(c-1) ◇ MS11 [dB] MC1
(c-2) ✱ MS11 [dB] MC2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Freq [GHz]

(5.13)



$Z = \sqrt{100(350)} = 187.1\Omega$ ✓

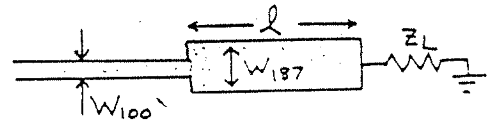
an SWR of 2 corresponds to a reflection coefficient magnitude of, $\Gamma_m = \frac{S-1}{S+1} = 1/3$

Then from (5.33) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\;\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 71\%$$

MICROSTRIP LAYOUT:

$\epsilon_r = 2.2$, $d = 0.159$ cm, $f = 4$ GHz



First try $w/d < 2$:

for W_{100}, $A_{100} = 2.213$, $W_{100}/d = 0.896 < 2$ (OK), $W_{100} = 0.142$ cm

for W_{197}, $A_{187} = 4.047$, $W_{187}/d = 0.140 < 2$ (OK), $W_{187} = 0.022$ cm

From (3.195), ϵ_e for W_{187} is $\epsilon_e = 1.66$.

Then the physical length of the $\lambda/4$ transformer is,

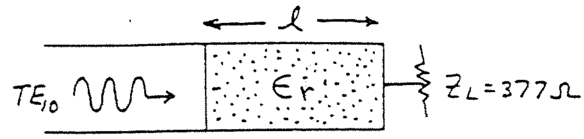
$$\ell = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_e}\,f} = 1.455 \text{ cm} \checkmark$$

(5.14) From (5.34) and (5.36), the partial reflection coefficients are,

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{150-100}{150+100} = 0.2 \;;\quad \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2} = \frac{225-150}{225+150} = 0.2$$

Since the approximate expression for Γ in (5.42) is identical to the numerator for the exact expression in (5.41), the greatest error will occur when the denominator of (5.41) departs from unity to the greatest extent. This occurs for $\theta = 0$ or $180°$. Then (5.41) gives the exact Γ as 0.384, while (5.42) gives the approximate $\Gamma = 0.4$. Thus the error is about 4%.

5.15



$a = 2.286$ cm

$k_0 = \frac{2\pi f}{c} = 209.4 \ m^{-1}$

In the air-filled guide,

$$\beta_a = \sqrt{k_0^2 - (\pi/a)^2} = 158.0 \ m^{-1} \checkmark$$

$$Z_a = \frac{k_0 \eta_0}{\beta_a} = \frac{(209.4)(377)}{158} = 499.6 \ \Omega \checkmark$$

So the matching section impedance must be,

$$Z_m = \sqrt{Z_a Z_L} = \sqrt{(499.6)(377)} = 434.0 \Omega$$

$$= \frac{k_m \eta_m}{\beta_m} = \frac{k_0 \eta_0}{\beta_m}$$

So the propagation constant of the matching section must be,

$$\beta_m = \frac{k_0 \eta_0}{Z_m} = \frac{(209.4)(377)}{434} = 181.9 \ m^{-1}$$

$$= \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$$

Solving for ϵ_r:

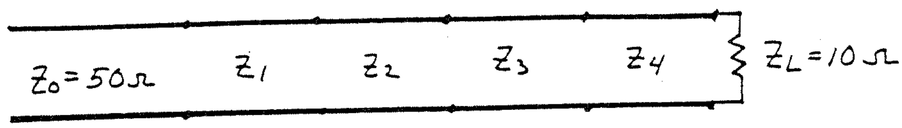
$$\epsilon_r = \frac{\beta_m^2 + (\pi/.02286)^2}{(209.4)^2} = \underline{1.185} \checkmark$$

The physical length of the matching section is,

$$\ell = \frac{\lambda_g}{4} = \frac{2\pi}{4\beta_m} = \frac{\pi}{2\beta_m} = 0.86 \text{ cm} \checkmark$$

(Note that this type of matching is not possible if $Z_L > Z_a$.)

(5.16)



Z_0/Z_L is not in Table 5.1, so we will use (5.53):

$n=0$: $\ln \frac{Z_1}{Z_0} = 2^{-4} C_0^4 \ln \frac{10}{50} = -0.10059$ $\Rightarrow Z_1 = 45.2\,\Omega$ ✓

$n=1$: $\ln \frac{Z_2}{Z_1} = 2^{-4} C_1^4 \ln \frac{10}{50} = -0.40236$ $\Rightarrow Z_2 = 30.2\,\Omega$ ✓

$n=2$: $\ln \frac{Z_3}{Z_2} = 2^{-4} C_2^4 \ln \frac{10}{50} = -0.60354$ $\Rightarrow Z_3 = 16.5\,\Omega$ ✓

$n=3$: $\ln \frac{Z_4}{Z_3} = 2^{-4} C_3^4 \ln \frac{10}{50} = -0.40236$ $\Rightarrow Z_4 = 11.1\,\Omega$ ✓

CHECK: $\ln \frac{Z_5}{Z_4} = 2^{-4} C_4^4 \ln \frac{10}{50} = -0.10059$ $\Rightarrow Z_5 = 10.04\,\Omega = Z_L$ ✓

If we interpolate between $Z_L/Z_0 = 4$ and $Z_L/Z_0 = 5$ in Table 5.1, we obtain $Z_1 = 45.1\,\Omega$, $Z_2 = 30.0\,\Omega$, $Z_3 = 16.5\,\Omega$, and $Z_4 = 11.1\,\Omega$ (after interchanging source and load ends). These results are close to the first solution, from (5.53).

From (5.55) and Example 5.6, the bandwidth is,

$$A \simeq \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.0503$$

(Note: this approximation actually gives better results than the "exact" value of A from $2^{-N} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$.

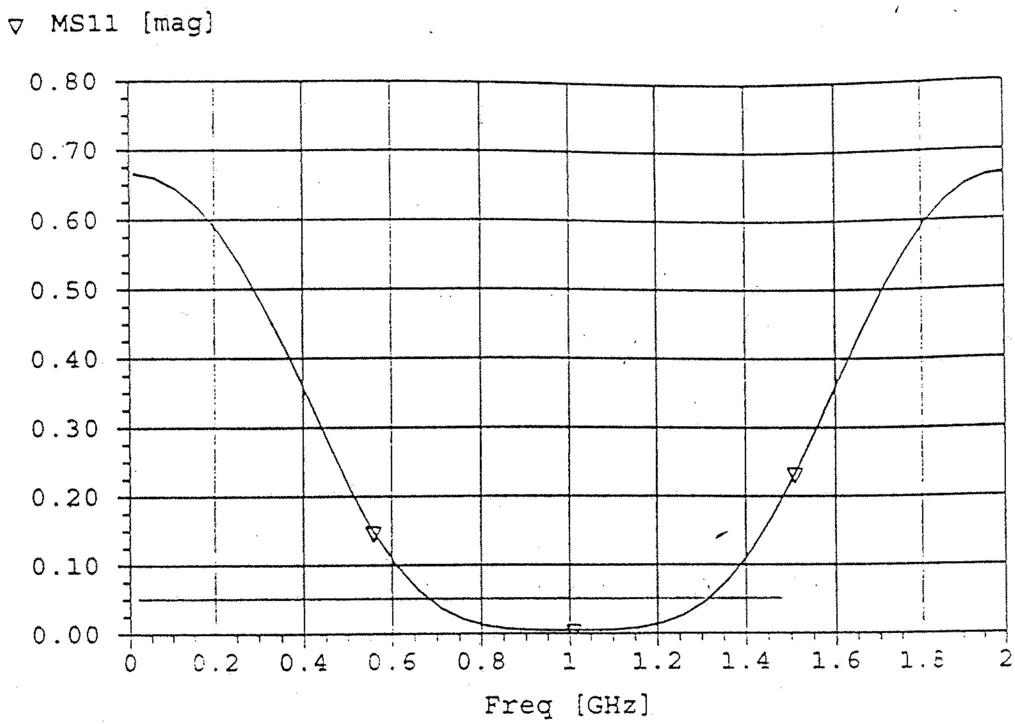
Then,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1}{2} \left| \frac{\Gamma_m}{A} \right|^{1/N} \right] = 67\%$$

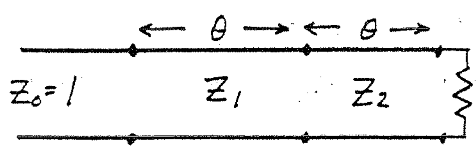
The calculated input reflection coefficient magnitude for this solution is plotted vs. frequency on the following page. The bandwidth from this graph is about,

$$\frac{\Delta f}{f_0} \approx \frac{1.32 - 0.68}{1} \times 100 = 64\%,$$

which is close to the above calculation.



5.17



From (5.50) the desired input reflection coefficient response is ($N=2$):

$$\Gamma(\theta) = 2A(1+\cos 2\theta)$$

From the above circuit, we have that $\Gamma(0) = \frac{R-1}{R+1} = 0.2$, so $A = 0.2/4 = 0.05$.

Now we calculate the input reflection coefficient of the above circuit using ABCD matrices and conversion to S-parameters:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_1 \sin\theta \\ jY_1 \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2 \sin\theta \\ jY_2 \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - Z_1 Y_2 \sin^2\theta & j(Z_1+Z_2)\cos\theta \sin\theta \\ j(Y_1+Y_2)\sin\theta\cos\theta & \cos^2\theta - Y_1 Z_2 \sin^2\theta \end{bmatrix}$$

Using Table 4.2 to convert to s-parameters gives the input reflection coefficient as,

$$\Gamma(\theta) = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell} = \frac{A+B-C-D}{S} + \frac{4\Gamma_\ell/S^2}{1 - \frac{-A+B-C+D}{S}\Gamma_\ell}$$

$$= \frac{(A+B-C-D)\left[S-\Gamma_\ell(-A+B-C+D)\right] + 4\Gamma_\ell}{S\left[S-\Gamma_\ell(-A+B-C+D)\right]}$$

where $S = A+B+C+D$, $\Gamma_\ell = \frac{R-1}{R+1}$.

This result can be equated to $2A(1+\cos 2\theta)$, and solved for Z_1 and Z_2, but this is a <u>very</u> lengthy procedure. Instead, we will first evaluate both expressions at $\theta = 90°$:

$$\Gamma(90°) = 0, \quad \text{and} \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}\Bigg|_{\theta=90°} = \begin{bmatrix} -Z_1Y_2 & 0 \\ 0 & -Y_1Z_2 \end{bmatrix}$$

So $\Gamma(\theta)$ reduces to the following equation:

$$(-Z_1Y_2 + Y_1Z_2)\left[-(Y_1Z_2 + Z_1Y_2) - \Gamma_\ell(Z_1Y_2 - Y_1Z_2)\right] + 4\Gamma_\ell = 0$$

$$(Z_1^2Y_2^2 - Y_1^2Z_2^2) + \Gamma_\ell(Z_1^2Y_2^2 + Y_1^2Z_2^2 + 2) = 0$$

$$(Z_1^4 - Z_2^4) + \Gamma_\ell(Z_1^4 + Z_2^4 + 2Z_1^2Z_2^2) = 0$$

$$(Z_1^2 - Z_2^2) + \Gamma_\ell(Z_1^2 + Z_2^2) = 0$$

$$Z_2^2 = Z_1^2\frac{1+\Gamma_\ell}{1-\Gamma_\ell} = Z_1^2 R \Rightarrow \underline{\underline{Z_2 = Z_1\sqrt{R}}} \checkmark \quad (\text{for } Z_0 = 1)$$

Another equation is harder to find, so we will make use of the fact that the transformer will be symmetric:

$$\frac{Z_1-1}{Z_1+1} = \frac{R-Z_2}{R+Z_2} = \frac{R/Z_2-1}{R/Z_2+1}$$

Thus, $\frac{R}{Z_2} = Z_1$ or $\underline{\underline{Z_1 = R^{1/4}}}$ (for $Z_0 = 1$)

If $R = 1.5$, these results reduce to,

$$Z_1 = (1.5)^{1/4} = 1.1067 \checkmark$$

$$Z_2 = 1.1067\sqrt{1.5} = 1.3554 \checkmark$$

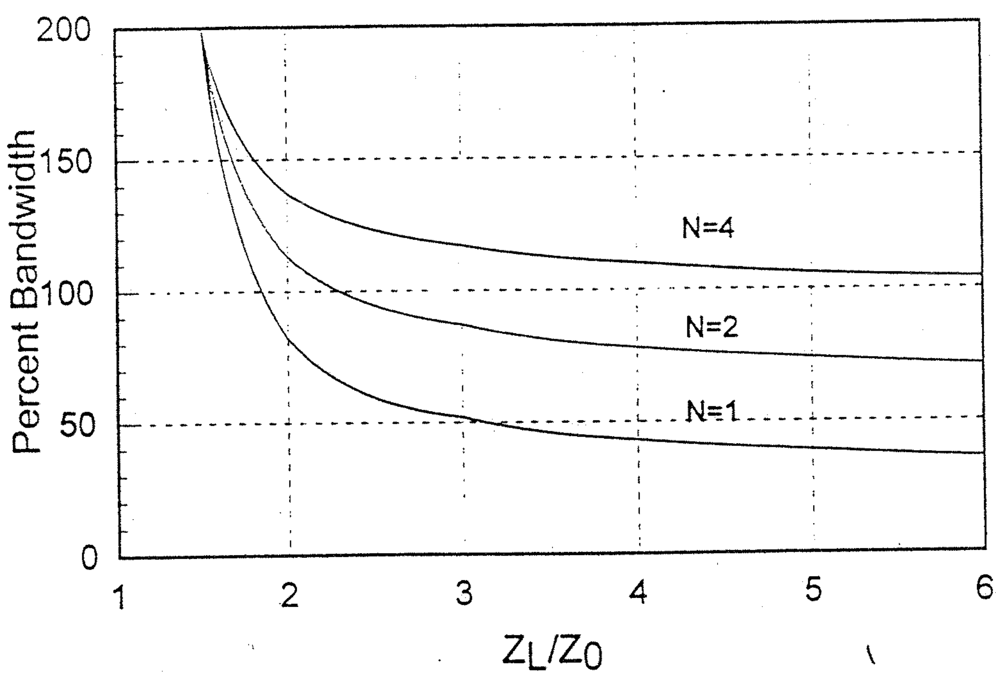
which agree with Table 5.1

5.18 From (5.55), the fractional bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right], \quad \text{with} \quad A = 2^{-N}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

Z_L/Z_0	N=1		N=2		N=4	
	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%
1.5	0.1000	200	0.0500	200	0.0125	200
2.0	0.1667	82	0.0833	113	0.0208	137
3.0	0.2500	52	0.1250	87	0.0313	117
4.0	0.3000	43	0.1500	78	0.0375	110
5.0	0.3333	39	0.1667	74	0.0417	106
6.0	0.3571	36	0.1786	71	0.0446	104

This data is plotted in the graph below.



5.19 $\quad x \doteq \sec\theta_m \cos\theta$

$n=1:$ $\quad T_1(x) = x = \sec\theta_m \cos\theta \qquad (5.60a)$ ✓

$n=2:$ $\quad T_2(x) = 2x^2-1 = 2\sec^2\theta_m \cos^2\theta - 1$

$$= 2\sec^2\theta_m \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right) - 1$$

$$= \sec^2\theta_m (1+\cos 2\theta) - 1 \qquad (5.60b)$$ ✓

$n=3:$ $\quad T_3(x) = 4x^3 - 3x = 4\sec^3\theta_m \cos^3\theta - 3\sec\theta_m \cos\theta$

$$= \sec^3\theta_m (3\cos\theta + \cos 3\theta) - 3\sec\theta_m \cos\theta \qquad (5.60c)$$ ✓

$n=4:$ $\quad T_4(x) = 8x^4 - 8x^2 + 1$

$$= 8\sec^4\theta_m \cos^4\theta - 8\sec^2\theta_m \cos^2\theta + 1$$

$$= \sec^4\theta_m (3 + 4\cos 2\theta + \cos 4\theta) - 4\sec^2\theta_m (1+\cos 2\theta) + 1$$

$$(5.60d)$$ ✓

5.20 $S_m = 1.2 \Rightarrow |\Gamma_m| = \frac{S_m - 1}{S_m + 1} = 0.091 = A$

From (5.61),

$$\Gamma(\theta) = 2e^{-j4\theta}\left[\Gamma_0 \cos 4\theta + \Gamma_1 \cos 2\theta + \tfrac{1}{2}\Gamma_2\right]$$

$$= A e^{-j4\theta} T_4(\sec\theta_m \cos\theta)$$

$$= A e^{-j4\theta}\left[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1\right]$$

From (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln Z_L/Z_0}{2\Gamma_m}\right|\right)\right]$$

$$= \cosh\left[\frac{1}{4}\cosh^{-1}\left(\frac{1}{2(.091)}\ln\frac{60}{40}\right)\right] = 1.0655$$

so, $\theta_m = \cos^{-1}(1/1.0655) = 20°$

Equating $\cos 4\theta$ terms:

$$2\Gamma_0 = A\sec^4\theta_m \Rightarrow \Gamma_0 = 0.0586 = \Gamma_4$$

Equating $\cos 2\theta$ terms:

$$2\Gamma_1 = A(4\sec^4\theta_m - 4\sec^2\theta_m) \Rightarrow \Gamma_1 = 0.02795 = \Gamma_3$$

Equating constant terms:

$$\Gamma_2 = A(3\sec^4\theta_m - 4\sec^2\theta_m + 1) = 0.0296$$

Then the characteristic impedances are,

$$Z_1 = Z_0\frac{1+\Gamma_0}{1-\Gamma_0} = 44.98\,\Omega \checkmark$$

$$Z_2 = Z_1\frac{1+\Gamma_1}{1-\Gamma_1} = 47.57\,\Omega \checkmark$$

$$Z_3 = Z_2\frac{1+\Gamma_2}{1-\Gamma_2} = 50.47\,\Omega \checkmark$$

$$Z_4 = Z_3\frac{1+\Gamma_3}{1-\Gamma_3} = 53.37\,\Omega \checkmark$$

CHECK: $Z_5 = Z_4\frac{1+\Gamma_4}{1-\Gamma_4} = 60.01\,\Omega = Z_L \checkmark$

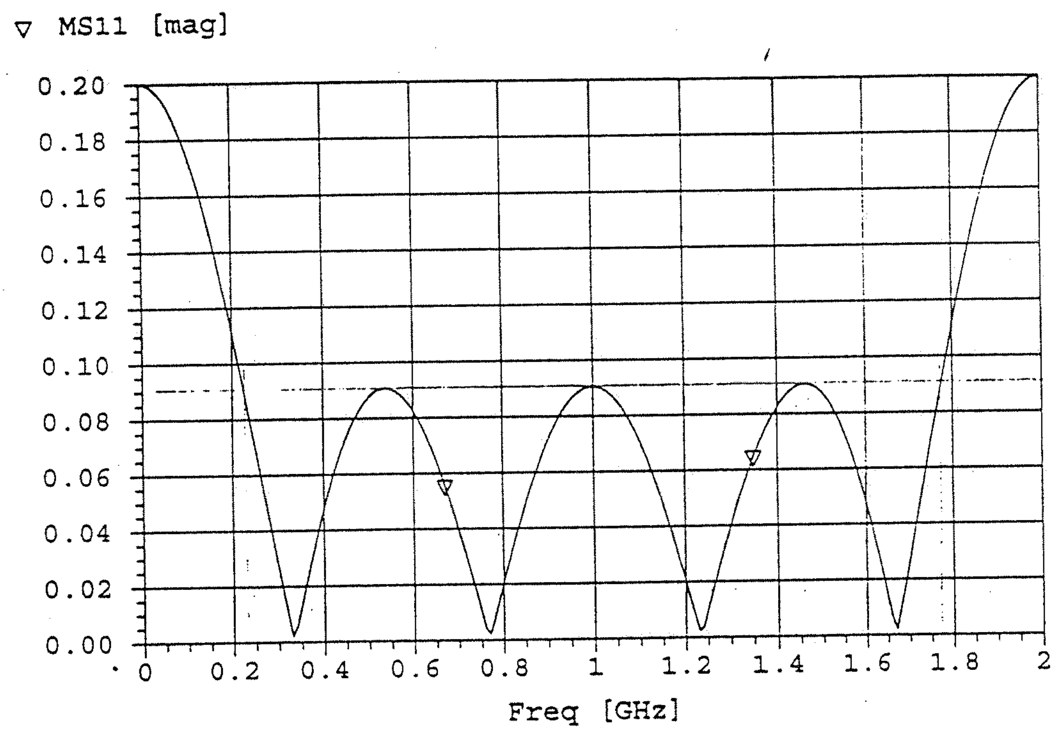
From (5.64) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4(20)}{180} = 156\% \checkmark$$

The calculated reflection coefficient magnitude versus frequency is shown below. The bandwidth from this graph gives a value of approximately,

$$\frac{\Delta f}{f_0} = \frac{1.78 - .23}{1} \times 100 = 155\%$$

in close agreement with the above calculation.



5.21 From (5.61) and (5.60b),

$$\Gamma(\theta) = A e^{-2j\theta} T_2(\sec\theta_m \cos\theta) = A e^{-2j\theta}\left[\sec^2\theta_m (1+\cos 2\theta) - 1\right]$$

$$\Gamma(0) = A T_2(\sec\theta_m) = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \frac{R-1}{R+1} = 0.2 \quad ; \quad A = \Gamma_m = 0.05$$

As in Problem 5.17, we will evaluate $\Gamma(\theta)$ for $\theta = 90°$. Then $\Gamma(90°) = \Gamma_m$. Also, as in Problem 5.17, from symmetry we have that $Z_1 Z_2 = R$. Then,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -Z_1^2/R & 0 \\ 0 & -R/Z_1^2 \end{bmatrix} \qquad (Z_0 = 1)$$

$$\Gamma(90°) = \Gamma_m = \frac{\left(-Z_1^2/R + R/Z_1^2\right)\left[-\left(Z_1^2/R + R/Z_1^2\right) - \Gamma_\ell\left(Z_1^2/R - R/Z_1^2\right)\right] + 4\Gamma_\ell}{-\left(Z_1^2/R + R/Z_1^2\right)\left[-\left(Z_1^2/R + R/Z_1^2\right) - \Gamma_\ell\left(Z_1^2/R - R/Z_1^2\right)\right]}$$

$$= \frac{(R^2 - Z_1^4)\left[-(Z_1^4 + R^2) - \Gamma_\ell(Z_1^4 - R^2)\right] + 4\Gamma_\ell R^2 Z_1^4}{(Z_1^4 + R^2)\left[(Z_1^4 + R^2) + \Gamma_\ell(Z_1^4 - R^2)\right]}$$

$$\Gamma_m (Z_1^4 + R^2)^2 + \Gamma_m \Gamma_\ell (Z_1^4 + R^2)(Z_1^4 - R^2) = -(R^2 - Z_1^4)(Z_1^4 + R^2) + \Gamma_\ell (Z_1^4 - R^2)^2 + 4\Gamma_\ell R^2 Z_1^4$$

$$Z_1^8 (\Gamma_m - 1)(\Gamma_\ell + 1) + 2 Z_1^4 R^2 (\Gamma_m - \Gamma_\ell) - R^4 (\Gamma_m + 1)(\Gamma_\ell - 1) = 0$$

For $\Gamma_m = 0.05$, $\Gamma_\ell = 0.2$, $R = 1.5$:

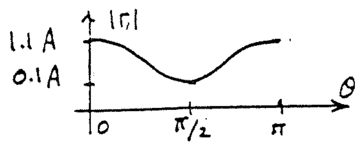
$$-1.140 Z_1^8 - 0.6750 Z_1^4 + 4.2525 = 0$$

$$Z_1^4 = \frac{0.675 \pm 4.455}{-2.280} = 1.65789 \Rightarrow Z_1 = 1.1347 Z_0 \checkmark$$

$$Z_2 = R/Z_1 = 1.3219 Z_0 \checkmark$$

These results agree with Table 5.2.

(5.22) $|\Gamma(\theta)| = A(0.1 + \cos^2\theta)\ , \quad 0 < \theta < \pi$



From (5.46a), for $N=2$,

$$|\Gamma(\theta)| = 2\left(\Gamma_0 \cos 2\theta + \tfrac{1}{2}\Gamma_1\right) = A(0.1+\cos^2\theta) \qquad \cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$

$$= A(0.6 + 0.5\cos 2\theta)$$

When $\theta = 0$, $\quad |\Gamma(0)| = 1.1A = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{1.5-1}{1.5+1} = 0.2 \Rightarrow A = 0.182$

Equating coefficients of $\cos 2\theta$:

$$2\Gamma_0 = 0.5A \quad \Rightarrow \quad \Gamma_0 = 0.0455$$

Equating constant terms:

$$\Gamma_1 = 0.6A = 0.109$$

so the characteristic impedances are,

$$Z_1 = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} = 1.095\, Z_0$$

$$Z_2 = Z_1 \frac{1+\Gamma_1}{1-\Gamma_1} = 1.245\, Z_1 = 1.363\, Z_0$$

CHECK: at $\theta = \pi/2$, the input impedance to the transformer will be,

$$Z_{in} = \frac{Z_1^2}{(Z_2^2/Z_L)} = \frac{Z_L Z_0 Z_1^2}{Z_2^2} = 0.968\, Z_0$$

So the input reflection coefficient is,

$$\Gamma_{in} = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right| = 0.016$$

which is reasonably close to $|\Gamma(\pi/2)| = 0.1A = 0.018$

(5.23)

$$\frac{d(\ln Z/Z_0)}{dz} = A \sin\frac{\pi z}{L}$$

$$\ln(Z/Z_0) = B - \frac{LA}{\pi}\cos\frac{\pi z}{L}$$ 兩邊同取 exp

$$Z(z) = C e^{-\frac{LA}{\pi}\cos\frac{\pi z}{L}}$$

$$Z(0) = Z_0 = C e^{-LA/\pi}, \quad Z(L) = Z_L = C e^{+LA/\pi}$$

Solve for C, A to get,

$$C = \sqrt{Z_0 Z_L}$$

$$A = \frac{-\pi}{2L}\ln(Z_0/Z_L) \checkmark$$

From (5.67),

$$\Gamma(\theta) = \frac{1}{2}\int_{z=0}^{L} e^{-2j\beta z}\frac{d}{dz}(\ln Z/Z_0)\,dz$$

$$= \frac{1}{2}\int_{z=0}^{L} A\sin\frac{\pi z}{L}\, e^{-2j\beta z}\,dz$$

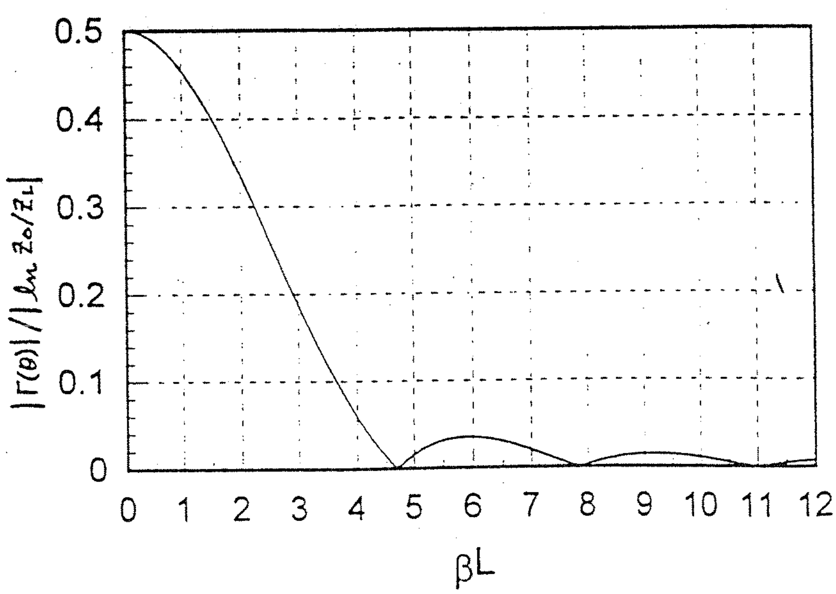
$$= \frac{A}{2}\left.\frac{e^{-2j\beta z}\left[-2j\beta\sin\frac{\pi z}{L} - \frac{\pi}{L}\cos\frac{\pi z}{L}\right]}{(\pi/L)^2 - 4\beta^2}\right|_0^L$$

$$= \frac{\pi A}{2L} e^{-j\beta L}\frac{(e^{-j\beta L} + e^{j\beta L})}{(\pi/L)^2 - 4\beta^2}$$

So,

$$|\Gamma(\theta)| = \frac{\pi^2}{2}\left|\ln\frac{Z_0}{Z_L}\right|\left|\frac{\cos\beta L}{\pi^2 - (2\beta L)^2}\right| \checkmark$$

This result is plotted as shown:



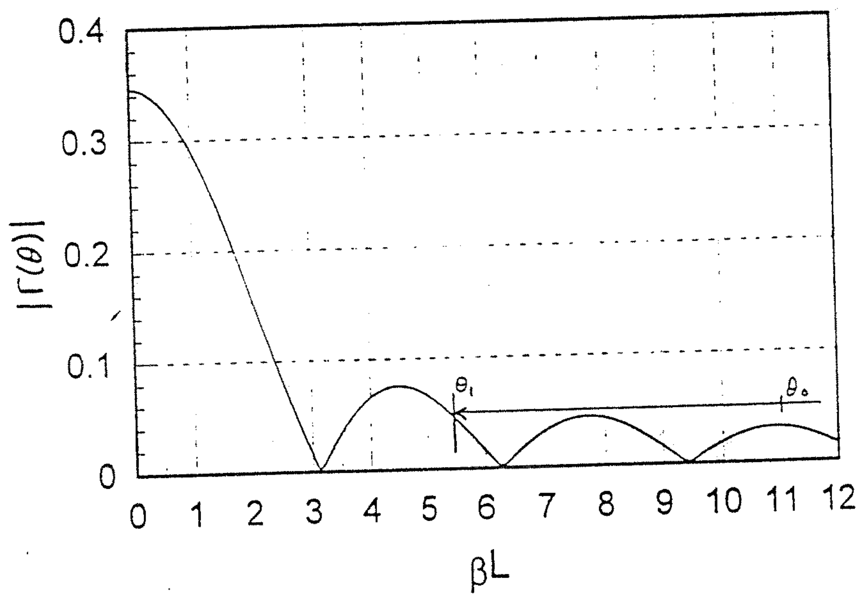
5.24 From (5.68), $Z(z) = Z_0 e^{az}$ for $0 < z < L$.

$$a = \frac{1}{L} \ln \frac{Z_L}{Z_0} = \frac{0.693}{L}$$

From (5.70),

$$|\Gamma(\theta)| = \frac{1}{2} \left| \ln \frac{Z_L}{Z_0} \right| \left| \frac{\sin \beta L}{\beta L} \right| = 0.346 \left| \frac{\sin \beta L}{\beta L} \right| \checkmark$$

This result is plotted in the graph shown below:



We see that the lower frequency limit for $|\Gamma| \leq 0.05$ is $\theta_1 = 5.5$. To obtain 100% bandwidth, we must have,

$$\frac{\theta_2 - \theta_1}{(\theta_1 + \theta_2)/2} = 1, \quad \text{or} \quad \theta_2 = 3\theta_1 = 16.5$$

Then at the center frequency,

$$\theta_0 = \frac{\theta_1 + \theta_2}{2} = 11.0 = \beta L$$

So,

$$L = \frac{11 \lambda_0}{2\pi} = 1.75 \lambda_0 \checkmark$$

From (5.64), θ_m for a Chebyshev transformer with 100% bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 1 \implies \theta_m = \pi/4.$$

Then from (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

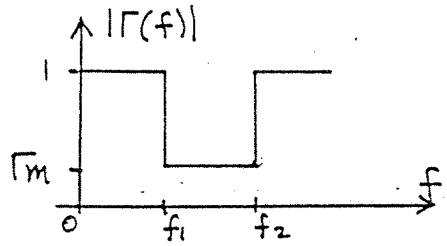
$$1.414 = \cosh\left[\frac{1}{N}(2.5846)\right] \implies N = 2.93 \Rightarrow \underline{N=3}$$

So $N=3$ sections would be required, for a length of $3\lambda_0/4$ at the center frequency. ✓
This is much shorter than the exponential taper matching section.

5.25 From Figure 5.22, the Bode-Fano limit for a parallel RC load is,

$$\int_0^\infty \ln\frac{1}{\Gamma(\omega)}\,d\omega \leq \frac{\pi}{RC}$$

The optimum reflection coefficient magnitude response will be as shown:



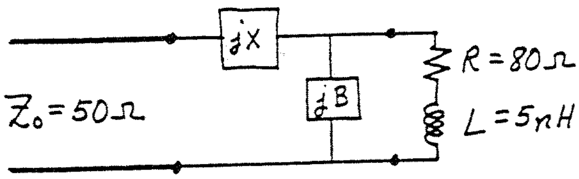
So, as in (5.80),

$$\ln\frac{1}{\Gamma_m} \leq \frac{\pi}{\Delta\omega RC} = \frac{\pi}{2\pi(10-2)\times10^9(100)(1.5\times10^{-12})} = 0.417$$

or,

$$\Gamma_m \geq 0.659 \implies RL < 3.6\,dB.$$

(5.26) L-section matching solution:



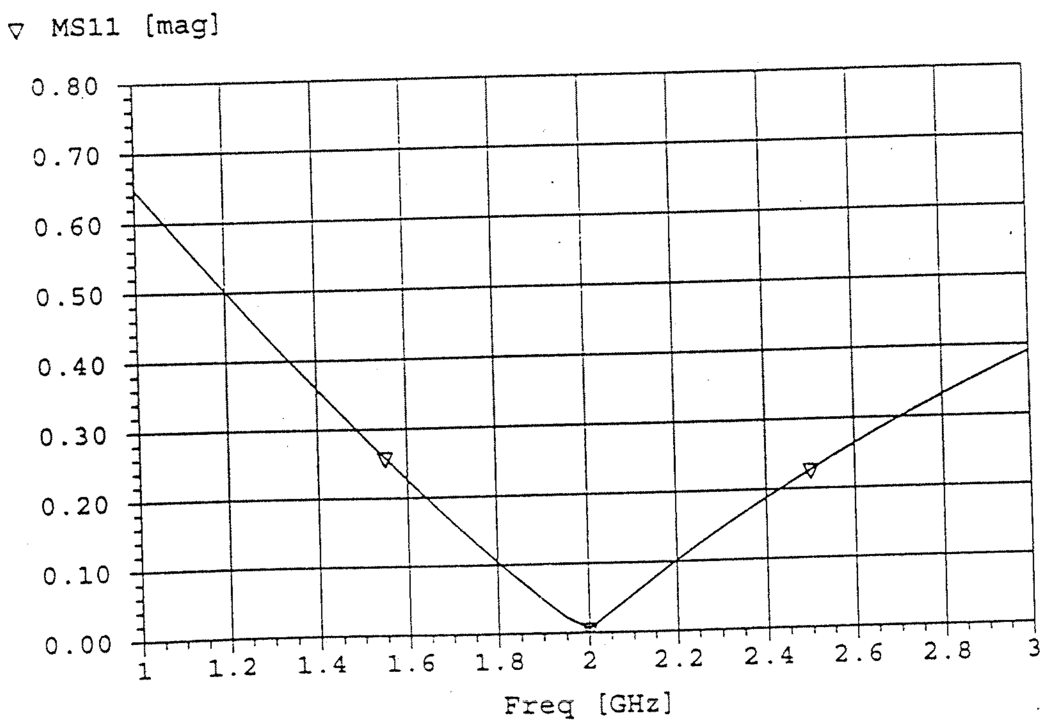
at $f = 2\,GHz$, $Z_L = 80 + j63\,\Omega$, $z_L = 1.6 + j1.26$ (INSIDE $1+jx$)

a Smith chart solution gives,

$jb = -j1.8 \Rightarrow$ INDUCTOR with $L = 22.1\,nH$. ✓

$jx = -j1.25 \Rightarrow$ CAPACITOR with $C = 1.27\,pF$. ✓

The input reflection coefficient magnitude is plotted below, where it is seen that the bandwidth for $|\Gamma| < 0.1$ is 20%.



Bode-Fano limit:

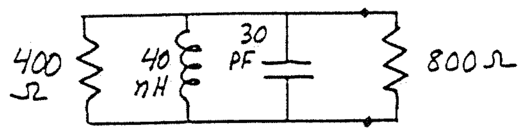
From Figure 5.22d, the Bode-Fano criteria gives a bandwidth limit of

$$\Delta\omega = \frac{\pi R}{L} \frac{1}{\ln 1/\Gamma_m} = 2.18 \times 10^{10} = \omega_2 - \omega_1$$

$$\frac{\Delta f}{f_0} = \frac{f_2 - f_1}{f_0} = \frac{2.18 \times 10^{10}}{2\pi\,(2 \times 10^9)} = 174\%$$

This is considerably more than the bandwidth of the L-section match.

6.1



From Table 6.1 for a parallel RLC circuit:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(40\times10^{-9})(30\times10^{-12})}} = 145 \text{ MHz}$$

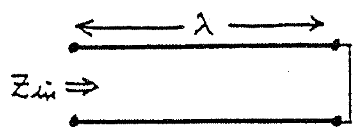
$$Q = \omega_0 RC = 2\pi f_0 RC = 2\pi(145\times10^6)(400)(30\times10^{-12}) = 10.9 \checkmark$$

$$Q_e = \frac{R_L}{\omega_0 L} = \frac{R_L}{R} Q = \frac{800}{400}(10.9) = 21.8 \checkmark$$

Then the loaded Q is, from (6.23),

$$Q_L = \frac{1}{\frac{1}{Q_e} + \frac{1}{Q}} = 7.27 \checkmark$$

6.2



$$l = \lambda = \frac{2\pi v_p}{\omega_0} \quad \text{for } \omega = \omega_0$$

This circuit has a series-type resonance, like the short-circuited λ/2 resonator. Thus, let

$$\beta l = \frac{\omega_0 l}{v_p} + \frac{\Delta\omega l}{v_p} = 2\pi\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then from (6.24) the input impedance is,

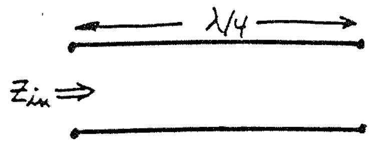
$$Z_{in} \simeq Z_0 \frac{\alpha l + j2\pi \frac{\Delta\omega}{\omega_0}}{1 + j2\pi\frac{\Delta\omega}{\omega_0}} \simeq Z_0\left(\alpha l + j2\pi\frac{\Delta\omega}{\omega_0}\right) = R + 2jL\Delta\omega$$

Thus, $R = Z_0 \alpha l$, $L = \frac{\pi Z_0}{\omega_0}$.

And,

$$Q = \frac{\omega_0 L}{R} = \frac{\pi Z_0}{Z_0 \alpha l} = \frac{\pi}{\alpha l} = \frac{\beta}{2\alpha} \quad \left(\text{since } l = \lambda = \frac{2\pi}{\beta} \text{ at res.}\right)$$

6.3



$$\ell = \frac{\lambda}{4} = \frac{\pi v_p}{2\omega_0} \quad \text{for } \omega = \omega_0$$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ line. So let,

$$\beta \ell = \frac{\omega_0 \ell}{v_p} + \frac{\Delta\omega \ell}{v_p} = \frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then, $$\tan \beta \ell = \tan \frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right) = -\cot \frac{\Delta\omega \pi}{2\omega_0} \simeq \frac{-2\omega_0}{\pi \Delta\omega}$$

The input impedance is,

$$Z_{in} = Z_0 \frac{1 + j \tan \beta\ell \tanh \alpha\ell}{\tanh \alpha\ell + j \tan \beta\ell} \simeq \frac{1 - j\frac{2\omega_0}{\pi\Delta\omega}\alpha\ell}{\alpha\ell - j\frac{2\omega_0}{\pi\Delta\omega}}$$

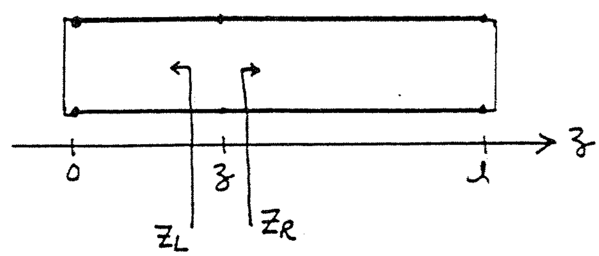
$$\simeq Z_0 \frac{\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}}{1 + j\frac{\pi\Delta\omega}{2\omega_0}\alpha\ell} \simeq Z_0\left(\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}\right) = R + 2jL\Delta\omega$$

$$\therefore R = Z_0 \alpha \ell \checkmark \quad , \quad L = \frac{\pi Z_0}{4\omega_0} \checkmark$$

Then, $$Q = \frac{\omega_0 L}{R} = \frac{\pi}{4\alpha\ell} = \frac{\beta}{2\alpha} \checkmark$$

(since $\ell = \frac{\lambda}{4} = \frac{\pi}{2\beta}$ at resonance)

(6.4)

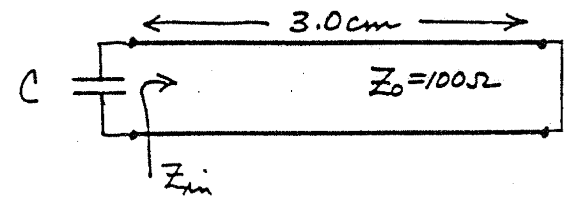


$\beta l = \pi$

$Z_L = j Z_0 \tan \beta z$

$Z_R = j Z_0 \tan \beta(l - z) = j Z_0 \tan(\pi - \beta z) = -j Z_0 \tan \beta z = Z_L^*$ ✓

(6.5)



$f_0 = 6$ GHz

$\beta = \frac{2\pi f}{c} = 125.7\ m^{-1}$ for an air-filled line

$\beta l = (125.7)(0.03) = 216°$ ✓

$Z_{in} = j Z_0 \tan \beta l = j(100) \tan 216° = j\,72.6\Omega = j\omega L$ ✓

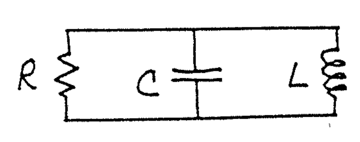
To achieve resonance we must have,

$$Z_{in} = (jX_c)^* = \frac{j}{\omega C}$$

So,

$$C = \frac{1}{\omega X_{in}} = 0.365\,pF \checkmark$$

The equivalent circuit at 6 GHz, with the shunt resistor, is as follows:



$R = 10{,}000\ \Omega$

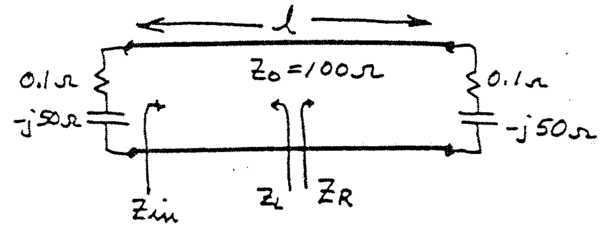
$C = 0.365\ pF$

$L = \frac{X_{in}}{\omega} = \frac{72.6}{2\pi(6 \times 10^9)} = 1.93\ nH$ ✓

So the Q is,

$$Q = \omega RC = 2\pi(6\times10^9)(10{,}000)(0.365\times10^{-12}) = 138. \checkmark$$

(6.6)



Since the resonator is symmetrical, at the midpoint of the line we must have, $Z_L = Z_R^* = Z_R$, or $\text{Im}\{Z_R\} = 0$:

Let $t = \tan \beta\ell/2$ and $Z_L = R_L + jX_L$. $(R_L = 0.1,\ X_L = -50.)$

$$Z_R = Z_0 \frac{Z_L + jZ_0 t}{Z_0 + jZ_L t} = Z_0 \frac{R_L + j(X_L + Z_0 t)}{(Z_0 - X_L t) + jR_L t}$$

$$= Z_0 \frac{R_L(Z_0 - X_L t) + R_L t(X_L + Z_0 t) + j(X_L + Z_0 t)(Z_0 - X_L t) - jR_L^2 t}{(Z_0 - X_L t)^2 + (R_L t)^2}$$

$\text{Im}\{Z_R\} = 0 \Rightarrow \quad (X_L + Z_0 t)(Z_0 - X_L t) - R_L^2 t = 0$ $\quad R_L^2 = 0.1^2$ 太小 略去

$$-X_L Z_0 t^2 + (Z_0^2 - X_L^2 - R_L^2)t + Z_0 X_L = 0$$

$$5000t^2 + 7500t - 5000 = 0$$

$$t^2 + 1.5t - 1 = 0 \qquad \tan\tfrac{\beta\ell}{2} = 0.5 \Rightarrow \beta\ell = 2\tan^{-1}0.5$$

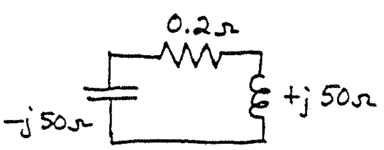
$$t = \frac{-1.5 \pm \sqrt{(1.5)^2 + 4}}{2} = -0.75 \pm 1.25 = \begin{cases} 0.50 \Rightarrow \beta\ell = 53.1^\circ \\ -2.00 \longrightarrow \beta\ell = -126.9^\circ = 53.1^\circ \end{cases}$$

So,

$$\ell = \frac{53.1^\circ}{360^\circ}\lambda = 0.148\lambda \qquad \tan\beta\ell = 1.332$$

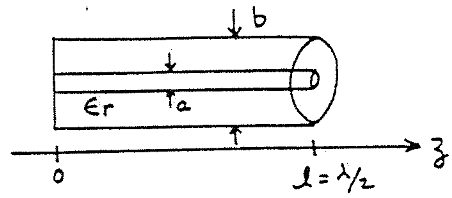
CHECK:

$$Z_{in} = 100\,\frac{(.1 - j50) + j133.2}{100 + j(.1 - j50)(1.332)} = 0.1 + j50\,\Omega \quad ✓$$



$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{50}{0.2} = 250.$$

6.7



$\beta \ell = \pi$
$\beta = \pi/\ell$
$\eta = \sqrt{\mu_0/\epsilon}$

From Section 2.2 the TEM fields of a coaxial line are,

$$\bar{E}^{\pm} = \hat{\rho}\frac{V_0}{\rho \ln b/a} e^{\mp j\beta z}, \quad \bar{H}^{\pm} = \pm\hat{\phi}\frac{V_0}{\eta \ln b/a} e^{\mp j\beta z}$$

$E_\rho = 0$ at $z=0$ in the resonator, so the standing wave fields can be written as,

$$E_\rho = \frac{V_0}{\rho \ln b/a}\left[e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2jV_0}{\rho \ln b/a} \sin \beta z$$

$$H_\phi = \frac{V_0}{\eta \ln b/a}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0}{\eta \ln b/a} \cos \beta z$$

From (1.84) and (1.86) the time-average stored electric and magnetic energies are,

$$W_e = \frac{\epsilon}{4}\int_v |\bar{E}|^2 dv = \frac{\epsilon}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\rho \ln b/a}\right)^2 \sin^2\frac{\pi z}{\ell}\, \rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi \epsilon V_0^2}{\ln b/a}$$

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\eta \ln b/a}\right)^2 \cos^2\frac{\pi z}{\ell}\, \rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi \mu_0 V_0^2}{\eta^2 \ln b/a} = \frac{\pi \epsilon V_0^2}{\ln b/a} = W_e \checkmark$$

6.8

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{\frac{R}{Z_0^2} + j\omega\left(\frac{L}{Z_0^2} - \frac{1}{\omega^2 C Z_0^2}\right)}$$

The input impedance of a parallel RLC circuit is,

$$Z_{in} = \frac{1}{\frac{1}{R'} + \frac{1}{j\omega L'} + j\omega C'} = \frac{1}{\frac{1}{R'} + j\omega\left(C' - \frac{1}{\omega^2 L'}\right)}$$

Thus the original circuit acts as a parallel $R'L'C'$ resonator with $R' = Z_0^2/R$, $C' = L/Z_0^2$, $L' = C Z_0^2$.
(This is the basis for using $\lambda/4$ lines as impedance and admittance inverters.)

6.9

From (6.40),

$$f_{101} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{.05}\right)^2 + \left(\frac{\pi}{.05}\right)^2} = 4.24\ \text{GHz} \quad (TE_{101}) ✓$$

$$f_{102} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{.05}\right)^2 + \left(\frac{2\pi}{.05}\right)^2} = 6.71\ \text{GHz} \quad (TE_{102}) ✓$$

at 4.24 GHz, $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0165\ \Omega$; $k = 88.8\ m^{-1}$ ✓

at 6.71 GHz, $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0207\ \Omega$; $k = 140.5\ m^{-1}$ ✓

From (6.46), with $a = b = d$,

$$(2\ell^2 a^3 b + 2bd^3 + \ell^2 a^3 d + ad^3) = 3a^4(1+\ell^2)$$

So,

$$Q_{101} = \frac{k^3 a^7 \eta_0}{2\pi^2 R_s}\,\frac{1}{3a^4(1+1)} = \frac{k^3 a^3 \eta_0}{12\pi^2 R_s} = 16{,}886. \quad (TE_{101}) ✓$$

$$Q_{102} = \frac{k^3 a^7 \eta_0}{2\pi^2 R_s}\,\frac{1}{3a^4(1+4)} = \frac{k^3 a^3 \eta_0}{30\pi^2 R_s} = 21{,}325. \quad (TE_{102}) ✓$$

(6.10) From Table 3.2, the magnetic fields of the TM_{11} waveguide mode are,

$$H_x^{\pm} = \frac{B^{\pm}}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} e^{\mp j\beta z}$$

$$H_y^{\pm} = \frac{B^{\pm}}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} e^{\mp j\beta z}$$

To have current maxima at $z=0,d$ the cavity fields must be,

$$H_x = \frac{A}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

$$H_y = \frac{A}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

The stored magnetic energy is,

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4} A^2 \frac{a}{2}\frac{b}{2}\frac{d}{2}\left(\frac{1}{b^2}+\frac{1}{a^2}\right) = \frac{abd\mu_0 A^2}{32}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

The power lost in the walls is,

$$P_\ell = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds = R_s\left\{\int_{x=0}^{a}\int_{z=0}^{d} |H_x(y=0)|^2 dz\,dx + \int_{y=0}^{b}\int_{z=0}^{d} |H_y(x=0)|^2 dy\,dz + \right.$$

$$\left. + \int_{x=0}^{a}\int_{y=0}^{b}\left[|H_x(z=0)|^2 + |H_y(z=0)|^2\right] dx\,dy\right\}$$

$$= \frac{A^2 R_s}{4}\,\frac{a^3d+b^3d+a^3b+ab^3}{a^2b^2}$$

Then,

$$Q = \frac{\omega_0(W_e+W_m)}{P_\ell} = \frac{2\omega_0 W_m}{P_\ell} = \frac{k_0\eta_0}{4R_s}\,\frac{abd\,(a^2+b^2)}{(a^3d+b^3d+a^3b+ab^3)}$$ ✓

6.11 From Section 3.3 the transverse fields of the TE_{10} mode in the two regions can be written as,

$$E_y = \begin{cases} A \sin\frac{\pi x}{a} \sin\beta_a z & \text{for } 0<z<d-t \\ B \sin\frac{\pi x}{a} \sin\beta_d(d-z) & \text{for } d-t<z<d \end{cases}$$

$$H_x = \begin{cases} -j\frac{A}{Z_a} \sin\frac{\pi x}{a} \cos\beta_a z & \text{for } 0<z<d-t \\ -j\frac{B}{Z_d} \sin\frac{\pi x}{a} \cos\beta_d(d-z) & \text{for } d-t<z<d \end{cases}$$

where $\beta_a = \sqrt{k_o^2-(\pi/a)^2}$, $\beta_d = \sqrt{\epsilon_r k_o^2-(\pi/a)^2}$

$Z_a = k_o\eta_o/\beta_a$, $Z_d = k\eta/\beta_d = k_o\eta_o/\beta_d$

Continuity of E_y, H_x at $z=d-t$:

E_y: $A \sin\beta_a(d-t) = B \sin\beta_d t$

H_x: $\frac{A}{Z_a} \cos\beta_a(d-t) = \frac{B}{Z_d} \cos\beta_d t$

Divide to obtain:

$$Z_a \tan\beta_a(d-t) = Z_d \tan\beta_d t$$

$$\beta_d \tan\beta_a(d-t) = \beta_a \tan\beta_d t$$

This equation can be solved for k_o. β_a and β_d are functions of k_o as given above.

6.12 TM modes: $(\nabla^2 + k^2)E_z = 0$

Let $E_z(x,y,z) = X(x)Y(y)Z(z)$.
Substitute into wave equation and divide by XYZ:

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + k^2 = 0$$

By the separation of variables argument,

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \quad \Rightarrow X(x) = A\cos k_x x + B\sin k_x x$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 \quad \Rightarrow Y(y) = C\cos k_y y + D\sin k_y y$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -k_z^2 \quad \Rightarrow Z(z) = E\cos k_z z + F\sin k_z z$$

with $k^2 = k_x^2 + k_y^2 + k_z^2$.

Now, $E_z = 0$ for $x = 0, a$ and $y = 0, b$. Therefore, $A = C = 0$ and $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$. To enforce the remaining boundary conditions, we need E_x or E_y:
From Maxwell's equations,

$$E_x = \frac{1}{k^2 - k_z^2}\frac{\partial^2 E_z}{\partial x \partial z} = \frac{1}{k^2 - k_z^2}(Bk_x \cos k_x x)(D\sin k_y y) \cdot (-k_z E\sin k_z z + k_z F\cos k_z z)$$

For $E_x = 0$ at $z = 0, d$ we must have $F = 0$, and $k_z = \frac{l\pi}{d}$.

Thus,
$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2,$$
which determines the resonant frequencies. The solution for TE modes is similar.

(6.13) From Table 3.5 the fields of the TM'_{nm0} mode are $(\beta = 0)$:

$$E_z = A \sin n\phi \, J_n(k_c \rho)$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2 \rho} A \cos n\phi \, J_n(k_c\rho)$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} A \sin n\phi \, J_n'(k_c\rho) \,, \qquad k_c = p_{nm}/a = k$$

The stored electric energy is,

$$W_e = \frac{\epsilon}{4}\int_v |\bar{E}|^2 dv = \frac{A^2\epsilon}{4}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \sin^2 n\phi \, J_n^2(k_c\rho)\,\rho \, d\rho \, d\phi \, dz$$

$$= \frac{A^2\epsilon}{4}\pi d \frac{a^2}{2} J_n'^2(p_{nm}) = \frac{A^2 a^2 \pi d \epsilon}{8} J_n'^2(p_{nm}) \quad \text{(using C.14)}$$

The power loss due to finite conductivity is,

$$P_c = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds$$

$$= \frac{R_s}{2}\left\{\int_{\phi=0}^{2\pi}\int_{z=0}^{d} |H_\phi(\rho=a)|^2 a \, d\phi \, dz + 2\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[|H_\rho|^2 + |H_\phi|^2\right]\rho \, d\rho \, d\phi\right\}$$

$$= \frac{A^2 R_s}{2}\left\{\frac{\pi a d}{\eta^2} J_n'^2(p_{nm}) + \frac{2\pi}{\eta^2}\frac{p_{nm}^2}{2k_c^2} J_n'^2(p_{nm})\right\}$$

$$= \frac{A^2 R_s \pi}{2\eta^2}(ad + a^2) J_n'^2(p_{nm})$$

Then, $$Q_c = \frac{2\omega W_e}{P_c} = \frac{\omega a^2 \pi d \epsilon (2\eta^2)}{4 R_s \pi a (d+a)} = \frac{a d k \eta}{2 R_s (d+a)} \checkmark$$

The power lost in the dielectric is,

$$P_d = \frac{\omega\epsilon''}{2}\int_v |\bar{E}|^2 dv = \frac{\omega\epsilon}{2}\tan\delta \int_v |\bar{E}|^2 dv = \frac{2kW_e}{\eta\epsilon}\tan\delta$$

So, $$Q_d = \frac{2\omega W_e}{P_d} = \frac{1}{\tan\delta} \checkmark \qquad \text{(as in (6.48))}$$

6.14 From Figure 6.10, maximum Q for the TE_{111} mode occurs for $2a/d \simeq 1.7$. From (6.53a) the resonant frequency of the TE_{111} mode is,

$$f_{111} = \frac{c}{2\pi}\sqrt{\left(\frac{p'_{11}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = \frac{c}{2\pi}\sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{1.7\pi}{2a}\right)^2} = \frac{1.55\times10^8}{a}$$

$$= 7\times10^9.$$

Thus, $a = 2.21$ cm ✓

$d = \frac{2a}{1.7} = 2.60$ cm ✓

$\sigma_{Ag} = 6.17\times10^7$ S/m ; $k = 146.6\ m^{-1}$; $ka = 3.24$

$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.021\,\Omega$ ✓

$\beta = \frac{\pi}{d} = 120.8\ m^{-1}$ $\qquad p'_{11} = 1.841$

From (6.57) the Q is

$$Q_c = \frac{(ka)^3 \eta a d \left[1 - \left(\frac{1}{p'_{11}}\right)^2\right]}{4(p'_{11})^2 R_s \left\{\frac{ad}{2}\left[1 + \left(\frac{\beta a}{p'_{11}}\right)^2\right] + \left(\frac{\beta a}{p'_{11}}\right)^2\left(1 - \frac{1}{p'^2_{11}}\right)\right\}}$$

$$= 15{,}336.$$

6.15 Choose coordinate system so that $b<a<d$. Then the dominant resonant mode is the TE_{101} mode:

$$f_{101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{d}\right)^2} = 5.2\ GHz$$

or,

$$\frac{1}{a^2}+\frac{1}{d^2} = \left(\frac{2f_{101}}{c}\right)^2 = (34.7)^2$$

The next two higher modes must be either the TM_{110}, TE_{102}, or TE_{011} modes:

$$\left(\frac{2f_{110}}{c}\right)^2 = \frac{1}{a^2}+\frac{1}{b^2} = (34.7)^2 + \frac{1}{b^2} - \frac{1}{d^2}$$

$$\left(\frac{2f_{102}}{c}\right)^2 = \frac{1}{a^2}+\frac{4}{d^2} = (34.7)^2 + \frac{3}{d^2}$$

$$\left(\frac{2f_{011}}{c}\right)^2 = \frac{1}{b^2}+\frac{1}{d^2}$$

Since $d>a$, $f_{011} < f_{110}$

Try $f_{011} = 6.5\ GHz$; $f_{110} = 7.2\ GHz$

Then we have, $\frac{1}{b^2} - \frac{1}{d^2} = 1100.$

$$\frac{1}{b^2} + \frac{1}{d^2} = 1878.$$

Solving gives,

$b = 2.60$ cm ✓

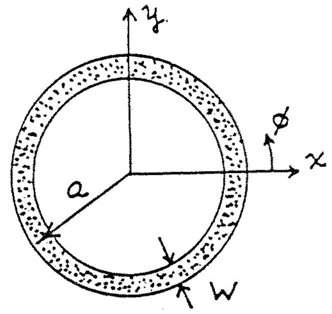
$d = 5.00$ cm ✓

$a = 3.53$ cm ✓

CHECK: $b<a<d$ OK ✓

$f_{102} = 7.35\ GHz > f_{110} = 7.2\ GHz$ ✓

(6.16)



$$e^{\pm j\beta a\phi} = e^{\pm jn\phi} \quad , n=1,2,3\ldots$$

FOR PERIODICITY

So, $\beta a = \frac{2\pi a}{\lambda_g} = \frac{2\pi a\sqrt{\epsilon_e} f}{c} = n$

$$f = \frac{nc}{2\pi a\sqrt{\epsilon_e}} \; ; \; n=1,2,3\ldots$$

(The ring circumference is $2\pi a = n\lambda_g$)

The above result assumes $a \gg W$, so that curvature effects can be neglected. This type of resonator is most often coupled using a gap feed to a microstripline.

(6.17) For TM_{nm0} modes we have $H_z=0$ and $\frac{\partial}{\partial z}=0$. The wave equation for E_z is,

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k^2\right)E_z = 0 \qquad \text{(from 4.134)} \; (\beta = k)$$

The general solution is,

$$E_z = (A_n\cos n\phi + B_n\sin n\phi)\,J_n(k\rho) \qquad \text{(finite at } \rho=0)$$

Since the choice of $\sin n\phi$ or $\cos n\phi$ (or any combination) depends only on the choice of the $\phi=0$ reference, we can let $B_n=0$.

Then,

$$E_z = A_n\cos n\phi\, J_n(k\rho)$$

We can find H_ϕ from (4.110d):

$$H_\phi = \frac{-j\omega\epsilon}{k^2}\frac{\partial E_z}{\partial\rho} = \frac{-j\omega\epsilon}{k}A_n\cos n\phi\, J_n'(k\rho)$$

For $H_\phi=0$ at $\rho=a$ we require $J_n'(ka)=0$, or $ka = p'_{nm}$.

So the resonant frequency is,

$$f_{nm0} = \frac{ck}{2\pi\sqrt{\epsilon_r}} = \frac{c\,p'_{nm}}{2\pi a\sqrt{\epsilon_r}}$$

and,

$$f_{110} = \frac{c\,p'_{11}}{2\pi a\sqrt{\epsilon_r}} = \frac{1.841c}{2\pi a\sqrt{\epsilon_r}} \quad ✓$$

This solution neglects the effect of fringing fields.

6.18 From (6.70), $\tan \beta L/2 = \alpha/\beta$,

with
$$\alpha = \sqrt{\left(\frac{2.405}{a}\right)^2 - k_o^2}$$
$$\beta = \sqrt{\epsilon_r k_o^2 - (2.405/a)^2}$$

The value of k_o at resonance must lie between $k_o = \frac{2.405}{a} = 602$, and $k_o = \frac{2.405}{a\sqrt{\epsilon_r}} = 100$.

We carry out a trial-and-error numerical search as follows:

k_o	α	β	$\tan \beta L/2 - \alpha/\beta$
110	592	275	−1.8
120	590	399	−1.02
150	583	672	.008
145	584	631	−.12
→ 149	583	664	−.0018

Thus, the resonant frequency is,

$$f_o = \frac{c k_o}{2\pi} = 7.11 \text{ GHz} \checkmark$$

(measured value is 7.8GHz)

(6.19) Following the analysis of Section 6.5, for $TE_{01\delta}$ mode:

$$H_z = H_0 J_0(k_c\rho) e^{\pm j\beta z}$$

$$E_\phi = \frac{j\omega\mu_0 H_0}{k_c} J_0'(k_c\rho) e^{\pm j\beta z} = A J_0'(k_c\rho) e^{\pm j\beta z}$$

$$H_\rho = \frac{\mp j\beta H_0}{k_c} J_0'(k_c\rho) e^{\pm j\beta z} = \frac{\mp A}{Z_{TE}} J_0'(k_c\rho) e^{\pm j\beta z}$$

for $|z| < L/2$, $\beta = \sqrt{\epsilon_r k_0^2 - k_c^2} = \sqrt{\epsilon_r k_0^2 - (p_{01}/a)^2}$; $Z_{TE} = \frac{\omega\mu_0}{\beta} = Z_d$

for $|z| > L/2$, $j\beta = \alpha = \sqrt{k_c^2 - k_0^2} = \sqrt{(p_{01}/a)^2 - k_0^2}$; $Z_{TE} = \frac{j\omega\mu_0}{\alpha} = Z_a$

So the standing wave fields can be written as,

$$E_\phi = \begin{cases} A J_0'(k_c\rho)[e^{-j\beta z} - e^{j\beta z}] = -2jA J_0'(k_c\rho)\sin\beta z & \text{for } |z| < L/2 \\ B J_0'(k_c\rho) e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

$$H_\rho = \begin{cases} \frac{A}{Z_d} J_0'(k_c\rho)[e^{-j\beta z} + e^{j\beta z}] = \frac{2A}{Z_d} J_0'(k_c\rho)\cos\beta z & \text{for } |z| < L/2 \\ \frac{B}{Z_a} J_0'(k_c\rho) e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

Continuity of E_ϕ and H_ρ at $z = L/2$ gives:

E_ϕ: $-2jA \sin\beta L/2 = B e^{-\alpha L/2}$

H_ρ: $\frac{2A}{Z_d}\cos\beta L/2 = \frac{B}{Z_a} e^{-\alpha L/2}$

dividing gives:

$$-jZ_d \tan\beta L/2 = Z_a$$

$$\frac{-j}{\beta}\tan\beta L/2 = j/\alpha$$

$$\tan\beta L/2 + \beta/\alpha = 0 \quad ✓$$

(6.20) assume $a>b$

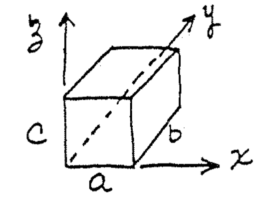
Because of the magnetic wall boundary conditions on the sidewalls, a rectangular dielectric waveguide along the z-axis would support TE modes with an H_z field of the form,

$$H_z = H_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b},$$

so the lowest order TE mode would be the TE_{11} mode. But $H_z \equiv 0$ for TM modes, so the lowest order TM mode would have,

$$H_x = H_0 \sin\frac{\pi x}{a}, \quad H_y = 0 \qquad (\text{if } a>b)$$

So the dominant mode of this resonator must be the $TM_{10\delta}$ mode. Thus we can write,



$$E_y = E_0 \sin\frac{\pi x}{a} e^{\pm j\beta z}$$

$$H_x = \frac{\pm E_0}{Z_{TM}} \sin\frac{\pi x}{a} e^{\pm j\beta z},$$

where,
$$\beta = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2} \qquad \text{for } |z| < c/2$$

$$j\beta = \alpha = \sqrt{(\pi/a)^2 - k_0^2} \qquad \text{for } z > c/2,$$

and $Z_{TM} = Z_d = \beta\eta/k = \beta\eta_0/\epsilon_r k_0$ for $|z| < c/2$,

$Z_{TM} = Z_a = -j\alpha\eta_0/k_0$ for $z > c/2$

Then the standing wave fields can be written as,

$$E_y = \begin{cases} A \sin\frac{\pi x}{a}\left[e^{-j\beta z} + e^{j\beta z}\right] = 2A \sin\frac{\pi x}{a} \cos\beta z & \text{for } |z| < c/2 \\ B \sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z > c/2 \end{cases}$$

$$H_x = \begin{cases} \frac{A}{Z_d} \sin\frac{\pi x}{a}\left[-e^{-j\beta z} + e^{j\beta z}\right] = \frac{2jA}{Z_d} \sin\frac{\pi x}{a} \sin\beta z & \text{for } |z| < c/2 \\ \frac{-B}{Z_a} \sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z > c/2 \end{cases}$$

Continuity of E_y, H_x at $z = c/2$:

$$2A \cos\beta c/2 = B e^{-\alpha c/2}$$

$$\frac{2jA}{Z_d} \sin\beta c/2 = \frac{-B}{Z_a} e^{-\alpha c/2}$$

divide to get:

$$\alpha \epsilon_r \tan\beta c/2 + \beta = 0$$

(6.21) From (6.72) we have that $f_o = \frac{c\ell}{2d}$, or

$$\ell = \frac{2df_o}{c} = \frac{2(.04)(94\times10^9)}{3\times10^8} = 25.$$

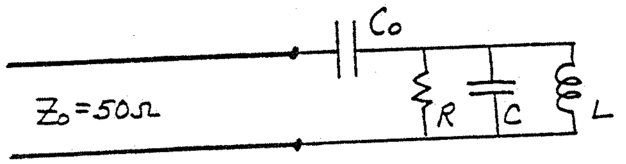
The surface resistivity is,

$$R_S = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{(2\pi)(94\times10^9)(4\pi\times10^{-7})}{2(5.8\times10^7)}} = 0.08\,\Omega$$

From (6.75) the Q is (due to conductor loss)

$$Q_c = \frac{\pi\ell\eta_o}{4R_s} = \frac{\pi(25)(377)}{4(.08)} = \underline{92,500.}$$

(6.22)



$R = 1000\,\Omega$
$L = 1.26\,nH$
$C = 0.804\,pF$

The simplest way to solve this problem is graphically, with a Smith chart. The admittance of the resonator at frequencies near resonance is,

$$Y_R = \frac{1}{R} + j\frac{2Q\Delta\omega}{R\omega_o},$$

where $\omega_o = \frac{1}{\sqrt{LC}} = 3.142\times10^{10}$ RPS ; $f_o = \frac{\omega_o}{2\pi} = 5.00$ GHz

$$Q = \frac{R}{\omega_o L} = 25.3$$

Normalized to Z_o, we have $y_R = Z_oY_R = 0.05 + j2.53\frac{\Delta\omega}{\omega_o}$. We can plot y_R on a Smith chart, versus $\Delta\omega/\omega_o$. For $\Delta\omega = 0$, $y_R = 0.05$. For $\Delta\omega = \pm0.1\omega_o$, $y_R = 0.05 \pm j0.253$. Next, convert this locus to z_R, an impedance locus. Then we see that a series capacitive reactance of $-jX_{c_o} = j4.2$ will yield an input impedance of $z_{in} = 1$. This corresponds to a resonator admittance $y_R = 0.05 - j0.22$. So the resonant frequency will be,

$$\Delta\omega = \frac{-0.22\,\omega_o}{2.53} = -0.0869\,\omega_o$$

$$\omega_r = \omega_o + \Delta\omega = (1-.0869)\omega_o = 0.913\,\omega_o$$

so, $f_r = \frac{\omega_r}{2\pi} = 4.566$ GHz (note lowering from f_0)

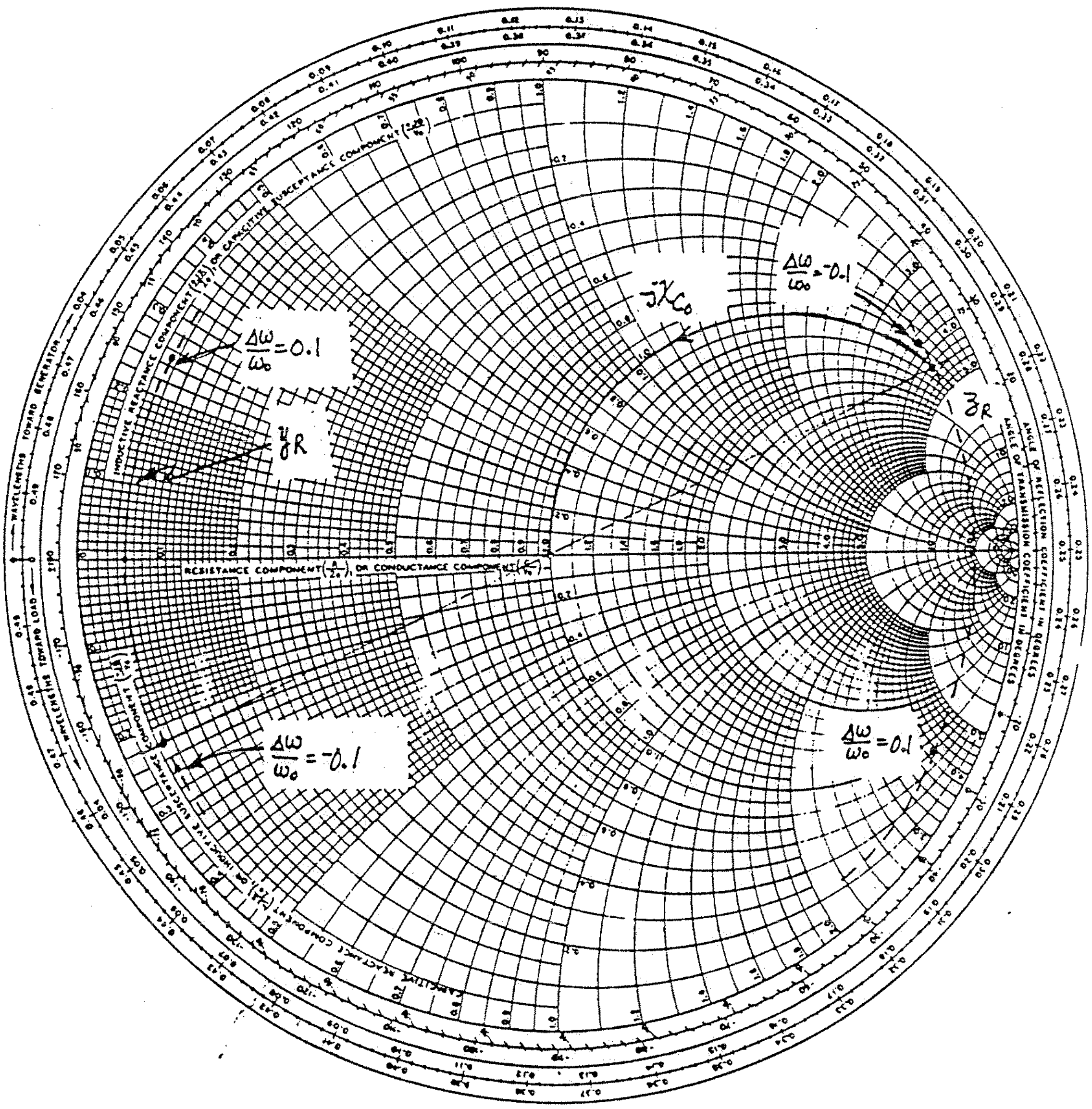
The coupling capacitor value is,

$$C_0 = \frac{1}{4.2 Z_0 \omega_r} = 0.166\ pF.$$

CHECK: at 4.566 GHz, $Y_R = (1 - j4.39) \times 10^{-3}$ S

$$Z_R = 49.2 + j216.5\ \Omega \simeq 50 + jX_{C_0}$$

$$\frac{1}{j\omega C_0} = -j210.$$



6.23 Assume TE_{101} mode, as in Section 6.7.

at 9 GHz, $k_o = 188.\ m^{-1}$; $\beta_o = 140.5\ m^{-1}$; $l = \frac{\lambda_g}{2} = \frac{\pi}{\beta_o} = 2.24$ cm.

$\frac{\omega_o}{2\pi} = f_o = 9$ GHz is the resonant frequency of the closed cavity, and does not include the effect of the coupling aperture. For a high-Q cavity, the actual resonant frequency, ω_1, will be close to ω_o. So we can approximately compute χ_L using ω_o. From (6.96),

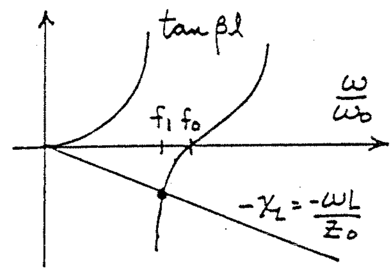
$$\chi_L = \sqrt{\frac{\pi - k_o \omega_1}{2Q\beta^2 C}} = 0.016 = \frac{\omega L}{Z_0} \Rightarrow \frac{L}{Z_0} = 2.83 \times 10^{-13}$$

Then solve (6.92) for ω:

$$\tan \beta l + \chi_L = 0$$

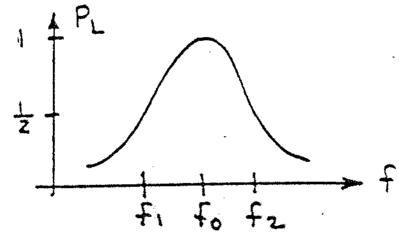
Numerical trial-and-error:

f	β	χ_L	$\tan \beta l + \chi_L$
9	140.	.0160	.01
8.9	137.7	.0158	-.04
8.97	139.65	.0159	.0025



Thus, $f_1 = 8.97$ GHz

6.24



assuming $Q \gg 1$, $f_0 = \frac{f_1+f_2}{2} = 8.2325$ GHz

$$BW = \frac{f_2 - f_1}{f_0} = 0.3\%$$

$$Q_L = \frac{1}{BW} = 329 \gg 1 \checkmark \quad \text{(loaded } Q\text{)}$$

at resonance,

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{r-1}{r+1} = 0.33 \Rightarrow r = \frac{1+\Gamma}{1-\Gamma} = 1.985$$

From (6.83), $Q = gQ_e = \dfrac{g}{\frac{1}{Q_L} - \frac{1}{Q}}$

Solve for Q:

$$\frac{Q}{Q_L} - 1 = g$$

$$Q = Q_L(g+1)$$

If we have a series resonator, $g = \frac{Z_0}{R} = \frac{1}{r} = 0.504$ (undercoupled)

$$Q = (1+g)Q_L = 495.$$

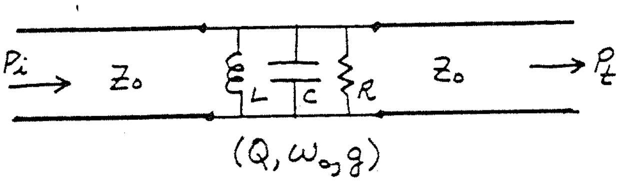
$$Q_e = \frac{Q}{g} = 982.$$

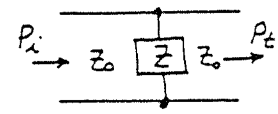
If we have a parallel resonator; $g = \frac{R}{Z_0} = r = 1.985$ (overcoupled)

$$Q = (1+g)Q_L = 982.$$

$$Q_e = \frac{Q}{g} = 495.$$

6.25





$$\frac{P_t}{P_i} = |S_{21}|^2 \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + Z_0/Z + 1} = \frac{2}{2 + Z_0/Z}$$

So, $$\frac{P_t}{P_i} = \frac{4}{|2 + Z_0/Z|^2}$$

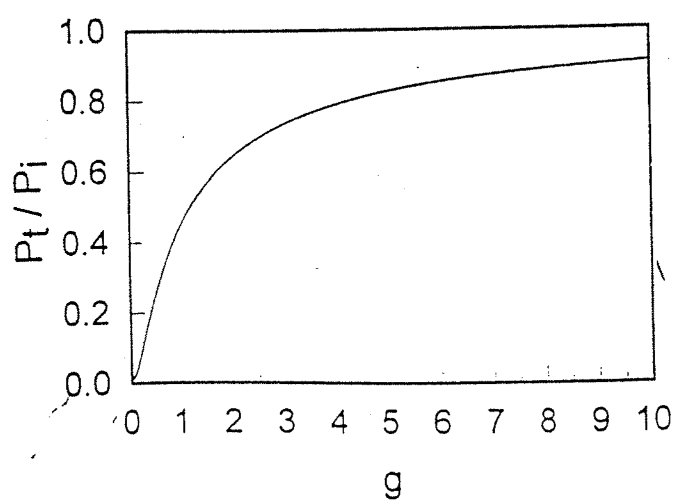
From Table 6.1, $\frac{Z_0}{Z} \simeq \frac{Z_0}{R} + j\frac{2Q\Delta\omega Z_0}{R\omega_0}$

For a parallel RLC resonator, $g = R/Z_0$, so

$$\frac{P_t}{P_i} = \frac{4}{\left(2+\frac{1}{g}\right)^2 + \left(\frac{2Q\Delta\omega}{g\omega_0}\right)^2} = \frac{4g^2}{(1+2g)^2 + \left(\frac{2Q\Delta\omega}{\omega_0}\right)^2}$$

at resonance $\Delta\omega = 0$, so this reduces to,

$$\frac{P_t}{P_i} = \frac{4g^2}{(1+2g)^2} = \left(\frac{2g}{1+2g}\right)^2$$



(6.26) The unperturbed TE_{101} cavity fields are,

$$E_y = A \sin\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

$$H_x = \frac{-jA}{Z} \sin\frac{\pi x}{a} \cos\frac{\pi z}{d} \qquad ; \; Z = k\eta/\beta$$

$$H_z = \frac{j\pi A}{k\eta a} \cos\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

Then the numerator in (6.102) is,

$$\int_{v_0} \left(\Delta\epsilon |\bar{E}_0|^2 + \Delta\mu |\bar{H}_0|^2\right) dv = (\mu_r - 1)\mu_0 \int_{x=0}^{a} \int_{y=0}^{b} \int_{z=0}^{t} \left(|H_x|^2 + |H_z|^2\right) dz\,dy\,dx$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \int_{z=0}^{t} \left(\frac{1}{Z^2}\cos^2\frac{\pi z}{d} + \frac{\pi^2}{k^2\eta^2 a^2}\sin^2\frac{\pi z}{d}\right) dz$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{1}{Z^2}\left(\frac{z}{2} + \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Big|_0^t + \frac{\pi^2}{k^2\eta^2 a^2}\left(\frac{z}{2} - \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Big|_0^t\right]$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{t}{2\eta^2} + \frac{\beta^2 - \pi^2/a^2}{k^2\eta^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right]$$

The denominator in (6.102) is $\frac{abd\epsilon_0 A^2}{2}$, so

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-(\mu_r - 1) ab\eta^2 [\bullet]}{abd}$$

$$= \frac{-(\mu_r - 1)}{d}\left(\frac{t}{2} + \frac{\beta^2 - \pi^2/a^2}{k^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right)$$

For $t \ll d$ this simplifies to,

$$\frac{\omega - \omega_0}{\omega_0} \simeq -(\mu_r - 1)\left(\frac{t}{d}\right)\left(\frac{\beta^2}{k^2}\right)$$

6.27 Following Example 6.8:

at $x = a/2,\ z = 0$:

$$E_y = 0$$

$$H_x = \frac{-jA}{Z}, \quad Z = k_0 \eta_0 / \beta$$

$$H_y = 0$$

Then,

$$\int_{\Delta v} (\mu |\bar{H}_0|^2 - \epsilon |\bar{E}_0|^2)\, dv = \mu_0 \frac{A^2}{Z^2} \Delta V \quad ; \ \Delta V = \pi l r_0^2$$

$$\int_{\Delta v} (\mu |\bar{H}_0|^2 + \epsilon |\bar{E}_0|^2)\, dv = \frac{V_0 \epsilon_0 A^2}{2}$$

So (6.109) reduces to,

$$\frac{\omega - \omega_0}{\omega_0} = \frac{2\mu_0 \Delta V}{Z^2 \epsilon_0 V_0} = \frac{2\eta_0^2 \Delta V \beta^2}{k_0^2 \eta_0^2 V_0} = \frac{2\beta^2}{k_0^2} \frac{\Delta V}{V_0}$$

(an increase in resonant frequency)

Chapter 7

7.1 This is a special case of a lossless reciprocal 3-port network; it was shown in general that such a network could not be matched at all ports (using the [S] matrix). Alternatively, we can argue as follows: If the input to each port is to be matched to its respective characteristic impedance, we must have,

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3} \quad \text{(port 1)}$$

$$\frac{1}{Z_2} = \frac{1}{Z_1} + \frac{1}{Z_3} \quad \text{(port 2)}$$

$$\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad \text{(port 3)}$$

It is not possible to satisfy these three equations simultaneously:

$$\det\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = -4 \neq 0$$

7.2

$$RL = -20\log|\Gamma| = -20\log|S_{11}| = -20\log(0.05) = 26.\ dB \checkmark$$

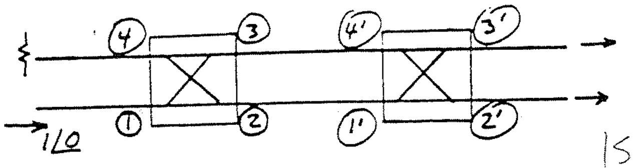
$$C = 10\log\frac{P_1}{P_3} = -20\log|S_{13}| = -20\log(0.1) = 20.\ dB \checkmark$$

$$D = 10\log\frac{P_3}{P_4} = 20\log\left|\frac{S_{13}}{S_{14}}\right| = 20\log\left(\frac{.1}{.05}\right) = 6.0\ dB \checkmark$$

$$I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}| = -20\log(0.05) = 26.\ dB \checkmark$$

isolation

(7.3)



$S_{13} = \beta e^{j\theta}$

$S_{24} = \beta e^{j\phi}$

$|S_{13}| = |S_{24}|$

$|S_{12}| = |S_{34}| = \alpha$

$C = 8.34\,dB \Rightarrow \beta = |S_{13}| = 0.383$

$-20 \log |S_{13}|$

$\alpha = \sqrt{1-\beta^2} = 0.924$

$\theta + \phi = \pi + 2n\pi$

If $V_1^+ = 1\angle 0^\circ$, then from (7.17),

$j = \cos 90^\circ + j \sin 90^\circ$

$V_3^- = j\beta V_1^+ = 0.383\angle 90^\circ$

$V_2^- = \alpha V_1^+ = 0.924\angle 0^\circ$

Then the outputs of the second coupler are,

$V_3'^- = j\beta V_1'^+ + \alpha V_4'^+ = j\beta V_2^- + \alpha V_3^-$

$= (0.383)(0.924)\angle 90^\circ + (0.924)(0.383)\angle 90^\circ = 0.707\angle 90^\circ$ ✓

$V_2'^- = \alpha V_1'^+ + j\beta V_4'^+ = \alpha V_2^- + j\beta V_3^-$

$= (0.924)(0.924)\angle 0 - (0.383)(0.383)\angle 0 = 0.707\angle 0^\circ$ ✓

Thus the outputs are identical to those for a single 3dB hybrid.

(7.4)

$P_1 = 4\,W, \quad C = 20\,dB, \quad D = 35\,dB, \quad IL = 0.5\,dB$

$P_1 = 4\,W = 10 \log \frac{4\,W}{\boxed{0.001\,W}} = 36.0\,dBm$

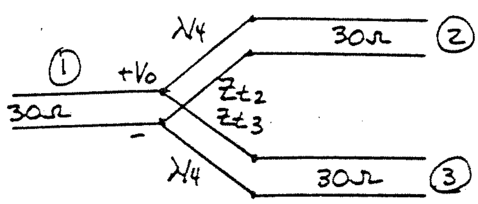
Insertion

$IL = 0.5\,dB = -10 \log \frac{P_2}{P_1} \Rightarrow P_2 = P_1 - IL = 35.5\,dBm = 3.55\,W$ ✓

$C = 20\,dB = -10 \log \frac{P_3}{P_1} \Rightarrow P_3 = P_1 - C = 16.0\,dBm = 0.04\,W$ ✓

$D = 35\,dB = 10 \log \frac{P_3}{P_4} \Rightarrow P_4 = P_3 - D = -19.0\,dBm = 0.0126\,mW$ ✓

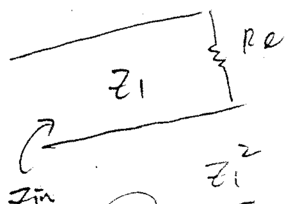
(7.5)



$$\frac{P_2}{P_3} = 3$$

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{3}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{3}{4Z_0}\right)$$



$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{1}{4Z_0}\right)$$

So, $Z_2 = 4Z_0/3 = 40\Omega$ ✓ $\quad Z_{in} = \frac{Z_1^2}{R_l}$

$Z_3 = 4Z_0 = 120\Omega$ ✓

The $\lambda/4$ matching transformers have impedances,

$Z_1 = \sqrt{R_l \cdot Z_{in}}$

$Z_{t2} = \sqrt{30(40)} = 34.6\Omega$ ✓

$Z_{t3} = \sqrt{30(120)} = 60.0\Omega$ ✓

Then the S-parameters are, (phase ref. at 30Ω ports)

$$S_{11} = \frac{30-30}{30+30} = 0$$

$$S_{22} = \frac{30\|120 - 40}{30\|120 + 40} = \frac{24-40}{24+40} = -0.25 \checkmark$$

$$S_{33} = \frac{30\|40 - 120}{30\|40 + 120} = \frac{17.1-120.}{17.1+120.} = -0.75 \checkmark$$

$$S_{21} = S_{12} = \sqrt{P_2/P_1}\, e^{j\theta} = \sqrt{3/4}\,\angle -90^\circ = 0.866\angle -90^\circ \checkmark$$

$$S_{31} = S_{13} = \sqrt{P_3/P_1}\, e^{j\theta} = \sqrt{1/4}\,\angle -90^\circ = 0.50\angle -90^\circ \checkmark$$

Since the network is lossless, we should have,

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

so, $S_{23} = S_{32} = \sqrt{1-(.25)^2-(.866)^2}\, e^{-2j\theta} = 0.433\angle -180^\circ$ ✓

(7.6)

T-NETWORK: From Table 4.1 the ABCD parameters are

$A = 1 + R_1/R_2$ $\qquad B = 2R_1 + R_1^2/R_2$

$C = 1/R_2$ $\qquad D = 1 + R_1/R_2$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \Rightarrow \cancel{1} + \cancel{\frac{R_1}{R_2}} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0 R_2} - \frac{Z_0}{R_2} - \cancel{1} - \cancel{\frac{R_1}{R_2}} = 0$$

$$R_1^2 + 2R_1R_2 - Z_0^2 = 0$$

$$R_2 = \frac{Z_0^2 - R_1^2}{2R_1}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \Rightarrow 2 + \frac{2R_1}{R_2} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0R_2} + \frac{Z_0}{R_2} = \frac{2}{\alpha}$$

$$2Z_0R_2 + 2Z_0R_1 + \underbrace{2R_1R_2 + R_1^2}_{= Z_0^2} + Z_0^2 = \frac{2}{\alpha} Z_0 R_2$$

$$R_2 + R_1 + Z_0 = R_2/\alpha$$

$$\left(\frac{1}{\alpha} - 1\right)(Z_0 - R_1) = 2R_1$$

$$Z_0\left(\frac{1}{\alpha} - 1\right) = R_1\left(1 + \frac{1}{\alpha}\right)$$

$$R_1 = Z_0 \frac{1-\alpha}{1+\alpha} \checkmark$$

$$R_2 = \frac{2\alpha}{1-\alpha^2} Z_0 \checkmark$$

For $Z_0 = 50\Omega$:

α (dB)	α	$R_1 (\Omega)$	$R_2 (\Omega)$
3	.708	8.6	141.9
10	.316	26.0	35.1
20	.100	40.9	10.1

Π-NETWORK: From Table 4.1 the ABCD parameters are,

$$A = 1 + R_2/R_1 \qquad B = R_2$$

$$C = \frac{2}{R_1} + \frac{R_2}{R_1^2} \qquad D = 1 + R_2/R_1$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \implies \frac{R_2}{Z_0} - \frac{2Z_0}{R_1} - \frac{R_2 Z_0}{R_1^2} = 0$$

$$R_2 R_1^2 - 2Z_0^2 R_1 - R_2 Z_0^2 = 0$$

$$R_2 = \frac{2Z_0^2 R_1}{R_1^2 - Z_0^2}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \implies 2 + \frac{2R_2}{R_1} + \frac{R_2}{Z_0} + \underbrace{\frac{2Z_0}{R_1} + \frac{Z_0 R_2}{R_1^2}}_{= R_2/Z_0} = \frac{2}{\alpha}$$

$$1 + \frac{R_2}{R_1} + \frac{R_2}{Z_0} = \frac{1}{\alpha}$$

$$R_1 Z_0 + R_2(Z_0 + R_1) = \frac{1}{\alpha} Z_0 R_1$$

$$2Z_0^2 \cancel{R_1} = \cancel{R_1} Z_0 \left(\frac{1}{\alpha} - 1\right)(R_1 - Z_0)$$

$$\frac{2Z_0\alpha}{1-\alpha} = R_1 - Z_0$$

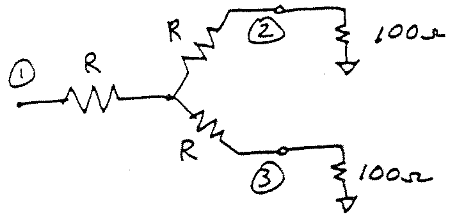
$$R_1 = Z_0\left(1 + \frac{2\alpha}{1-\alpha}\right) = \frac{1+\alpha}{1-\alpha} Z_0 \checkmark$$

$$R_2 = \frac{1-\alpha^2}{2\alpha} Z_0 \checkmark$$

For $Z_0 = 50\,\Omega$:

α (dB)	α	R_1 (Ω)	R_2 (Ω)
3	.708	292.5	17.6
10	.316	96.2	71.2
20	.100	61.1	247.5

7.7



DESIGN:

$Z_0 = 100\,\Omega$

$R = 33.3\,\Omega$ ✓

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

CASE a) ports 2 & 3 matched to $100\,\Omega$ $(V_2^+ = V_3^+ = 0)$;

If $V_1^+ = 1$,

$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}$$

$$V_3 = V_3^+ + V_3^- = \tfrac{1}{2}$$

$$P_3 = V_3^2/Z_0 = 0.25/Z_0$$

CASE b) port 3 matched, $\Gamma = 0.3$ at port 2 $(V_3^+ = 0)$:

If $V_1^+ = 1$,

$$V_2^- = \tfrac{1}{2}[V_1^+ + V_3^+] = \tfrac{1}{2}$$

$$V_2^+ = \Gamma V_2^- = (0.3)(\tfrac{1}{2}) = 0.15$$

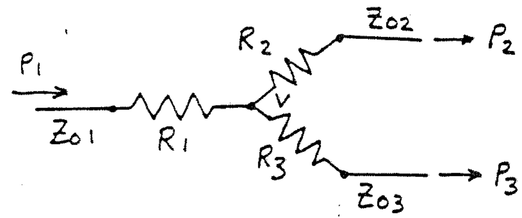
$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}(1.15) = 0.575\ \text{v}$$

$$V_3 = V_3^+ + V_3^- = 0.575\ \text{v}$$

$$P_3 = V_3^2/Z_0 = 0.331/Z_0$$

$$\frac{P_3\ (\text{PORT 2 MISMATCHE})}{P_3\ (\text{PORT 2 MATCHED})}(dB) = 10\log\left(\frac{0.331}{0.25}\right) = \underline{1.2\,dB}$$

(7.8)



$\alpha = \dfrac{P_2}{P_3}$

let $Z_1 = R_1 + Z_{01}$
$Z_2 = R_2 + Z_{02}$
$Z_3 = R_3 + Z_{03}$

For $\alpha = \frac{P_2}{P_3}$ we must have,

$$\frac{Z_{02} Z_3^2}{Z_2^2 Z_{03}} = \alpha \qquad \ldots (1)$$

For all three ports to be matched we must have,

$$Z_{01} = R_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad \ldots (2)$$

$$Z_{02} = R_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} \qquad \ldots (3)$$

$$Z_{03} = R_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \ldots (4)$$

We assume that Z_{01} and α are given; then we have 5 unknowns ($R_1, R_2, R_3, Z_{02}, Z_{03}$), and 4 equations. So we need one more equation: we will choose the condition that,

$$\boxed{Z_{02} Z_{03} = Z_{01}^2} \qquad \ldots (5)$$

which will ensure that when $\lambda/4$ transformers are used to match Z_{02} and Z_{03} to a final output impedance of Z_{01}, phase tracking will be maintained for ports 2 and 3. (See Problem 7.10). Now use (1) and (5) to eliminate Z_3 and Z_{03} from (2), (3), (4). Then (2) reduces to,

$$R_1 = \frac{Z_{01}\left[Z_{02} + \sqrt{\alpha}\,(Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

$$Z_1 = \frac{Z_{01}\left[2 Z_{02} + \sqrt{\alpha}\,(2 Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

and (3) and (4) then reduce to,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-\alpha Z_{01}Z_{02}\right] = 2Z_{02}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

and,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-Z_{01}Z_{02}\right] = 2Z_{01}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

Then we obtain a quartic equation for Z_{02}:

$$Z_{02}^4+Z_{01}(2\sqrt{\alpha}-1)Z_{02}^3+Z_{01}^2(\alpha-1)Z_{02}^2+Z_{01}^3(\alpha-2\sqrt{\alpha})Z_{02}-\alpha Z_{01}^4=0$$

after finding Z_{02} (either numerically or using the formula for a quartic equation), the above equations can be used to find Z_{03}, R_2, R_3, and R_1.

EXAMPLE: let $Z_{01}=1$, $\alpha=2$:

Then, $$Z_{02}^4+1.828\,Z_{02}^3+Z_{02}^2-0.828\,Z_{02}-2=0$$

The solution for Z_{02} was computed numerically using an HP-15C calculator to be,

$$Z_{02}=0.8935$$

Then,

$Z_2 = 1.041$; $R_2 = 0.1478$

$Z_3 = 1.6477$; $R_3 = 0.5285$; $Z_{03} = 1.1192$

$R_1 = 0.3621$

CHECK: $Z_{in} = R_1 + Z_2 \| Z_3 = 1.000$ ✓

NOTE: There are other choices for the relation between Z_{02} and Z_{03}, or other pairs of variables. One possibility that may give a simpler solution would be to let $Z_1=\alpha^{1/3}Z_2$.

(7.9) From (7.37), $K^2 = P_3/P_2 = 1/3 \Rightarrow K = 0.577$

$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}} = 131.7\ \Omega$ ✓

$Z_{02} = K^2 Z_{03} = 43.9\ \Omega$ ✓

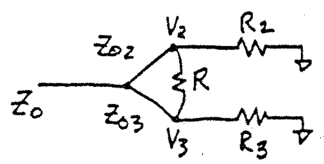
$R = Z_0(K + 1/K) = 115.5\ \Omega$ ✓

The output impedances are,

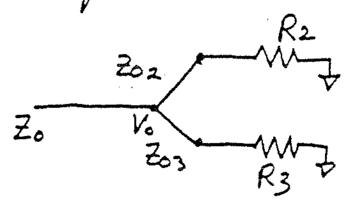
$R_2 = Z_0 K = 28.9\ \Omega$ ✓

$R_3 = Z_0/K = 86.7\ \Omega$ ✓

(7.10)



assuming the output ports are matched, no power should be dissipated in resistor R (for lossless power division). Therefore $V_2 = V_3$, and the resistor R can be removed:



Input matching requires that,

$$\frac{1}{Z_0} = \frac{R_2}{Z_{02}^2} + \frac{R_3}{Z_{03}^2}$$

Power division requires that $P_3 = K^2 P_2$:

$$P_2 = \tfrac{1}{2}|V_0|^2 R_2/Z_{02}^2$$
$$P_3 = \tfrac{1}{2}|V_0|^2 R_3/Z_{03}^2 = K^2 P_2$$

Thus,

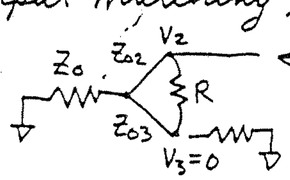
$$K^2 \frac{R_2}{Z_{02}^2} = \frac{R_3}{Z_{03}^2}$$

and,

$$Z_{02}^2 = Z_0 R_2(1+K^2) \qquad \ldots (1)$$
$$Z_{03}^2 = Z_0 R_3(1+1/K^2) \qquad \ldots (2)$$

For output matching,

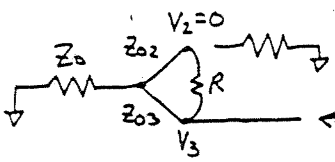
PORT 2:



$$\leftarrow R_2 = R \,\|\, \frac{Z_{02}^2}{Z_0} = \frac{R R_2(1+K^2)}{R + R_2(1+K^2)}$$

$$R_2(1+K^2) = RK^2 \qquad \ldots (3)$$

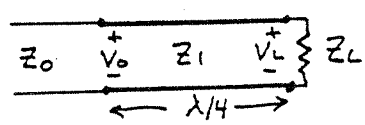
PORT 3:



$$\leftarrow R = R \,\|\, \frac{Z_{03}^2}{Z_0} = \frac{R R_3(1+1/K^2)}{R+R_3(1+1/K^2)}$$

$$R_3(1+K^2) = R \qquad \cdots (4)$$

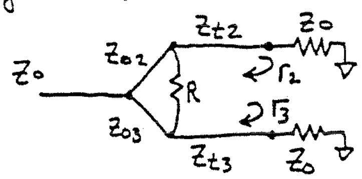
From (3) and (4) we have that $R_2 = K^2 R_3$. We need one more condition to obtain the design equations. This condition is that the output impedances R_2, R_3 be chosen so that output matching transformers have the same transfer phase at both ports:



It is easy to show that,

$$\frac{V_L}{V_0} = -j\,\frac{1+\Gamma}{1-\Gamma} \; ; \quad \Gamma = \frac{Z_L - Z_1}{Z_L + Z_1}$$

So the phase of V_L/V_0 does not change when Γ is replaced by $-\Gamma$, or when Z_L and Z_1 are interchanged (Z_L, Z_1 real).



$$\Gamma_2 = \frac{Z_0/Z_{t2} - 1}{Z_0/Z_{t2} + 1}$$

$$\Gamma_3 = \frac{Z_0/Z_{t3} - 1}{Z_0/Z_{t3} + 1}$$

So we choose $Z_{t2} Z_{t3} = Z_0^2 = \sqrt{Z_0 R_2}\,\sqrt{Z_0 R_3} = K Z_0 R_3$, which leads to the design equations,

$R_2 = K Z_0$ ✓ $\qquad Z_{02} = Z_0\sqrt{K(1+K^2)}$ ✓

$R_3 = Z_0/K$ ✓ $\qquad Z_{03} = Z_0\sqrt{\dfrac{1+K^2}{K^3}}$ ✓

$R = Z_0(K + 1/K)$ ✓

7.11 Setting $A_{10}^- = 0$ from (7.40b):

$$\left(\epsilon_0 \alpha_e + \frac{\mu_0 \alpha_m}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{\mu_0 \pi^2 \alpha_m}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

For a round aperture, $\alpha_e = 2r_0^3/3$; $\alpha_m = 4r_0^3/3$:

$$\left(\epsilon_0 + \frac{2\mu_0}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\mu_0\pi^2}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

Since $Z_{10} = k_0\eta_0/\beta$, this simplifies as follows:

$$\left(k_0^2 + 2\beta^2\right)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\cos^2\frac{\pi s}{a} = 0$$

$$\left(3k_0^2 - \frac{2\pi^2}{a^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\left(1 - \sin^2\frac{\pi s}{a}\right) = 0$$

$$3k_0^2 \sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}$$

$$\frac{6a^2}{\lambda_0^2}\sin^2\frac{\pi s}{a} = 1, \text{ or } \sin\frac{\pi s}{a} = \frac{\lambda_0}{\sqrt{6}\,a} < 1 \text{ for } a > \lambda_0/2 \checkmark$$

7.12 at $f = 11$ GHz, $k_0 = 230.4\ m^{-1}$; $\beta = 116.4\ m^{-1}$; $Z_{10} = k_0\eta_0/\beta = 746.2\ \Omega$

$P_{10} = ab/Z_{10} = 1.67 \times 10^{-7}$ W

From (7.41) the position of the coupling aperture is,

$$\sin\frac{\pi s}{a} = \pi\sqrt{\frac{2}{4\pi^2 - k_0^2 a^2}} = 0.867 \Rightarrow s = 0.334a = \underline{0.528\text{ cm}}$$

Then, $\sin^2\frac{\pi s}{a} = 0.752$, $\cos^2\frac{\pi s}{a} = 0.248$

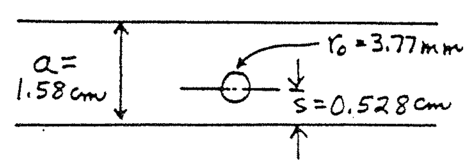
For $C = 20$ dB, $\left|\frac{A_{10}^-}{A}\right| = 0.1$. From (7.40b) we have,

$$0.1 = \frac{\omega}{P_{10}}\left[\frac{2}{3}\epsilon_0 \sin^2\frac{\pi s}{a} + \frac{4\mu_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] r_0^3$$

$$0.1 P_{10} = \left[\frac{2}{3}\frac{k_0}{\eta_0}\sin^2\frac{\pi s}{a} + \frac{4k_0\eta_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] r_0^3$$

$$1.67 \times 10^{-8} = 3.12 \times 10^{-1} r_0^3$$

$$r_0 = \underline{3.77\text{ mm}}$$



7.13 at $f = 17\ \text{GHz}$, $k_0 = 356.\ \text{m}^{-1}$, $\beta = 295.3\ \text{m}^{-1}$,

$Z_{10} = k_0\eta_0/\beta = 454\ \Omega$, $P_{10} = ab/Z_{10} = 2.75 \times 10^{-7}\ \text{W}$

From (7.44) the necessary skew angle is, for $s = a/2$,

$$\cos\theta = k_0^2/2\beta^2 = 0.728 \Rightarrow \underline{\theta = 43^\circ}$$

From (7.45) the aperture radius is,

$$\left|\frac{A_{10}^-}{A}\right| = 0.0316 = \frac{4k_0^2 r_0^3}{3ab\beta} = 4.58 \times 10^6 r_0^3 \Rightarrow \underline{r_0 = 1.90\ \text{mm}}$$

7.14 $N = 4$ (5-holes)

$k_0 = 366.5\ \text{m}^{-1}$, $\beta = 307.9\ \text{m}^{-1}$, $Z_{10} = k_0\eta_0/\beta = 448.8\ \Omega$

$P_{10} = ab/Z_{10} = 2.78 \times 10^{-7}\ \Omega$

From (7.40a), with $s = a/2$,

$$|K_f| = \frac{2k_0}{3\eta_0 P_{10}}\left|1 - 2\beta^2/k_0^2\right| = 9.59 \times 10^5$$

From (7.55),

$$C = 20\text{dB} = -20\log|K_f| - 20\log k - 20\log\sum_{n=0}^{N} C_n^N$$

For $N = 4$, $\sum_{n=0}^{N} C_n^N = 1+4+6+4+1 = 16$, so

$$20 = -119.6 - 20\log k - 24.1$$

$$k = 6.53 \times 10^{-9}$$

From (7.54) the aperture radii are,

$$r_0 = k^{1/3} = 1.87\ \text{mm} = r_4$$

$$r_1 = (4k)^{1/3} = 2.97\ \text{mm} = r_3$$

$$r_2 = (6k)^{1/3} = 3.97\ \text{mm}$$

The spacing between the apertures is $\lambda_g/4 = 5.1\ \text{mm}$.
(Smaller apertures would result if $s = a/4$ were used.)

7.15 $N=4$ (5 hole)

$k_0 = 366.5\ m^{-1}$, $\beta = 307.9\ m^{-1}$, $Z_0 = 448.8\ \Omega$

$P_{10} = 2.78 \times 10^{-7}\ W$

From (7.40a,b) with $s = a/2$,

$$|K_f| = \frac{2k_0}{3\eta_0 P_{10}}\left|1 - \frac{2\beta^2}{k_0^2}\right| = 9.59\times10^5$$

From (7.59),

$$30dB = D_{min} = 20\log T_N(\sec\theta_m)$$

$$T_4(\sec\theta_m) = \cosh\left[4\cosh^{-1}(\sec\theta_m)\right] = 31.6$$

$$\sec\theta_m = 1.587$$

From (7.57),

$$C = 20dB = -20\log|K_f| - 20\log k - 30$$

$$k = 3.30\times10^{-9}$$

From (7.56) and (6.60d):

$$2\left[r_0^3\cos 4\theta + r_1^3\cos 2\theta + r_2^3\right] = k\left[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1\right]$$

Then,

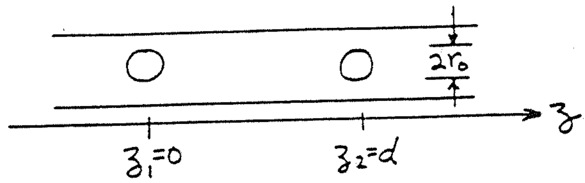
$$2r_0^3 = k\sec^4\theta_m \Rightarrow r_0 = r_4 = 2.19\ mm$$

$$2r_1^3 = k\left[4\sec^4\theta_m - 4\sec^2\theta_m\right] \Rightarrow r_1 = r_3 = 2.93\ mm$$

$$2r_2^3 = k\left[3\sec^4\theta_m - 4\sec^2\theta_m + 1\right] \Rightarrow r_2 = 2.54\ mm.$$

The spacing between the apertures is $\lambda_g/4 = 5.1\ mm$.

7.16



The incident TE_{10} fields are, for $x=0$, $y=b/2$, $z=z_n$:

$$E_y = A \sin\frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_x = \frac{-A}{Z_{10}} \sin\frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_z = \frac{j\pi A}{\beta a Z_{10}} \cos\frac{\pi x}{a} e^{-j\beta z} = \frac{j\pi A}{\beta a Z_{10}} e^{-j\beta z_n}$$

$P_{10} = ab/Z_{10}$; $Z_{10} = k_0\eta_0/\beta$.

From (4.124)-(4.125) the equivalent polarization currents are: $(\hat{n}=\hat{x})$

$$\bar{P}_e = 0$$

$$\bar{P}_m = -\alpha_m H_z \delta(x)\delta(y-b/2)\delta(z-z_n) e^{-j\beta z_n}$$

Then the amplitudes of the forward and reverse coupled waves from a single aperture are,

$$A_{10}^+ = \frac{1}{P_{10}}\int_v \bar{H}_{10}^- \cdot j\omega\mu_0 \bar{P}_m \, dv = \frac{-j\omega\mu_0\alpha_m}{P_{10}}\left(\frac{j\pi A}{\beta a Z_0}\right) e^{-j\beta z_n} H_z^- = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}$$

$$A_{10}^- = \frac{1}{P_{10}}\int_v \bar{H}_{10}^+ \cdot j\omega\mu_0 \bar{P}_m \, dv = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0} e^{-2j\beta z_n}$$

Then the total forward and backward wave amplitudes from two apertures at $z_1=0$ and $z_2=d$ are,

$$A_{10}^+ = \frac{j\pi^2\alpha_m A(2)}{P_{10}a^2 k_0\eta_0} \qquad A_{10}^- = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}\left(1+e^{-2j\beta d}\right)$$

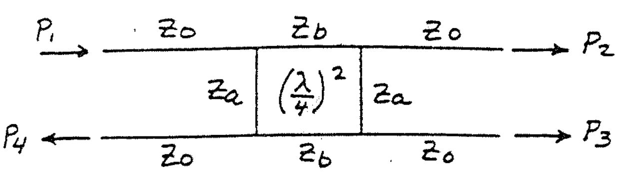
For $A_{10}^-=0$, we must have $(1+e^{-2j\beta d})=0$, or $\underline{d=\lambda_g/4}$.

Then the coupling factor is,

$$C = 20\log\left|\frac{A}{A_{10}^+}\right| = 20\log\left|\frac{P_{10}a^2 k_0\eta_0}{2\pi^2\alpha_m}\right|$$

$$= 20\log\left|\frac{3a^3 b\beta}{8\pi^2 r_0^3}\right| \text{ dB}$$

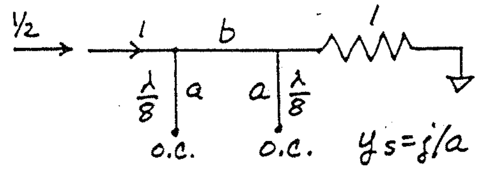
(7.17)



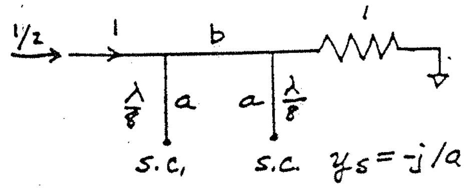
$\alpha = P_2/P_3$

let $a = \frac{Z_a}{Z_0}$, $b = \frac{Z_b}{Z_0}$

Following the analysis of Section 7.5, the even and odd circuits are: (in normalized form)



EVEN MODE



ODD MODE

The ABCD matrices are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} = \begin{bmatrix} -b/a & jb \\ j/b - jb/a^2 & -b/a \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} = \begin{bmatrix} b/a & jb \\ j/b - jb/a^2 & b/a \end{bmatrix}$$

$$\Gamma_e \overset{S_{11}^0}{=} \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{-2b/a + j(b + 1/b - b/a^2)}$$

$$\Gamma_o \overset{S_{11}^0}{=} \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{2b/a + j(b + 1/b - b/a^2)}$$

The reflection at port 1 is then,

$$\underset{S_{11}}{B_1} = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right)\frac{-2j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)}{\left(\frac{2b}{a}\right)^2 + \left(b + \frac{1}{b} - \frac{b}{a^2}\right)^2} = 0$$

Thus, $b - \frac{1}{b} + \frac{b}{a^2} = 0$ (can't have $b + \frac{1}{b} - \frac{b}{a^2} = 0$, or else $\underset{?}{\underline{\overset{S_{12}}{B_2} = \overset{S_{13}}{B_3} = 0}}$)

So, $a = \frac{b}{\sqrt{1-b^2}}$ Then $\frac{b}{a^2} = \frac{1}{b} - b$

Then,

$$T_e \overset{S_{e2}^e}{=} \frac{2}{A+B+C+D} = \frac{2}{-2\frac{b}{a} + j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{\frac{-b}{a} + jb}$$

$$\Gamma_o \overset{S_{21}^{o}}{} = \frac{2}{A+B+C+D} = \frac{2}{\frac{2b}{a} + j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{\frac{b}{a} + jb}$$

So the output wave amplitudes at ports 2 and 3 are,

$$\underset{S_{21}}{B_2} = \tfrac{1}{2}(\Gamma_e + \Gamma_o) = \tfrac{1}{2}\left[\frac{1}{-b/a + jb} + \frac{1}{b/a + jb}\right] = \frac{-j}{b(1+1/a^2)}$$

$$\underset{S_{31}}{B_3} = \tfrac{1}{2}(\underset{S_{12}^{e} - S_{12}^{o}}{\Gamma_e - \Gamma_o}) = \tfrac{1}{2}\left[\frac{1}{-b/a + jb} - \frac{1}{b/a + jb}\right] = \frac{-1/a}{b(1+1/a^2)}$$

This shows a 90° phase shift between ports 2 and 3.

For $P_2/P_3 = \alpha$,

$$P_2 = \alpha P_3$$

$$|\overset{S_{12}}{B_2}|^2 = \alpha |\overset{S_{13}}{B_3}|^2$$

$$1 = \frac{\alpha}{a^2} \Rightarrow a = \sqrt{\alpha} \text{ or } Z_a = \sqrt{\alpha}\, Z_0 \checkmark$$

Then, $b = \frac{a}{\sqrt{1+a^2}} = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} = \sqrt{\frac{\alpha}{1+\alpha}}$, or $Z_b = \sqrt{\frac{\alpha}{1+\alpha}}\, Z_0 \checkmark$

CHECK: When $\alpha = 1$, $Z_a = Z_0 \checkmark$, $Z_b = Z_0/\sqrt{2} \checkmark$

at the isolated port,

$$B_4 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right)(\cdot) = 0$$

So there is isolation.

EXAMPLE: $\alpha = 3$ (6dB), $Z_0 = 50\Omega$

Then $Z_a = 87\Omega \checkmark$

$Z_b = 43\Omega \checkmark$

(7.18) $b = 0.32$ cm, $\epsilon_r = 2.2$, $Z_{oe} = 70\,\Omega$, $Z_{oo} = 40\,\Omega$

Thus, $\sqrt{\epsilon_r}\, Z_{oe} = 104\,\Omega$; $\sqrt{\epsilon_r}\, Z_{oo} = 59\,\Omega$.

From Figure (7.29),

$$S/b = 0.075 \Longrightarrow S = 0.24 \text{ mm} \checkmark$$

$$W/b = 0.67 \Longrightarrow W = 2.1 \text{ mm} \checkmark$$

(7.19) $\epsilon_r = 10$; $d = 0.16$ cm ; $W = 0.16$ cm ; $S = 0.064$ cm

$W/d = 1$

$S/d = 0.4$

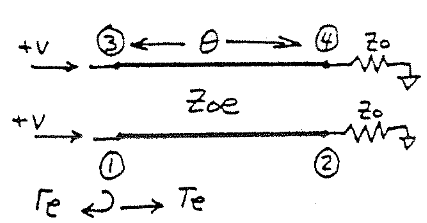
From Figure 7.30,

$Z_{oe} = 61\,\Omega$

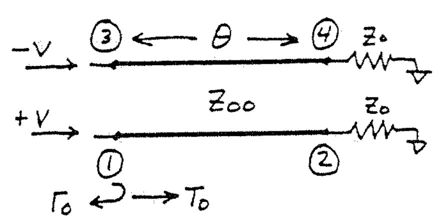
$Z_{oo} = 35\,\Omega$

41.5

(7.20)



EVEN MODE



ODD MODE

For $\theta = \pi/2$,

$$\Gamma_e = \frac{Z_{oe}^2/Z_o - Z_o}{Z_{oe}^2/Z_o + Z_o} = \frac{Z_{oe}^2 - Z_o^2}{Z_{oe}^2 + Z_o^2} \qquad \Gamma_o = \frac{Z_{oo}^2/Z_o - Z_o}{Z_{oo}^2/Z_o + Z_o} = \frac{Z_{oo}^2 - Z_o^2}{Z_{oo}^2 + Z_o^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & jZ_{oe}\sin\theta \\ \frac{j}{Z_{oe}}\sin\theta & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & jZ_{oo}\sin\theta \\ \frac{j}{Z_{oo}}\sin\theta & \cos\theta \end{bmatrix}$$

$$T_e = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right)\sin\theta} \qquad T_o = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{oo}}{Z_o} + \frac{Z_o}{Z_{oo}}\right)\sin\theta}$$

For a unit amplitude wave incident at port 1, the output wave amplitudes are,

$B_1 = \frac{1}{2}(\Gamma_e + \Gamma_o)$ $\qquad$ $B_2 = \frac{1}{2}(T_e + T_o)$

$B_3 = \frac{1}{2}(\Gamma_e - \Gamma_o)$ $\qquad$ $B_4 = \frac{1}{2}(T_e - T_o)$

So the reflection at port 1 is,

$$B_1 = \frac{1}{2}\left[\frac{Z_{0e}^2 - Z_0^2}{Z_{0e}^2 + Z_0^2} + \frac{Z_{0o}^2 - Z_0^2}{Z_{0o}^2 + Z_0^2}\right] = \frac{Z_{0o}^2 Z_{0e}^2 - Z_0^4}{(Z_0^2 + Z_{0e}^2)(Z_0^2 + Z_{0o}^2)} = 0$$

Thus, $\underline{Z_0 = \sqrt{Z_{0e} Z_{0o}}}$ ✓ (so all ports are matched ✓)

Then, $\left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}}\right) = \left(\frac{Z_0}{Z_{0o}} + \frac{Z_{0o}}{Z_0}\right)$

So, $T_e = T_o$, and $\underline{B_4 \equiv 0}$ ✓

The output waves at ports 2 and 3 are,

$$B_2 = \frac{2}{2\cos\theta + j\left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}}\right)\sin\theta} = \frac{1}{\cos\theta + j\,\frac{Z_{0e}+Z_{0o}}{2Z_0}\sin\theta}$$

$$B_3 = \frac{1}{2}\left[\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}} - \frac{Z_{0o}-Z_{0e}}{Z_{0o}+Z_{0e}}\right] = \frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}$$

Let $C = B_3 = \frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}$

Then $\sqrt{1-C^2} = \frac{2Z_0}{Z_{0e}+Z_{0o}}$, and so,

$$B_2 = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

For $\theta = \pi/2$, the midband responses are,

$$B_3 = C \quad ✓$$

$$B_2 = -j\sqrt{1-C^2} \quad ✓$$

which agree with (7.85) – (7.86).

7.21 $\quad C = 10^{-19.1/20} = 0.1109 \quad ; \; f = 8\text{GHz} \quad ; \; Z_0 = 60\,\Omega$

From (7.87),

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} = 67.1\,\Omega \qquad Z_{0o} = Z_0 \sqrt{\frac{1-C}{1+C}} = 53.7\,\Omega$$

For a stripline with $\epsilon_r = 2.2$, $b = 0.32$ cm,

$$\sqrt{\epsilon_r}\, Z_{0e} = 99.5\,\Omega \;, \qquad \sqrt{\epsilon_r}\, Z_{0o} = 79.7\,\Omega$$

From Figure 7.29,

$$S/b = 0.36 \implies S = 1.15 \text{ mm}$$
$$W/b = 0.60 \implies W = 1.92 \text{ mm}$$

The line lengths are,

$$\ell = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_r}\, f} = 6.32 \text{ mm}$$

7.22 $\quad C = 5\text{dB} = 10^{-5/20} = 0.562 \quad ; \; f_0 = 8\text{GHz} \; ; \; Z_0 = 60\,\Omega$

From (7.87),

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} = 113.3\,\Omega \qquad Z_{0o} = Z_0 \sqrt{\frac{1-C}{1+C}} = 31.8\,\Omega$$

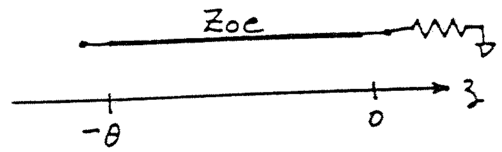
Then, $\sqrt{\epsilon_r}\, Z_{0e} = 168.1\,\Omega \qquad \sqrt{\epsilon_r}\, Z_{0o} = 47.2\,\Omega \qquad (\epsilon_r = 2.2)$

From Figure 7.29,

$$S/b \cong 0.009 \implies S = 0.029 \text{ mm } (!)$$
$$W/b \cong 0.34 \implies W = 1.09 \text{ mm}$$

This design is probably not practical due to the extremely close spacing of the lines.

7.23 For V_{1e} or V_{1o} at port 1, we first find V_{2e} or V_{2o}:



$$\Gamma_e = \frac{Z_o - Z_{oe}}{Z_o + Z_{oe}}$$

$$1+\Gamma_e = \frac{2Z_o}{Z_o + Z_{oe}}$$

$$V(z) = V^+\left(e^{-j\beta z} + \Gamma_e e^{j\beta z}\right)$$

$$V_{1e} = V(-\theta) = V^+\left(e^{j\theta} + \Gamma_e e^{-j\theta}\right)$$

$$V_{2e} = V(0) = V^+(1+\Gamma_e)$$

So,
$$V_{2e} = \frac{V_{1e}(1+\Gamma_e)}{e^{j\theta} + \Gamma_e e^{-j\theta}} = \frac{2Z_o V_{1e}}{(Z_o+Z_{oe})e^{j\theta} + (Z_o - Z_{oe})e^{-j\theta}}$$

$$= \frac{Z_o V_{1e}}{Z_o\cos\theta + jZ_{oe}\sin\theta} = \frac{Z_o V_{1e}\sec\theta}{Z_o + jZ_{oe}\tan\theta}$$

Similarly,
$$V_{2o} = \frac{Z_o V_{1o}\sec\theta}{Z_o + jZ_{oo}\tan\theta}$$

Then using (7.74) and the results following (7.79) gives,

$$V_4 = V_{2e} - V_{2o} = \frac{Z_o V\sec\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta}\left[\frac{Z_o + jZ_{oe}\tan\theta}{Z_o + jZ_{oe}\tan\theta} - \frac{Z_o + jZ_{oo}\tan\theta}{Z_o + jZ_{oo}\tan\theta}\right]$$

$$V_2 = V_{2e} + V_{2o} = \frac{2Z_o V\sec\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = \frac{\frac{2Z_o}{Z_{oe}+Z_{oo}}V}{\frac{2Z_o}{Z_{oe}+Z_{oo}}\cos\theta + j\sin\theta}$$

$$= \frac{V\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}\,V \qquad \text{since } \sqrt{1-C^2} = \frac{2Z_o}{Z_{oe}+Z_{oo}}$$

$V_3 = 2jV_1 \sin\theta\, e^{-jN\theta}\left[C_1 \cos(N-1)\theta - \ldots + \frac{1}{2}C_M\right]$, $M = \frac{N+1}{2}$

$C_0 = \left|\frac{V_2}{V_1}\right|_{\theta=\pi/2}$

(7.24) $N=3$, $C=25\,dB$, $Z_0=50\,\Omega$, BINOMIAL

From (7.90) and Example 7.8:

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \frac{1}{2}C_2\right] \quad ; \quad C_3 = C_1$$

$$= C_1 \sin 3\theta + (C_2 - C_1)\sin\theta \quad ; \text{ for } \theta=\pi/2,\ C = C_0 = C_2 - 2C_1$$

$$\frac{dC}{d\theta} = \left[3C_1\cos 3\theta + (C_2-C_1)\cos\theta\right]\Big|_{\pi/2} \equiv 0$$

$$\frac{d^2C}{d\theta^2} = \left[-9C_1\sin 3\theta - (C_2-C_1)\sin\theta\right]\Big|_{\pi/2} = 10C_1 - C_2 = 0$$

at midband ($\theta=\pi/2$), $C_0 = 10^{-25/20} = 0.0562 = C_2 - 2C_1$

solving for C_1, C_2:

$$0.0562 = 10C_1 - 2C_1 = 8C_1$$
$$C_1 = C_3 = 0.00703$$
$$C_2 = 10C_1 = 0.0703$$

Using (7.87) gives the even and odd mode characteristic impedances as,

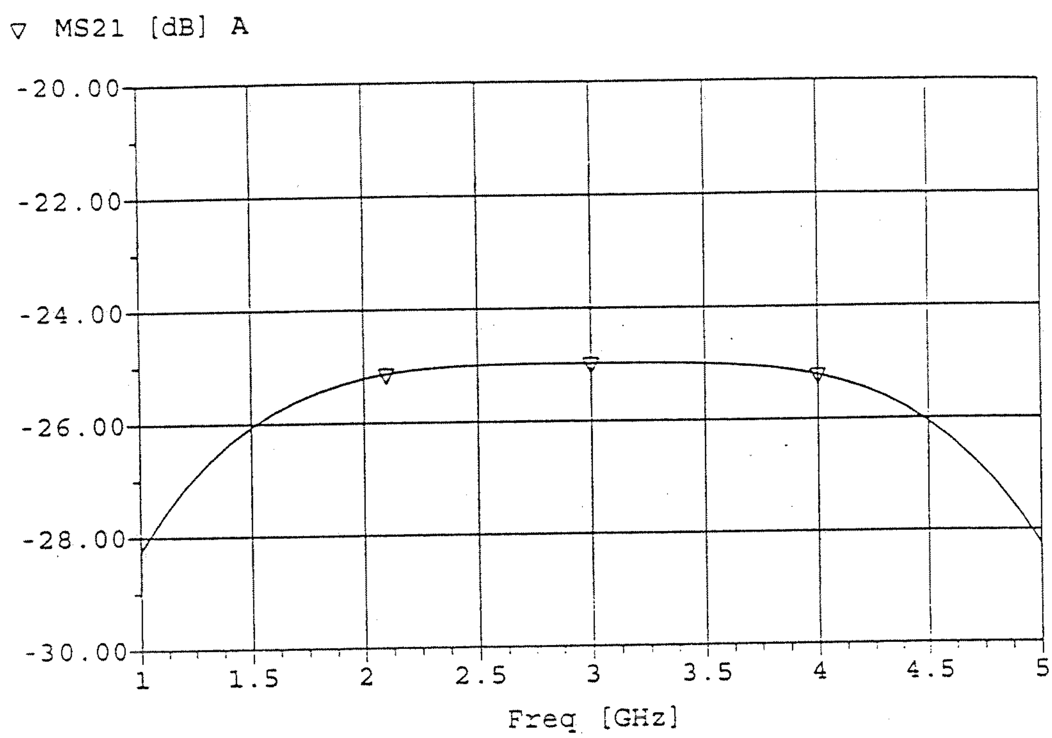
$$Z_{0e}^1 = Z_{0e}^3 = Z_0\sqrt{\frac{1+C_1}{1-C_1}} = 50.35\,\Omega \checkmark$$

$$Z_{0o}^1 = Z_{0o}^3 = Z_0\sqrt{\frac{1-C_1}{1+C_1}} = 49.65\,\Omega \checkmark$$

$$Z_{0e}^2 = Z_0\sqrt{\frac{1+C_2}{1-C_2}} = 53.65\,\Omega \checkmark$$

$$Z_{0o}^2 = Z_0\sqrt{\frac{1-C_2}{1+C_2}} = 46.60\,\Omega \checkmark$$

The coupling vs. frequency is plotted on the following page.



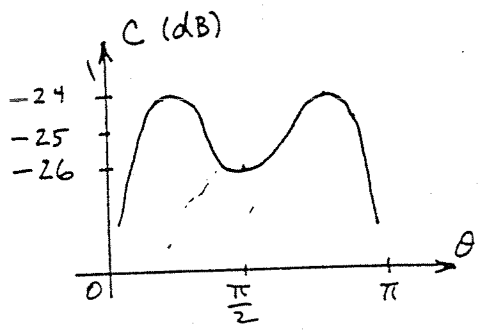
(7.25) $N = 3$, $C = 25\,dB$, $Z_0 = 50\Omega$, 1 dB EQUAL RIPPLE

From (7.90) and Example 7.8,

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \tfrac{1}{2}C_2\right]$$

$$= C_1 \sin 3\theta + (C_2 - C_1)\sin\theta$$

Ordinarily, we would try to equate this to a Chebyshev polynomial, but this will not work here because C is a polynomial in $\sin\theta$. The desired response, based on the above form of C, is as shown:



So at $\theta = \pi/2$, $C_0 = -26\,dB = 0.0501$
$= C_2 - 2C_1$

Then there is only one parameter to determine, from the ripple level. We can easily do this by trial-and-error. The maximum value of C is $C_{MAX} = -24\text{dB} = 0.0631$. So we compute C vs. θ, for various values of C_1, with $C_2 = 0.0501 + 2C_1$, using a simple computer program, and pick out C_{MAX}:

C_1	C_{MAX}	
0.01	0.0520	
0.02	0.0638	
0.019	0.0625	
0.0195	0.0632	← close to desired ripple

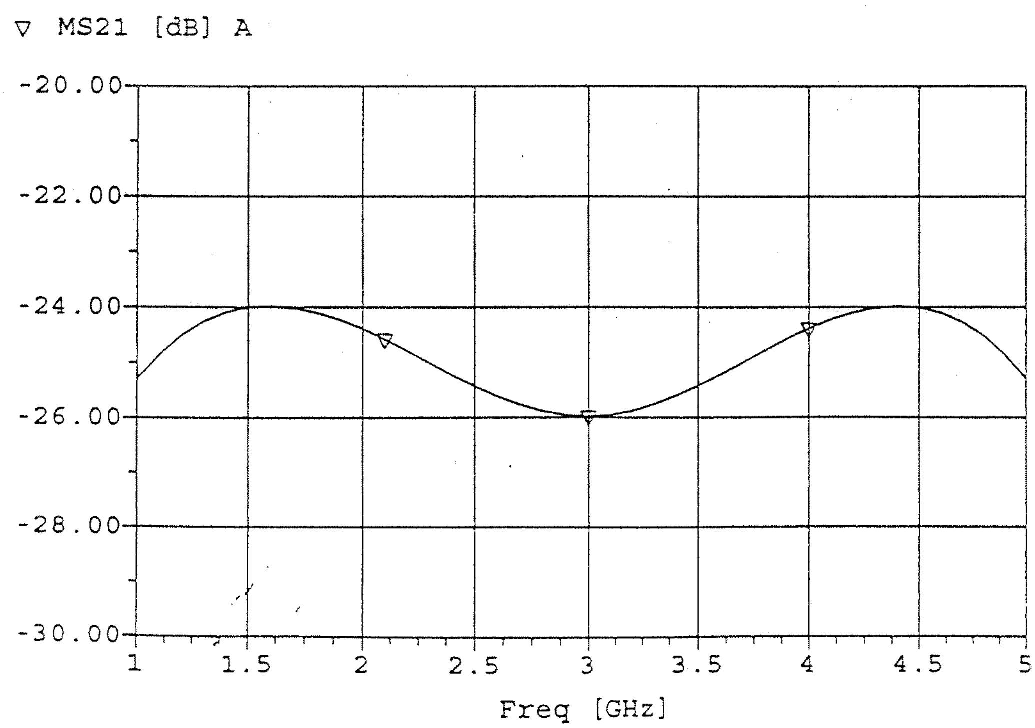
Thus, $C_1 = 0.0195$; $C_2 = 0.0891$

Then, $Z_{0e}^1 = 50.98\Omega$; $Z_{0o}^1 = 49.03\Omega$

$Z_{0e}^2 = 54.67\Omega$; $Z_{0o}^2 = 45.73\Omega$

$Z_{0e}^3 = 50.98\Omega$; $Z_{0o}^3 = 49.03\Omega$

The coupling vs frequency is plotted below. Note that the bandwidth is much greater than that of Problem 7.24.



7.26 From (7.98) and (7.99) we can show that,

$$Z_{e4} = Z_o \sqrt{\frac{1+C}{1-C}}$$

Equating this to (7.97a) gives:

$$Z_o \sqrt{\frac{1+C}{1-C}} = \frac{Z_{oo} + Z_{oe}}{3 Z_{oo} + Z_{oe}} = \frac{1 + \frac{Z_{oe}}{Z_{oo}}}{3 + \frac{Z_{oe}}{Z_{oo}}} Z_{oe}$$

Now solve (7.99) for Z_{oe} in terms of Z_{oo}:

$$3C(Z_{oe}^2 + Z_{oo}^2) + 2C Z_{oe} Z_{oo} = 3(Z_{oe}^2 - Z_{oo}^2)$$

$$3(C-1) Z_{oe}^2 + 2C Z_{oe} Z_{oo} + 3(C+1) Z_{oo}^2 = 0$$

$$Z_{oe} = \frac{-2C Z_{oo} \pm \sqrt{4C^2 Z_{oo}^2 - 36(C^2-1) Z_{oo}^2}}{6(C-1)} = \frac{-C - \sqrt{9-8C^2}}{3(C-1)} Z_{oo}$$

(choose negative root since Z_{oe} and Z_{oo} are positive, and $C<1$)
Substituting for Z_{oe}/Z_{oo} in the above expression gives,

$$Z_o \sqrt{\frac{1+C}{1-C}} = \frac{2C-3-\sqrt{9-8C^2}}{8C-9-\sqrt{9-8C^2}} Z_{oe}$$

or

$$Z_{oe} = Z_o \sqrt{\frac{1+C}{1-C}} \frac{[8C-9-\sqrt{9-8C^2}][2C-3+\sqrt{9-8C^2}]}{(2C-3)^2 - (9-8C^2)}$$

$$= Z_o \sqrt{\frac{1+C}{1-C}} \frac{(24C^2 - 42C + 18) + 6(C-1)\sqrt{9-8C^2}}{12C(C-1)}$$

$$= Z_o \sqrt{\frac{1+C}{1-C}} \frac{4C-3+\sqrt{9-8C^2}}{2C} \quad \checkmark$$

To find Z_{oo}, simply replace C by $-C$ (because of symmetry of (7.98) and (7.99):

$$Z_{oo} = Z_o \sqrt{\frac{1-C}{1+C}} \frac{4C+3-\sqrt{9-8C^2}}{2C} \quad \checkmark$$

7.27 $f = 5\,GHz$, $\epsilon_r = 10$, $d = 1\,mm$

$C = 10^{-3/20} = 0.708$

assume $Z_0 = 50\,\Omega$ (not stated in problem!)

From (7.100) we have,

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1-C}{1+C}}} Z_0 = 176.4\,\Omega$$

$$Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1+C}{1-C}}} Z_0 = 52.5\,\Omega$$

(These results are very approximate; SuperCompact gives $Z_{0e} = 121\,\Omega$ and $Z_{0o} = 21\,\Omega$ for this design)

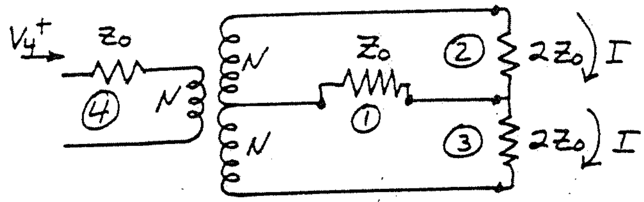
From Figure 7.30,

$$S/d \cong 0.075 \Rightarrow S = 0.075\,mm$$

$$W/d \cong 0.07 \Rightarrow W = 0.07\,mm$$

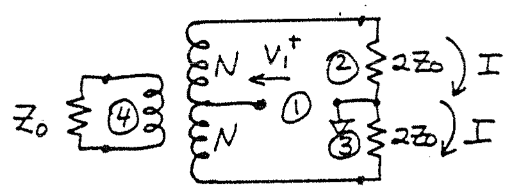
(SuperCompact gives 0.071 mm and 0.075 mm for S and W, respectively, starting with the values calculated above for Z_{0e} and Z_{0o}. These are reasonably close to our values for S and W, even though Z_{0e}, Z_{0o} are not close to the SuperCompact values.)

(7.28) First consider an incident wave at port 4, with the other ports matched:



There is no voltage drop across the resistor at port 1, so $S_{41}=S_{14}=0$. The load impedance across the secondary is $4Z_0$, so the input impedance at port 4 is $Z_{in}=(N/2N)^2(4Z_0)=Z_0$, so $S_{44}=0$. The voltages at ports 2 and 3 have the same magnitude, but opposite signs (relative to the center terminal). Power conservation then gives $S_{24}=S_{42}=1/\sqrt{2}$; $S_{34}=S_{43}=-1/\sqrt{2}$.

Now consider an incident wave at port 1, with matched loads at the other ports:



This excites the transformer in an "odd mode", so $V_4=0$. (consistent with $S_{41}=0$). Ports 2 and 3 are now equally excited, so $S_{21}=S_{12}=S_{31}=S_{13}=1/\sqrt{2}$. The input impedance at port 1 is Z_0, so $S_{11}=0$.

Finally, the unitary properties of the S-matrix for a lossless network lead to $S_{22}=S_{33}=0$. Thus, the S-matrix is similar in form to (7.101).

7.29 From (7.101) the [S] matrix of a 180° (3dB) hybrid is,

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

For V_1 at port 1 and V_4 at port 4, the output voltages are (note: hybrid is matched, so $V_1 = V_1^+$, $V_4 = V_4^+$)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ 0 \\ 0 \\ V_4 \end{bmatrix}$$

$$= \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 \\ V_1 - V_4 \\ V_1 + V_4 \\ 0 \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \quad \text{(difference)} \\ \checkmark \quad \text{(sum)} \\ \checkmark \end{matrix}$$

7.30 $\alpha = \beta = \sqrt{2}/2$ for $C = 3\,dB$

From (7.115a), $\beta = \dfrac{2\sqrt{k}}{k+1} \Rightarrow k = 0.1716$ ✓

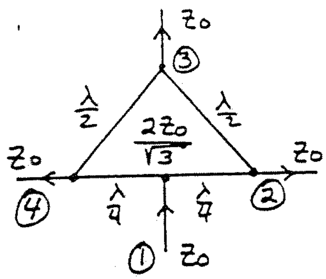
From (7.115b), $\alpha = \dfrac{1-k}{1+k} \Rightarrow k = 0.1716$ ✓

Then,

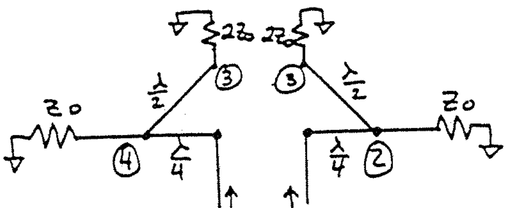
$$Z_{oe}(0) = Z_{oo}(0) = Z_o = 50\,\Omega$$
$$Z_{oe}(L) = Z_o/k = 291\,\Omega$$
$$Z_{oo}(L) = k Z_o = 8.6\,\Omega$$

a Klopfenstein taper can be used for these taper variations.

7.31



First, let $V_1^+ = 1v$ at port 1, with matched loads at other ports. Then we can bisect the network as follows:



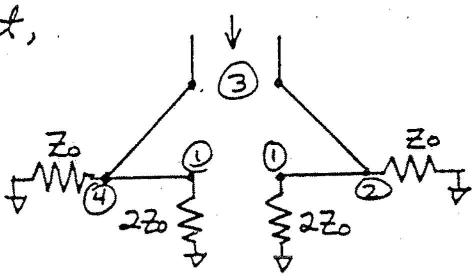
The input impedance of one of these halves is,

$\left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{(Z_0 + 2Z_0)}{2Z_0^2} = 2Z_0$, so $Z_{in}^{(1)} = Z_0$, and $S_{11} = 0$. ✓

Because of the $\lambda/2$ line, the voltage magnitude at port 3 is equal to the voltage magnitude at port 4. By power conservation, $P_2 = P_4 = P_3 = P_{in}/3$. Thus,

$S_{41} = S_{21} = 1/\sqrt{3} \underline{/-90^\circ}$, $S_{31} = 1/\sqrt{3} \underline{/-270^\circ}$. $(|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1)$

Next, let $V_3^+ = 1v$ at port 3, with other ports matched, and bisect,



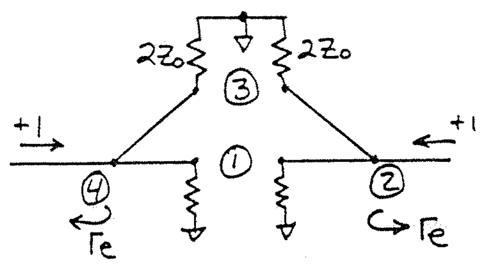
The input impedance of one of these halves is

$Z_0 \| \left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{1}{2Z_0} = Z_0 \| \frac{2Z_0}{3} = \frac{2Z_0}{5}$, so $Z_{in}^{(3)} = Z_0/5$, and $S_{33} = -2/3$.

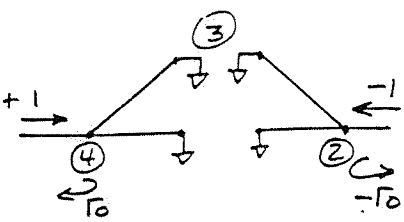
So the power delivered to each half is $\frac{1}{2} P_{inc}(1 - |S_{33}|^2) = 5/18\, P_{inc}$. Of this, 2/5 goes to port 4 and 3/5 goes to port 1.

So, $S_{43} = S_{23} = 1/3 \underline{/-180^\circ}$. The total power to port 1 is then $1/3\, P_{inc}$, so $S_{13} = 1/\sqrt{3} \underline{/-270^\circ}$. (Then $|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = \frac{1}{3} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} = 1$)

Now drive ports 2 and 4 with even and odd excitations:



EVEN MODE



ODD MODE

$Z_{in}^{e} = 2Z_0 \| \frac{2Z_0}{3} = Z_0/2$

$\Gamma_e = \frac{1/2 - 1}{1/2 + 1} = -1/3$

$Z_{in}^{o} = 0$

$\Gamma_o = -1$

Then, $S_{22} = S_{44} = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{1}{2}(-1/3 - 1) = -2/3$

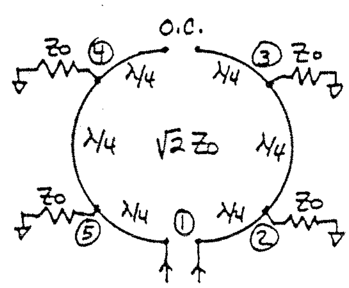
$S_{24} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1}{2}(-1/3 + 1) = 1/3$

So the complete S-matrix is,

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{3}\angle{-90} & 1/\sqrt{3}\angle{-270} & 1/\sqrt{3}\angle{-90} \\ 1/\sqrt{3}\angle{-90} & -2/3 & -1/3 & 1/3 \\ 1/\sqrt{3}\angle{-270} & -1/3 & -2/3 & -1/3 \\ 1/\sqrt{3}\angle{-90} & 1/3 & -1/3 & -2/3 \end{bmatrix} \checkmark$$

This checks with an analysis using SuperCompact.

7.32 Let $V_1^+ = 1\angle 0$



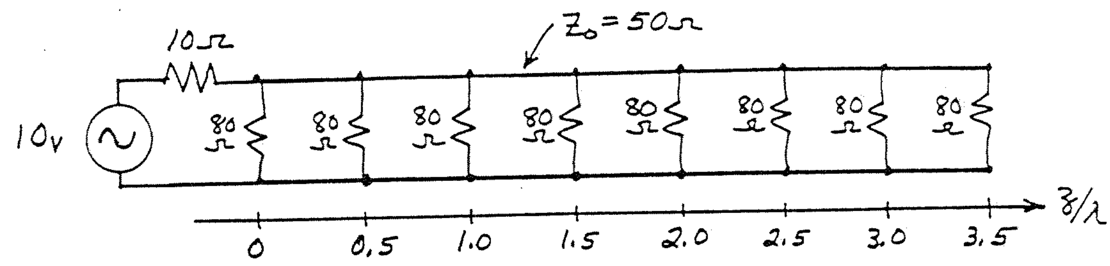
Bisecting the network places an effective short circuit at ports 3 and 4, due to the $\lambda/4$ O.C. stubs. Thus $S_{41} = S_{31} = 0$. Then there is an effective open circuit in parallel with the Z_0 loads at ports 2 and 5. So the input impedance of one of the halves is $(\sqrt{2}Z_0)^2/Z_0 = 2Z_0$. The total input impedance at port 1 is then Z_0, so $S_{11} = 0$, and the input power divides evenly to ports 2 and 5. Thus,

$V_1^- = 0;\ V_2^- = V_5^- = 0.707\angle{-90°};\ V_3^- = V_4^- = 0.$

CHECK: $|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1$ ✓

Chapter 8

8.1



The input impedance is $80\Omega/8 = 10\Omega$, so $V(0) = 5v$. In between the resistors, we must use the transmission line equations. For $0 < z < \lambda/2$, we have,

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x}) \quad , \text{ where } x = z - \lambda/2$$

$$V(z=0) = V(x = -\lambda/2) = V^+(-1-\Gamma) = 5v$$

Thus,

$$V(x) = \frac{-5}{1+\Gamma}(e^{-j\beta x} + \Gamma e^{j\beta x}) \quad , \quad \Gamma = \frac{80/7 - 50}{80/7 + 50} = -0.628$$

$$|V(z)| = \frac{5}{1+\Gamma}\left|1 + \Gamma e^{-2j\beta z}\right| \quad \text{for } 0 < z < \lambda/2$$

So $\quad |V(\lambda/2)| = +5v \quad (V(\lambda/2) = -5v)$. The peak occurs for $z = \lambda/4$:

$$|V(z = \lambda/4)| = \frac{5}{1+\Gamma}(1-\Gamma) = 5\,\frac{Z_0}{Z_L} = \frac{5(50)}{80/7} = 21.8v$$

Intermediate values can also be calculated in the same manner. For $\lambda/2 < z < \lambda$ we repeat the above procedure, but with $Z_L = 80/6$. Thus we have,

$|V(z = 0.75\lambda)| = 18.8v$ ✓

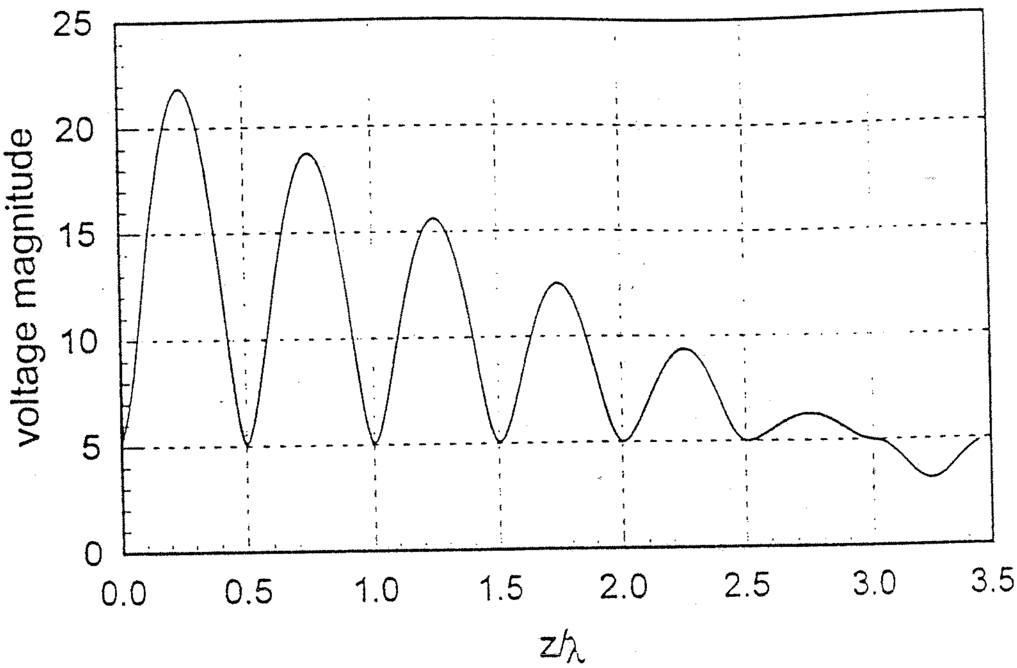
$|V(z = 1.75\lambda)| = 12.5v$ ✓

$|V(z = 2.25\lambda)| = 9.37v$ ✓

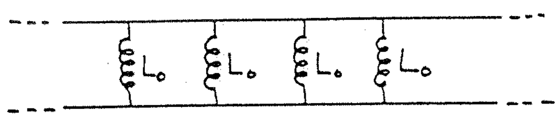
$|V(z = 2.75\lambda)| = 6.25v$ ✓

$|V(z = 3.25\lambda)| = 3.12v$ ✓

$|V(z)|$ is plotted on the following page.



8.2



$Z_o = 100\Omega$; $d = 1.0\,cm$

$k = k_o$; $L_o = 3\,nH$

let $\theta = k_o d$, $\quad b = \frac{-Z_o}{\omega L_o} = \frac{-Z_o}{c k_o L_o} = \frac{-111}{k_o}$

From (8.9) a passband occurs when,

$$|\cos\beta d| = \left|\cos\theta - \frac{b}{2}\sin\theta\right| \leq 1 \,,$$

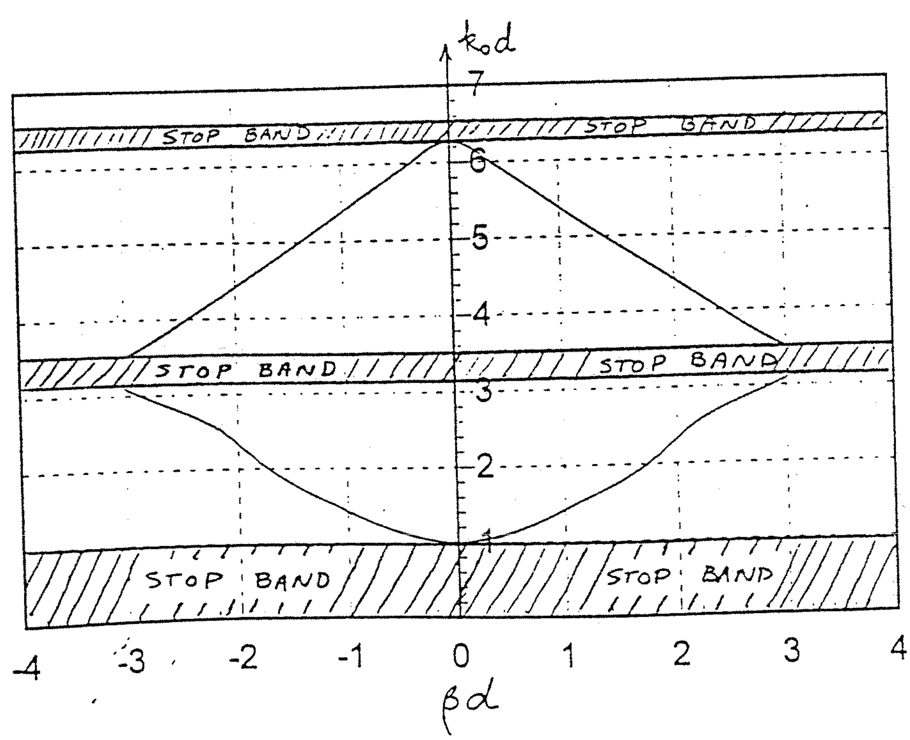
and a stopband occurs when,

$$\cosh\alpha d = \left|\cos\theta - \frac{b}{2}\sin\theta\right| \geq 1$$

so we can compute the following data table:

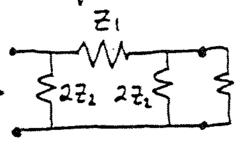
	k_0	θ	$\cos\theta - b/2 \sin\theta$	βd (rad)
STOP	10	5.7°	1.55	—
	30	17.2°	1.50	—
	100	57.3°	1.007	—
PASS	110	63.0°	0.903	0.444
	150	85.9°	0.441	1.11
	200	114.6°	-0.164	1.74
	250	143.2°	-0.535	2.14
	300	171.9°	-0.964	2.87
	310	177.6°	-0.992	3.02
STOP	320	183.3°	-1.02	—
	340	194.5°	-1.009	—
PASS	350	200.5°	-0.992	3.02
	360	206.3°	-0.965	2.87
	400	229.2°	-0.758	2.43
	450	257.8°	-0.332	1.91
	500	286.5°	0.177	1.39
	550	315.1°	0.637	0.88
	600	343.8°	0.934	0.36
	625	358.1°	0.996	0.08

These passbands and stopbands are plotted below:



(8.3) $Z_{i\pi}$ can easily be derived from $Z_i = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{B}{C}}$.

Verify as follows:

$Z_{in} \Rightarrow$ 

$$Z_i = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1+Z_1/4Z_2}} = \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}$$

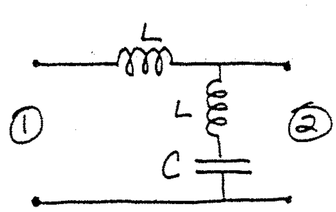
let $$Z = 2Z_2 \| Z_i = \frac{\frac{2Z_2^2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}}{2Z_2 + \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}} = \frac{2Z_2\sqrt{Z_1}}{2\sqrt{Z_2+Z_1/4}+\sqrt{Z_1}}$$

$$Z_{in} = 2Z_2 \| (Z_1+Z) = \frac{\frac{4Z_1Z_2\sqrt{Z_2+Z_1/4}+2Z_2\sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4}+\sqrt{Z_1}}}{2Z_2 + \frac{2Z_1\sqrt{Z_2+Z_1/4}+\sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4}+\sqrt{Z_1}}}$$

$$= \frac{4Z_1Z_2\sqrt{Z_2+Z_1/4}+Z_2\sqrt{Z_1}(2Z_2+Z_1)(2)}{4Z_2\sqrt{Z_2+Z_1/4}+2Z_2\sqrt{Z_1}+2Z_1\sqrt{Z_2+Z_1/4}+\sqrt{Z_1}(2Z_2+Z_1)}$$

$$= \frac{Z_2\left[\sqrt{Z_1}(2Z_2+Z_1)+2Z_1\sqrt{Z_2+Z_1/4}\right]}{2\sqrt{Z_1}(Z_2+Z_1/4)+Z_1\sqrt{Z_2+Z_1/4}+2Z_2\sqrt{Z_2+Z_1/4}}$$

$$= \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}} = Z_i \quad \checkmark$$

(8.4) 

$$\frac{1}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega C}{1-\omega^2 LC}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1-2\omega^2 LC}{1-\omega^2 LC} & j\omega L \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} \quad \checkmark$$

From (8.27),

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{j\omega L(1-2\omega^2 LC)}{j\omega C}} = \sqrt{\frac{L}{C}(1-2\omega^2 LC)} \quad \checkmark$$

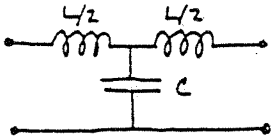
$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{j\omega L}{j\omega C(1-2\omega^2 LC)}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{L}{C(1-2\omega^2 LC)}}$$

From (8.31); $\cosh\gamma = \sqrt{AD} = \sqrt{\dfrac{1-2\omega^2 LC}{1-\omega^2 LC}}$.

8.5 $R_o = 50\Omega$, $f_c = 50$ MHz, $f_\infty = 52$ MHz, LOW-PASS

Design equations from Table 8.2:

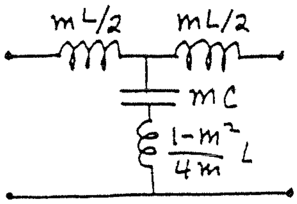
CONSTANT-k SECTION;



$L = 2R_o/\omega_c = 3.18\times10^{-7}$ H ; $L/2 = 159$ nH

$C = 2/\omega_c R_o = 127.$ pF

m-DERIVED SECTION:



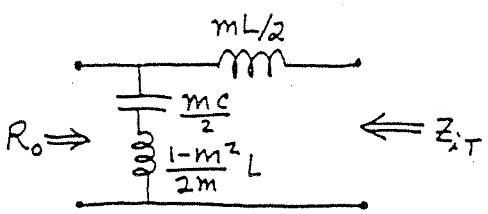
$m = \sqrt{1-(f_c/f_\infty)^2} = 0.275$

$\dfrac{mL}{2} = 43.7$ nH

$mC = 34.9$ pF

$\dfrac{1-m^2}{4m}L = 267.$ nH

MATCHING SECTIONS: ($m = 0.6$)

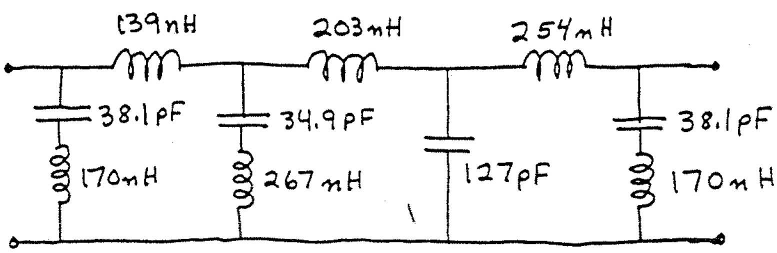


$\dfrac{mL}{2} = 95.4$ nH

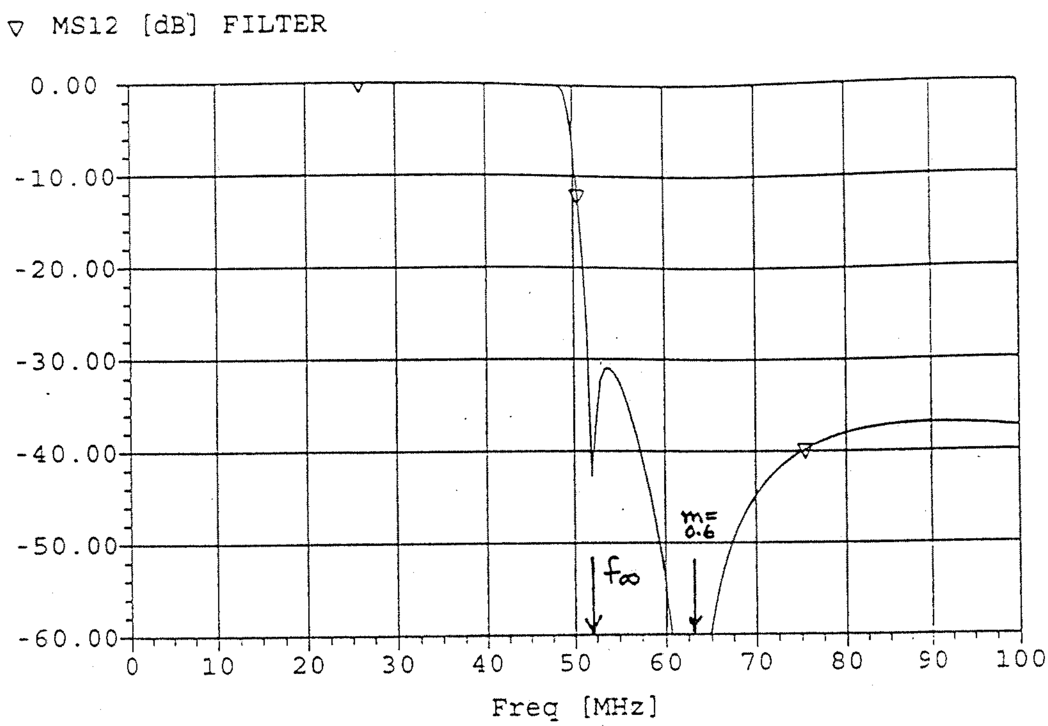
$\dfrac{mC}{2} = 38.1$ pF

$\dfrac{1-m^2}{2m}L = 170.$ nH

COMPLETE FILTER:



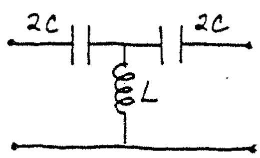
The calculated response is shown on the following page.



(8.6) $R_o = 75\Omega$, $f_c = 50$ MHz , $f_\infty = 48$ MHz , HIGH-PASS

Design equations are given in Table 8.2

CONSTANT-k SECTION:

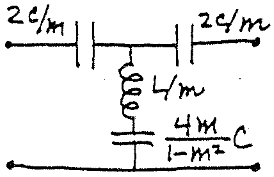


$L = R_o/2\omega_c = 119.$ nH ✓

$C = 1/2R_o\omega_c = 21.2$ pF ✓

$2C = 42.4$ pF ✓

m-DERIVED SECTION:



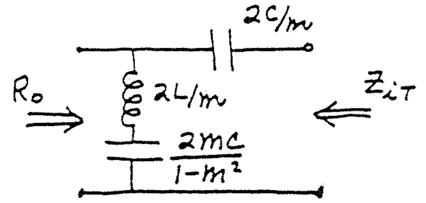
$m = \sqrt{1-(f_\infty/f_c)^2} = 0.280$ ✓

$\frac{2C}{m} = 151.$ pF ✓

$L/m = 425.$ nH ✓

$\frac{4mC}{1-m^2} = 25.8$ pF ✓

MATCHING SECTION: (m=0.6)

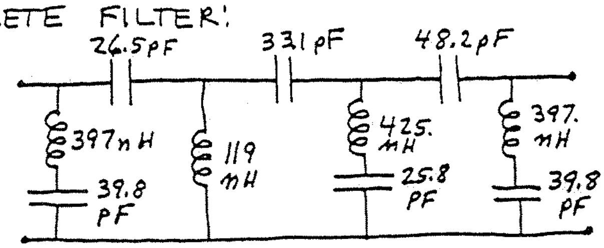


$\frac{2C}{m}$ = 70.7 pF ✓

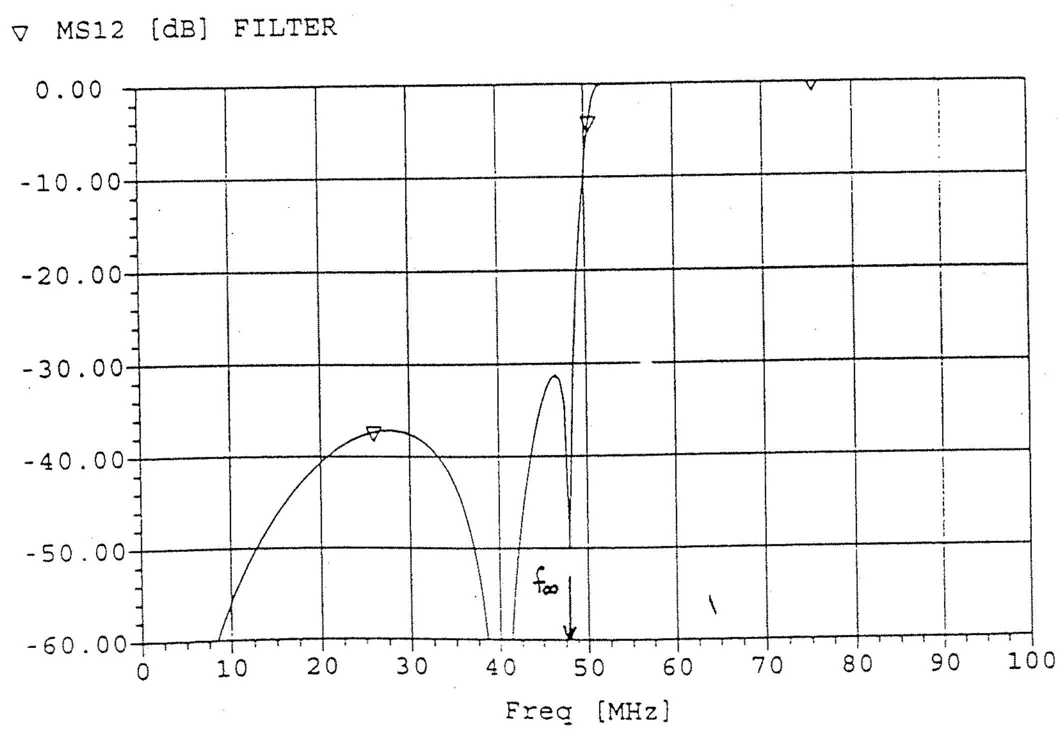
$\frac{2L}{m}$ = 397. nH. ✓

$\frac{2mC}{1-m^2}$ = 39.8 pF ✓

COMPLETE FILTER:

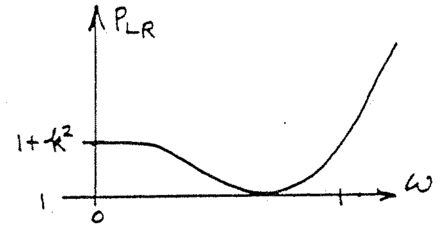


The calculated filter response is shown below.



8.7 From (8.61),

$$P_{LR} = 1 + k^2 T_N^2(\omega) = 1 + k^2(2\omega^2 - 1)^2 \quad \text{for } N = 2$$



$P_{LR} = 1 = 0\,dB$

$P_{LR} = 1 + k^2 = 1\,dB = 1.259$

so $k = \pm 0.509$

(We must choose $k = -0.509$, otherwise L, C are not real.)

Then from (8.63),

$$R = 1 + 2k^2 - 2k\sqrt{1+k^2} = 2.66 \checkmark$$

We also have that,

$$4k^2 = \frac{1}{4R} L^2 C^2 R^2 \implies L = \frac{-4k}{C\sqrt{R}}$$

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$$

$$-16k^2 R = R^2 C^2 + \frac{16k^2}{C^2 R} + 8kR\sqrt{R}$$

$$R^2 C^4 + (16k^2 R + 8kR\sqrt{R})C^2 + \frac{16k^2}{R} = 0$$

$$7.08C^4 - 6.64C^2 + 1.56 = 0$$

Thus,

$$C = 0.685 \checkmark$$

$$L = 1.822 \checkmark$$

(R, L, C check with results given in Matthai, Young, and Jones.)

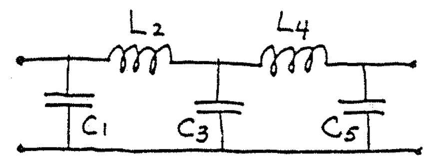
8.8 $f_0 = 3$ GHz, LOW-PASS, M.F., $Z_0 = 75\,\Omega$, $\alpha = 20$ dB at 5 GHz

Following Example 8.4:

$$\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{5}{3} - 1 = 0.667,$$

so from Figure 8.26 we see that $N=5$ will give $\alpha > 20$ dB. Then from Table 8.3, the LP prototype values are,

$g_1 = 0.618$
$g_2 = 1.618$
$g_3 = 2.000$
$g_4 = 1.618$
$g_5 = 0.618$



Use (8.67) to scale cutoff frequency and impedance:

$$C_1 = \frac{g_1}{R_0 \omega_c} = 0.437 \text{ pF} \checkmark$$

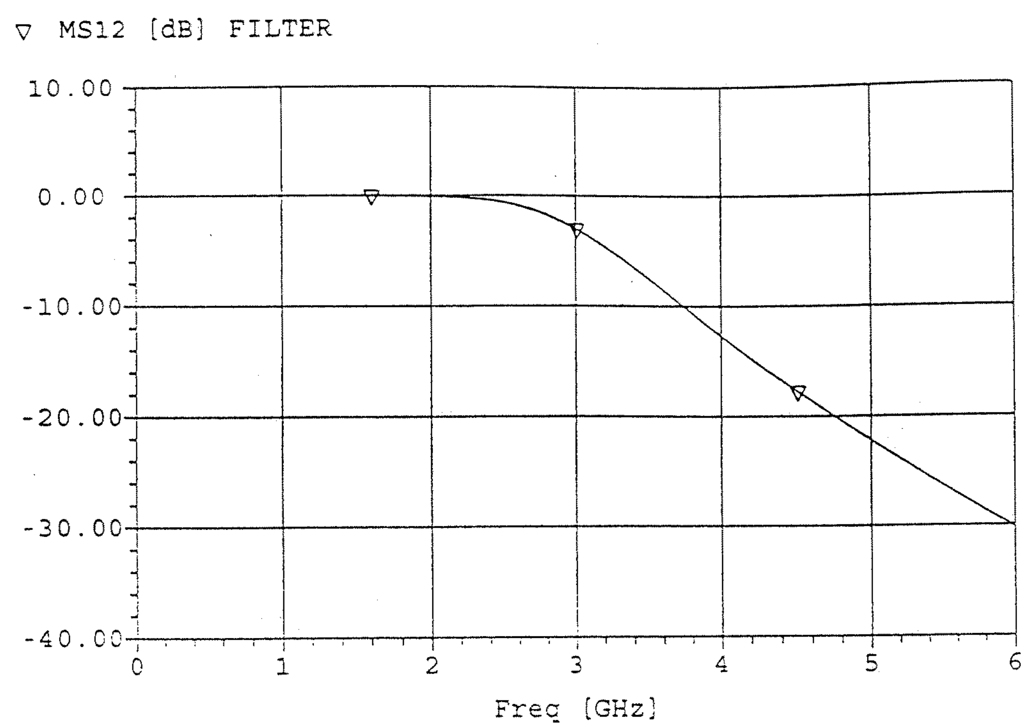
$$L_2 = \frac{R_0 g_2}{\omega_c} = 6.44 \text{ nH} \checkmark$$

$$C_3 = \frac{g_3}{R_0 \omega_c} = 1.41 \text{ pF} \checkmark$$

$$L_4 = \frac{R_0 g_4}{\omega_c} = 6.44 \text{ nH} \checkmark$$

$$C_5 = \frac{g_5}{R_0 \omega_c} = 0.437 \text{ pF} \checkmark$$

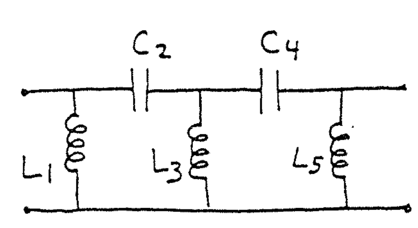
The calculated filter response is shown on the following page. Note that the insertion loss at 5 GHz is more than 20dB.



(8.9) $f_0 = 1\,GHz$, HIGH PASS, 3dB E.R., $N=5$, $Z_0 = 50\,\Omega$

at $f = 0.6\,GHz$, $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{1}{.6} - 1 = 0.667$, so from Figure 8.27b, the attenuation for $N=5$ should be about 41 dB. From Table 8.4 the prototype values are,

$g_1 = 3.4817$
$g_2 = 0.7618$
$g_3 = 4.5381$
$g_4 = 0.7618$
$g_5 = 3.4817$



Impedance and frequency scaling using (8.70):

$$L_1 = \frac{Z_0}{\omega_c g_1} = 2.28\ mH \quad \checkmark$$

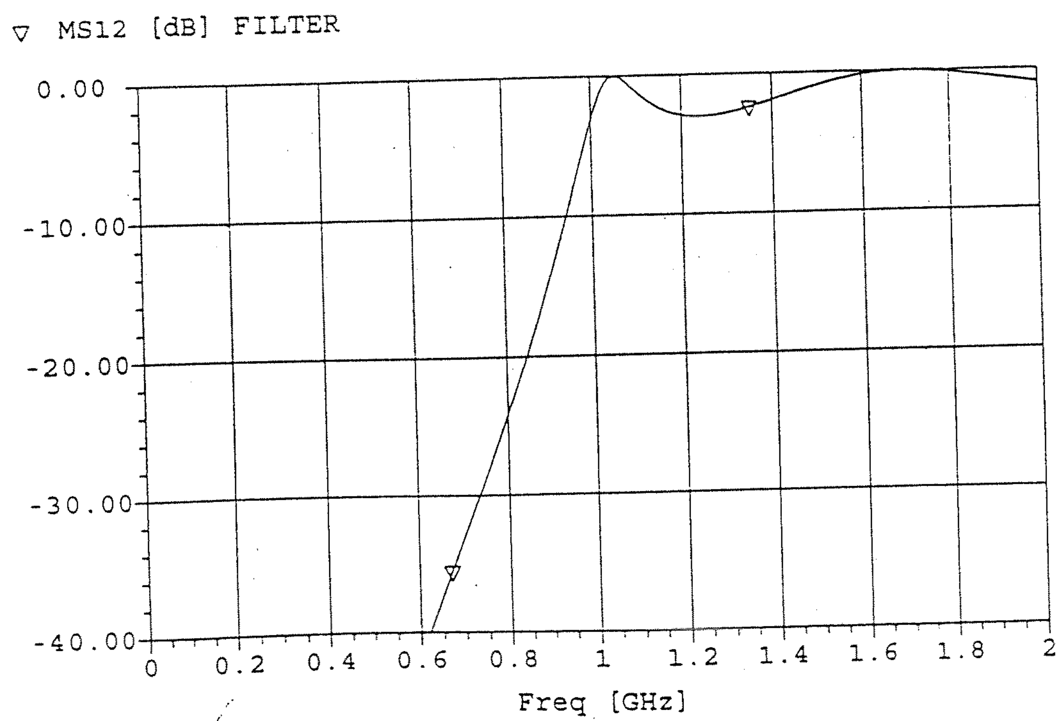
$$C_2 = \frac{1}{Z_0 \omega_c g_2} = 4.18\ pF \quad \checkmark$$

$$L_3 = \frac{Z_0}{\omega_c g_3} = 1.75\ mH \quad \checkmark$$

$$C_4 = \frac{1}{Z_0 \omega_c g_4} = 4.18\ pF \quad \checkmark$$

$$L_5 = \frac{Z_0}{\omega_c g_5} = 2.28\ mH \quad \checkmark$$

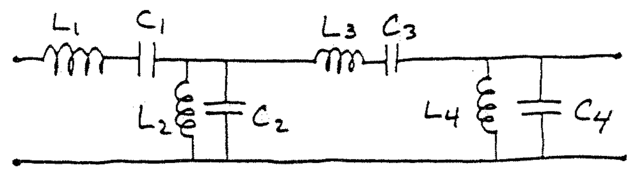
The calculated filter response is shown below. Note that the insertion loss at $f = 0.6\ GHz$ is just a bit more than 40dB.



8.10 $f_o = 2\,GHz$, B.P., M.F.G.D., $\Delta = 0.05$, $N = 4$, $Z_o' = 50\Omega$

From Table 8.5 the prototype element values are,

$g_1 = 1.0598$
$g_2 = 0.5116$
$g_3 = 0.3181$
$g_4 = 0.1104$



From Table 8.6 and (8.64) the scaled element values are,

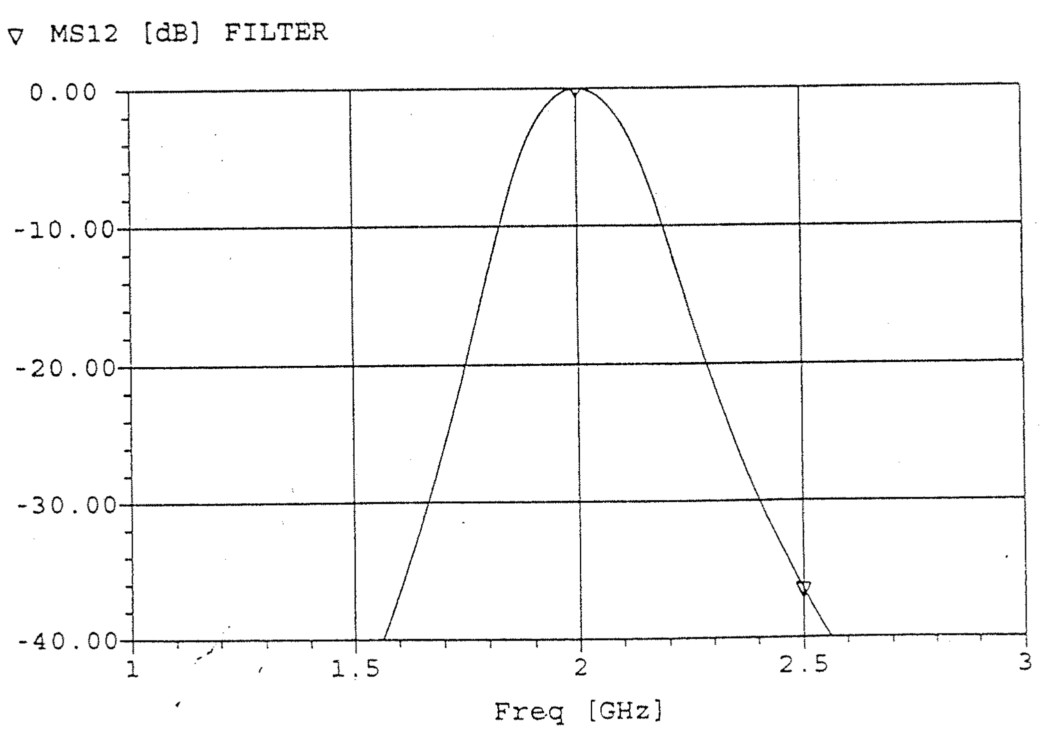
$L_1 = \dfrac{g_1 Z_o}{\omega_o \Delta} = 84.3\,nH$ ✓ $\qquad C_1 = \dfrac{\Delta}{\omega_o g_1 Z_o} = 0.075\,pF$ ✓

$L_2 = \dfrac{\Delta Z_o}{\omega_o g_2} = 0.388\,nH$ ✓ $\qquad C_2 = \dfrac{g_2}{\omega_o \Delta Z_o} = 16.3\,pF$ ✓

$L_3 = \dfrac{g_3 Z_o}{\omega_o \Delta} = 25.3\,nH$ ✓ $\qquad C_3 = \dfrac{\Delta}{\omega_o g_3 Z_o} = 0.25\,pF$ ✓

$L_4 = \dfrac{\Delta Z_o}{\omega_o g_4} = 1.80\,nH$ ✓ $\qquad C_4 = \dfrac{g_4}{\omega_o \Delta Z_o} = 3.51\,pF$ ✓

The calculated filter response is shown below.



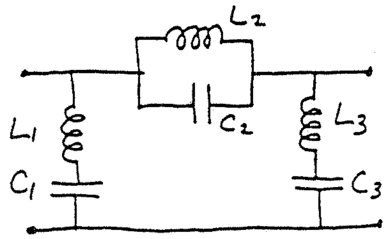
8.11 $f_0 = 3\text{GHz}$, $Z_0 = 75\Omega$, $N=3$, B.S., 0.5dB E.R.

First use (8.75) to transform 3.1 GHz to a L.P. prototype response frequency:

$$\omega \leftarrow \Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1} = 0.1\left(\frac{3.1}{3} - \frac{3}{3.1}\right)^{-1} = 1.52$$

So, $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.52$, and Figure 8.27a gives an attenuation of 11 dB for $N=3$. From Table 8.4, the prototype values are,

$g_1 = 1.5963$
$g_2 = 1.0967$
$g_3 = 1.5963$



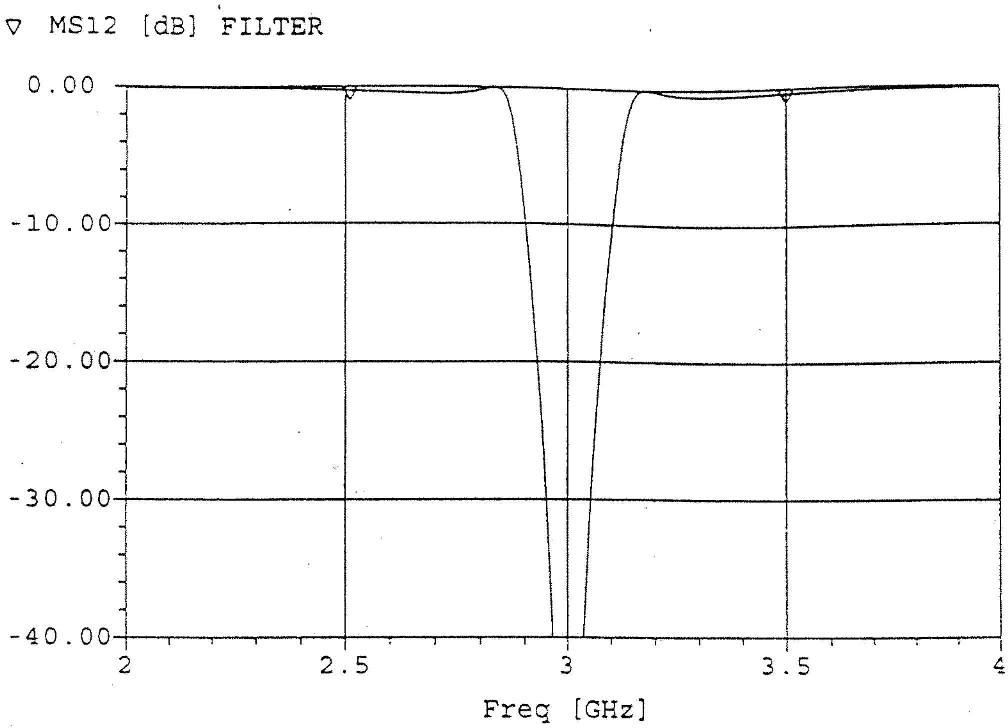
From Table 8.6 and (8.64) the scaled element values are,

$$L_1 = \frac{Z_0}{\omega_0 g_1 \Delta} = 24.9\ \text{nH} \checkmark \qquad C_1 = \frac{g_1 \Delta}{\omega_0 Z_0} = 0.113\ \text{pF} \checkmark$$

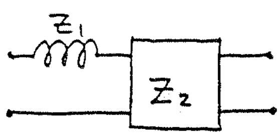
$$L_2 = \frac{g_2 \Delta Z_0}{\omega_0} = 0.436\ \text{nH} \checkmark \qquad C_2 = \frac{1}{Z_0 \omega_0 g_2 \Delta} = 6.45\ \text{pF} \checkmark$$

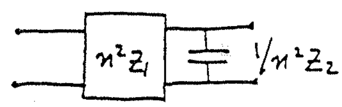
$$L_3 = \frac{Z_0}{\omega_0 g_3 \Delta} = 24.9\ \text{nH} \checkmark \qquad C_3 = \frac{g_3 \Delta}{Z_0 \omega_0} = 0.113\ \text{pF} \checkmark$$

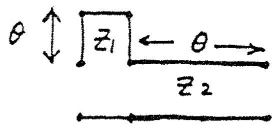
The calculated response for this filter is shown on the following page. Note that the insertion loss at 3.1 GHz is about 10dB.



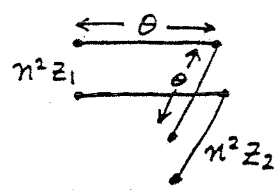
(8.12)







$Z_s = jZ_1 \tan\theta$



$Y_s = \frac{j}{n^2Z_2}\tan\theta$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jZ_1\tan\theta \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & jZ_2\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j(Z_1+Z_2)\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jn^2Z_2\sin\theta \\ \frac{j\sin\theta}{n^2Z_1} & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{j\tan\theta}{n^2Z_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & jn^2Z_1\sin\theta \\ \frac{j}{n^2}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\theta & \cos\theta \end{bmatrix}$$

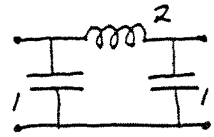
So these two matrices are equal if,

$$Z_1 + Z_2 = n^2 Z_1$$

or, $n^2 = 1 + Z_2/Z_1$ ✓

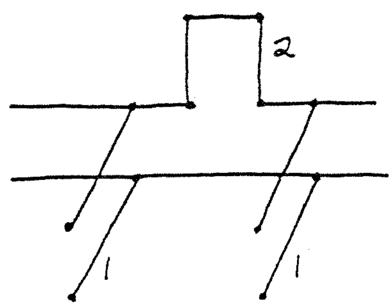
8.13 $f_0 = 6$ GHz, $N=3$, M.F., $Z_0 = 50\Omega$

From Table 8.3 the L.P. prototype is,

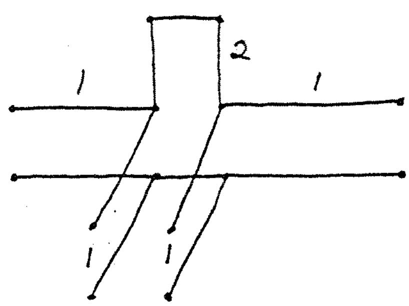


(choosing a π-circuit simplifies the problem)

Richards' transform:

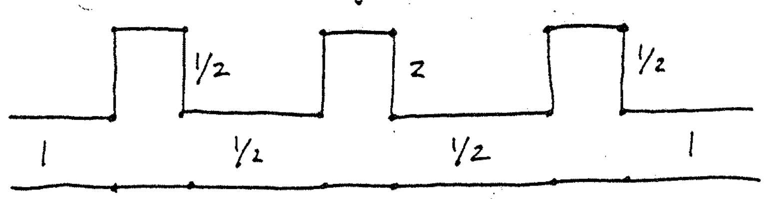


add unit elements at ends:

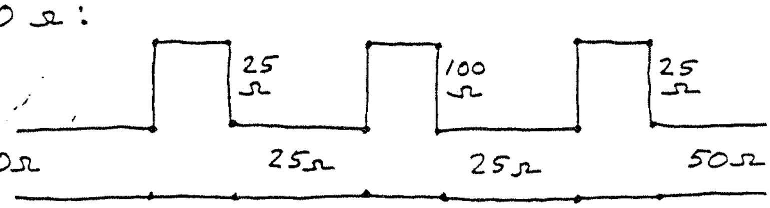


apply first Kuroda identity (twice):

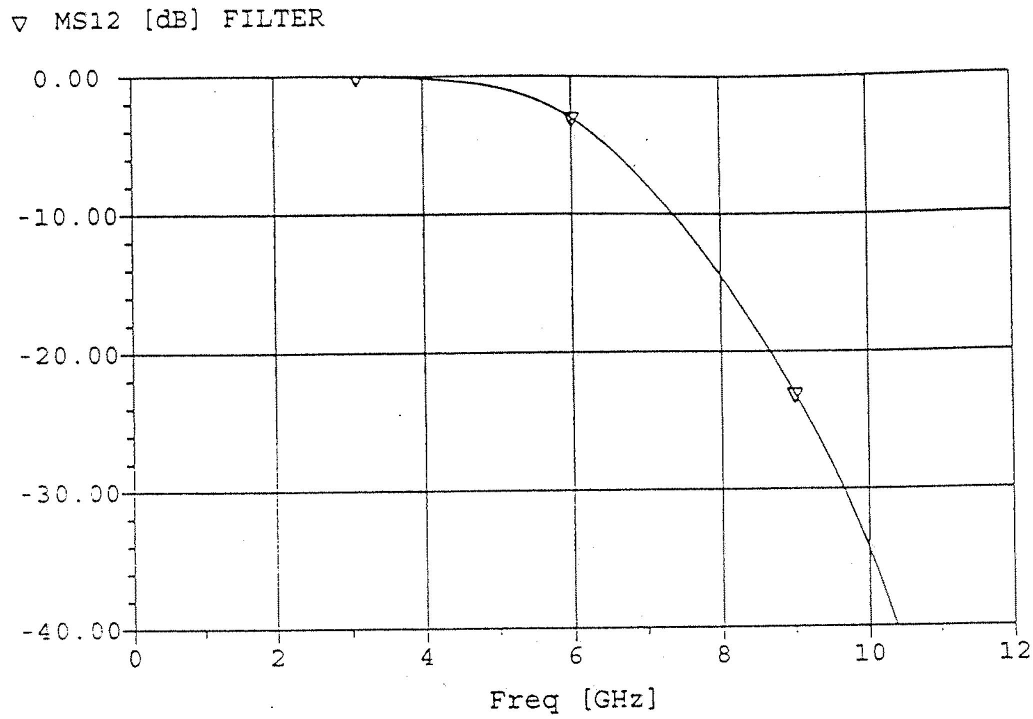
$Z_1 = 1$
$Z_2 = 1$
$n^2 = 1 + \frac{Z_2}{Z_1}$
$= 2$



Scale to 50Ω:



all line lengths and stub lengths are $\lambda/8$ long at 6 GHz. The calculated filter response is shown below.

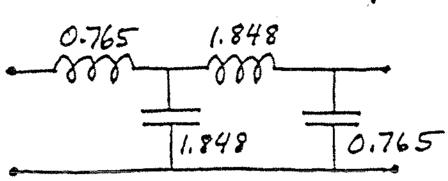


(8.14) $f_0 = 8$ GHz, $N=4$, L.P., M.F., $Z_0 = 50\,\Omega$

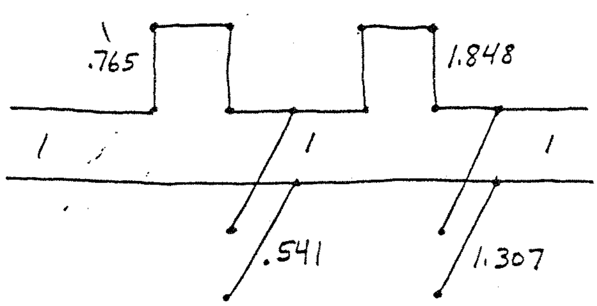
From Table 8.3 the L.P. prototype is,

$L = \frac{g_1 Z_0}{\omega}$

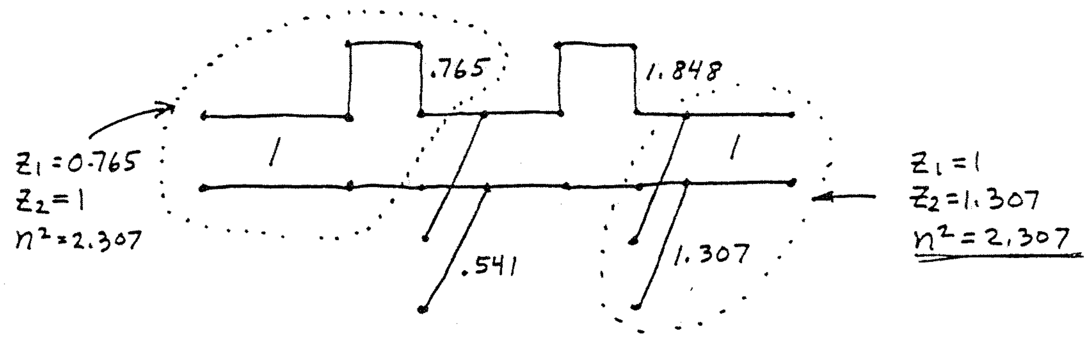
$Z_1 = j\omega L_1 \Rightarrow Z_1 = g_1 Z_0$



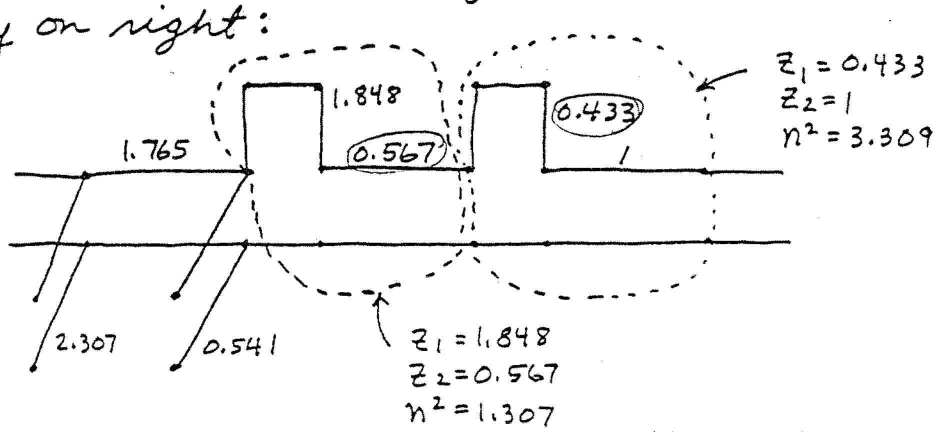
applying Richard's transform:



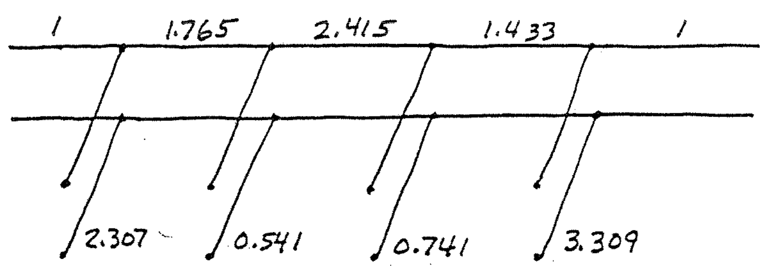
add unit elements:



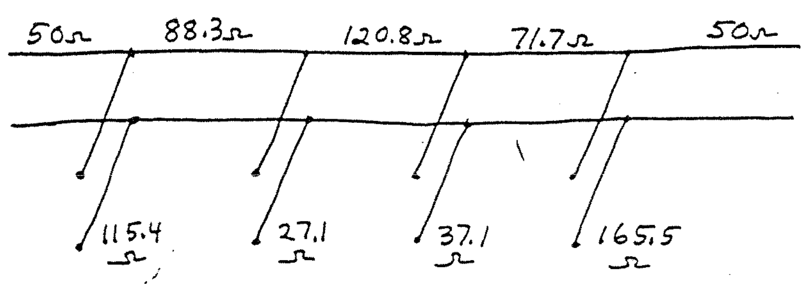
Use the second Kuroda identity on left; first Kuroda identity on right:



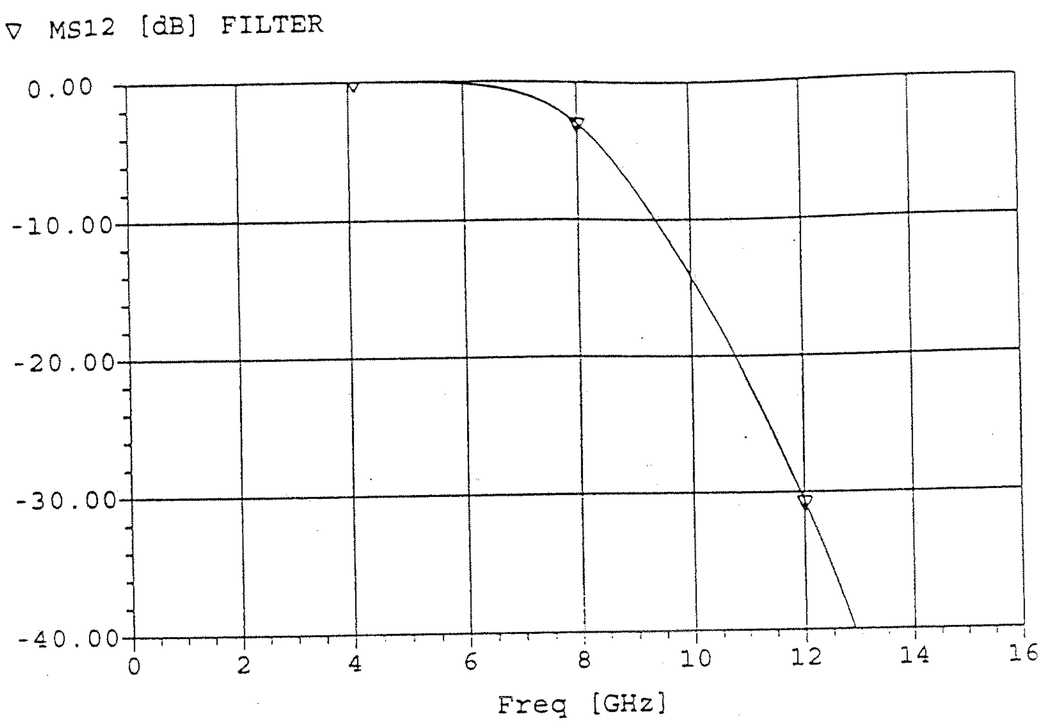
Now use the second Kuroda identity twice:



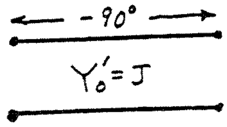
Scale to 50Ω:



all lines are $\lambda/8$ long at 8 GHz. The calculated filter response is shown on the following page.

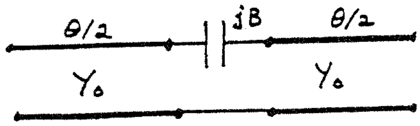

▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
0
2
4
6
8
10
12
14
16
Freq [GHz]

(8.15) Quarter-wave line ($-\lambda/4$ long, since $\theta < 0$)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix}$$

ADMITTANCE INVERTER:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 & -j/B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) & j\left(\frac{1}{Y_0}\sin\theta - \frac{1}{B}\cos^2\frac{\theta}{2}\right) \\ jY_0\left(\sin\theta + \frac{Y_0}{B}\sin^2\frac{\theta}{2}\right) & \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) \end{bmatrix} \checkmark$$

Equivalence requires the following conditions:

A, D: $\cos\theta + \frac{Y}{2B}\sin\theta = 0 \Rightarrow \theta = -\tan^{-1}\left(\frac{2B}{Y_0}\right) < 0$ ✓

B: $\frac{Y_0}{J} = -\sin\theta + \frac{Y_0}{B}\cos^2\theta/2$

C: $\frac{J}{Y_0} = -\sin\theta - \frac{Y_0}{B}\sin^2\theta/2$

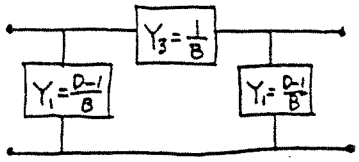
$\ominus$ $\frac{Y_0}{J} - \frac{J}{Y_0} = \frac{Y_0}{B}$

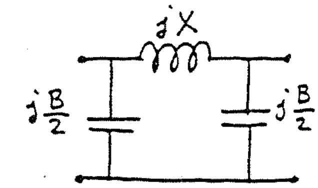
or, $$B = \frac{Y_0}{\frac{Y_0}{J} - \frac{J}{Y_0}} = \frac{J}{1-(J/Y_0)^2} \checkmark$$

also, $$\tan|\theta| = \frac{2B}{Y_0} = \frac{2(J/Y_0)}{1-(J/Y_0)^2} = \frac{2\tan\left|\frac{\theta}{2}\right|}{1-\tan^2\left|\frac{\theta}{2}\right|}$$

so, $$\tan\left|\frac{\theta}{2}\right| = J/Y_0 \checkmark$$

(8.16) The easiest way to do this problem is to use the π-network of Table 4.1, with the shunt and series element values given in terms of the ABCD parameters:





Then using the ABCD parameters for a transmission line gives the equivalent circuit elements as,

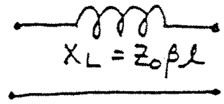
$$jX = B = jZ_0 \sin \beta\ell$$

$$j\frac{B}{2} = \frac{D-1}{B} = \frac{\cos\beta\ell - 1}{jZ_0 \sin\beta\ell} = \frac{j}{Z_0} \tan \beta\ell/2$$

For $\beta\ell < \pi/4$ and large Z_0, these results reduce to:

$X \simeq Z_0\beta\ell$ ✓

$\frac{B}{2} \simeq 0$ ✓

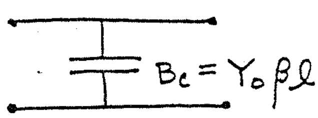


✓

For $\beta\ell < \pi/4$ and small Z_0, these results reduce to:

$X \simeq 0$

$\frac{B}{2} \simeq \frac{\beta\ell}{2Z_0}$



✓

(NOTE: This problem can also be done using y-parameters, but a sign change for y_{12} will be required because of the reference direction for I_2 for the ABCD parameters.)

(8.17) $f_0 = 4\,GHz$, $N=5$, L.P., 0.5dB E.R., $Z_0 = 100\Omega$

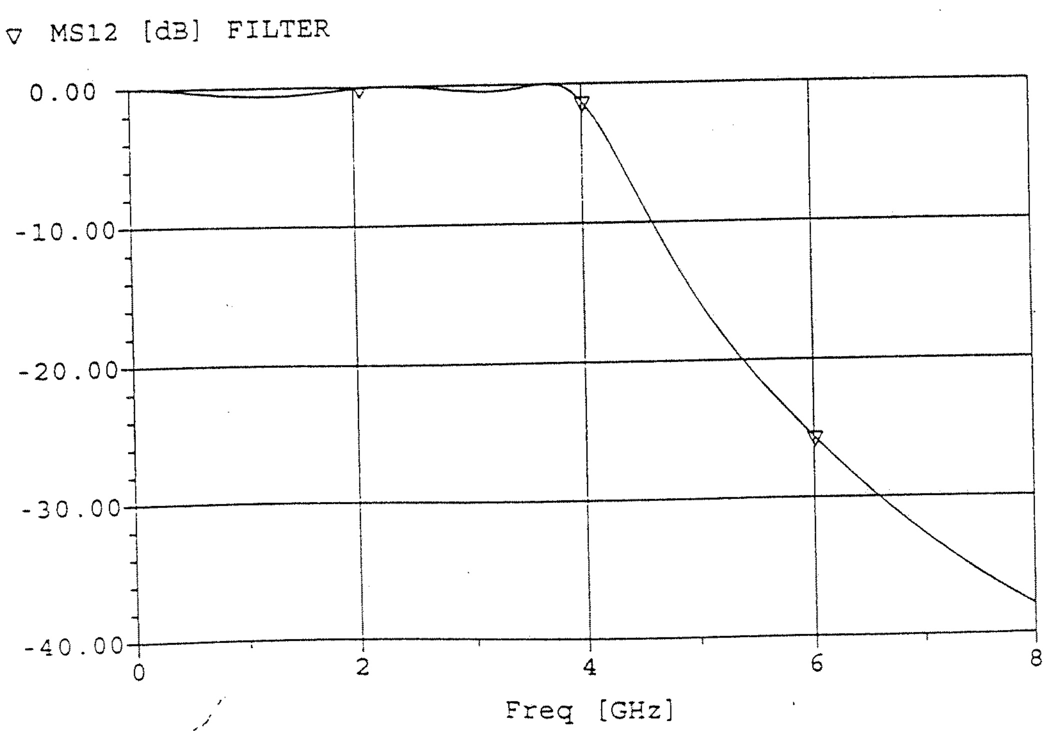
From Table 8.4 and (8.86), with $Z_l = 15\Omega$ and $Z_h = 200\Omega$,

$$g_1 = 1.7058 = C_1 \Longrightarrow \beta l_1 = g_1 Z_l/Z_0 = 14.7^\circ \checkmark$$
$$g_2 = 1.2296 = L_2 \Longrightarrow \beta l_2 = g_2 Z_0/Z_h = 35.2^\circ \checkmark$$
$$g_3 = 2.5408 = C_3 \Longrightarrow \beta l_3 = g_3 Z_l/Z_0 = 21.8^\circ \checkmark$$
$$g_4 = 1.2296 = L_4 \Longrightarrow \beta l_4 = g_4 Z_0/Z_h = 35.2^\circ \checkmark$$
$$g_5 = 1.7058 = C_5 \Longrightarrow \beta l_5 = g_5 Z_l/Z_0 = 14.7^\circ \checkmark$$

NOTE: $\beta l < 45^\circ$ for all cases ✓ (lengths at $f = 4\,GHz$)

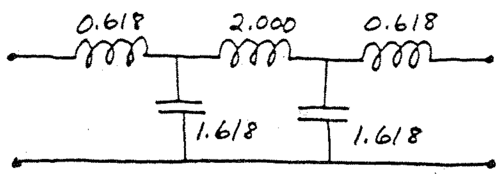
	l_1	l_2	l_3	l_4	l_5	
100Ω	15Ω	200Ω	15Ω	200Ω	15Ω	100Ω

The calculated filter response is shown below:



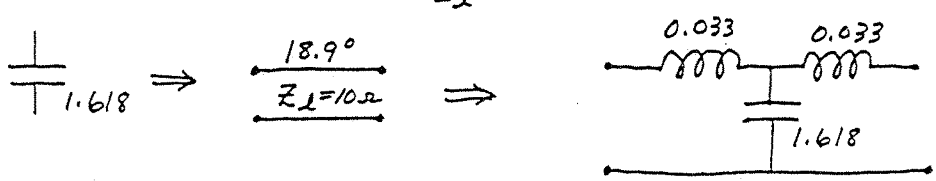
8.18 $f_o = 2GHz$, L.P., M.F., $Z_o = 50\Omega$

From Table 8.3 the LP prototype is,



The shunt capacitors can be implemented with a length of Z_ℓ line. Using (8.83b) gives,

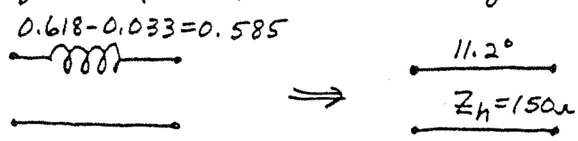
$$B = \frac{1.618}{Z_o} = \frac{\sin \beta\ell}{Z_\ell} \Rightarrow \beta\ell = 18.9°$$



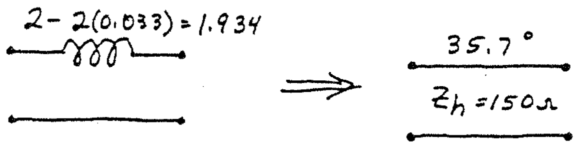
The model for this line (see Figure 8.39) shows inductors on either side, with values given by (8.83a):

$$\frac{X}{2Z_o} = \frac{Z_\ell}{Z_o} \tan \beta\ell/2 = 0.033$$

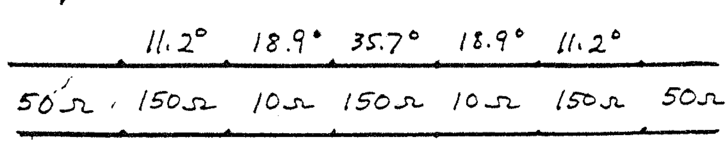
Then the end inductors of value 0.618 can be implemented as lengths of Z_h line. Using (8.83a) gives,



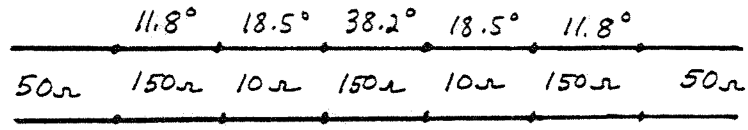
and the middle inductor of value 2.000 can be implemented as,



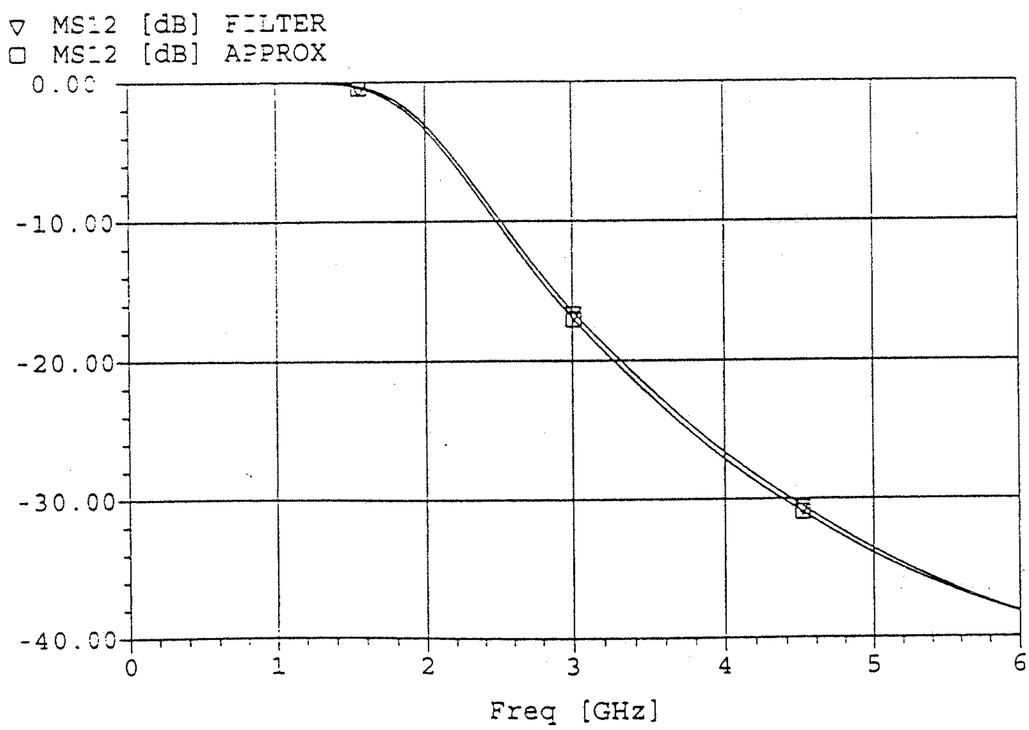
The final filter:



For comparison, the design using the approximations of (8.84) and (8.85) is,



Note that only the middle section differs very much in these two designs. The calculated filter response is shown below for both designs. Note that there is very little difference.



(8.19) 3.00–3.50 GHz, B.P., M.F., $N=3$, $Z_0 = 50\Omega$

$f_0 = 3.25$ GHz ; $\Delta = \dfrac{3.5-3.0}{3.25} = 0.154$

Use (8.71) to transform 2.9 GHz to normalized LP form:

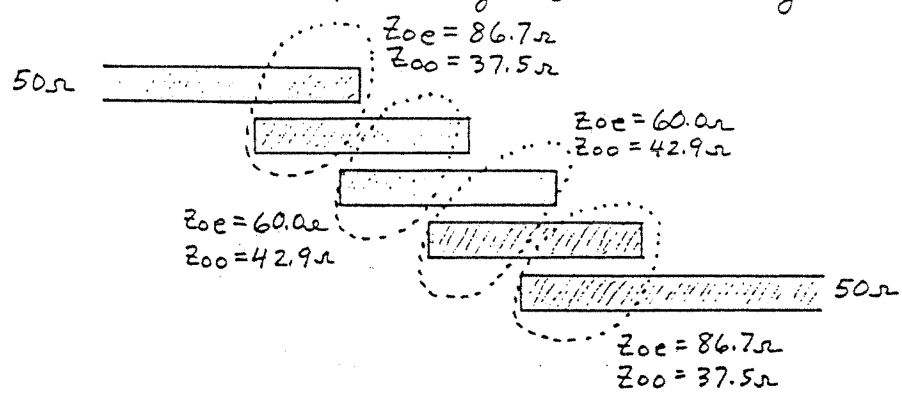
$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{0.154}\left(\frac{2.9}{3.25} - \frac{3.25}{2.9}\right) = -1.48$$

Then, $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.48$, and Figure 8.26 gives $\alpha = 10.5$ dB ✓

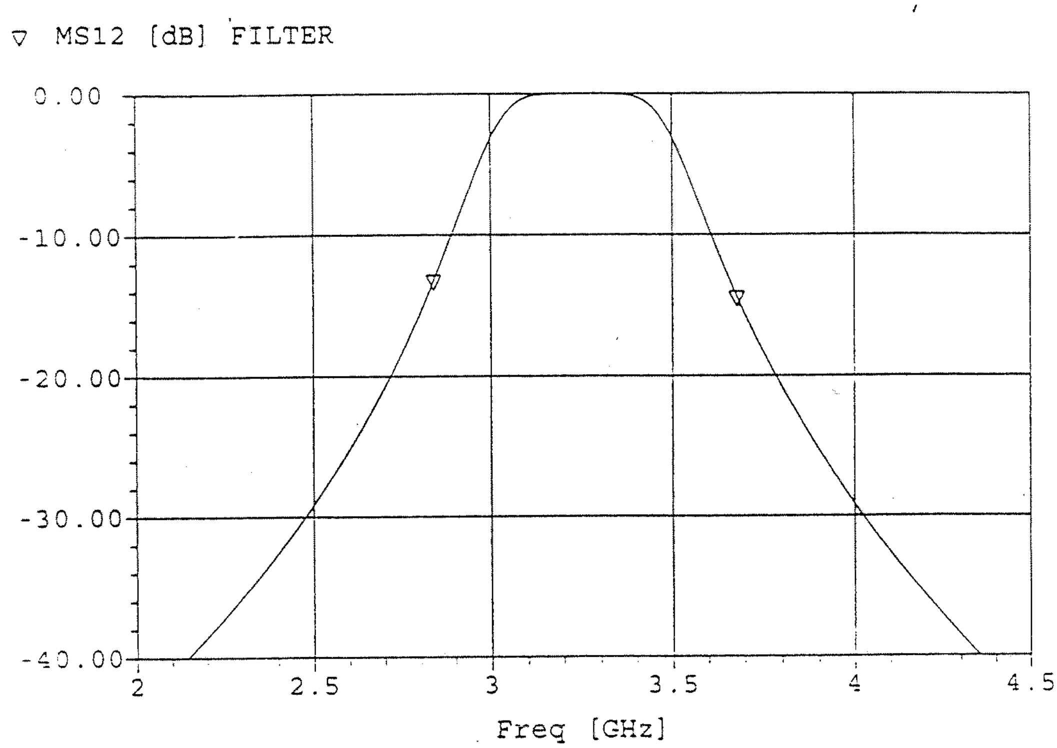
The L.P. prototype values are given in Table 8.3, and $Z_0 J_n$ can be found from (8.121). Then Z_{oe} and Z_{oo} follow from (8.108).

n	g_n	$Z_0 J_n$	$Z_{oe}(\Omega)$	$Z_{oo}(\Omega)$
1	1.000	0.492	86.7	37.5
2	2.000	0.171	60.0	42.9
3	1.000	0.171	60.0	42.9
4	1.000	0.492	86.7	37.5

all lines are $\lambda/4$ long at 3.25 GHz.



The calculated filter response is shown on the following page. The insertion loss at 2.9 GHz is a bit less than 10 dB.



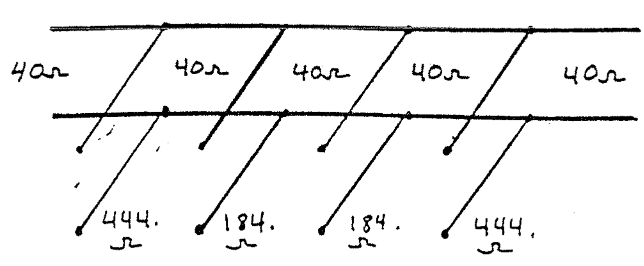
(8.20) $f_0 = 3$ GHz, B.S., M.F., $N=4$, $\Delta = 0.15$, $Z_0 = 40\Omega$.

We find the g_n values from Table 8.3. Then the $\lambda/4$ o.c. stub characteristic impedances can be found from (8.130):

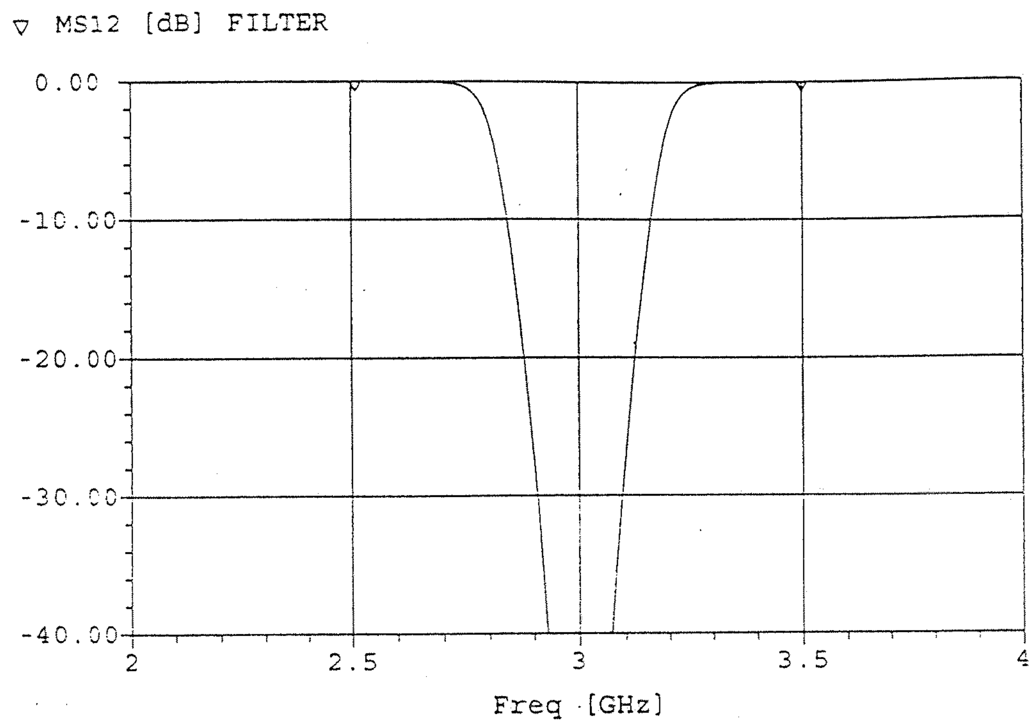
n	g_n	$Z_{0n}(\Omega)$
1	0.765	444.
2	1.848	184.
3	1.848	184.
4	0.765	444.

(Z_{01} and Z_{04} may be too high to be practical)

The final filter is shown below:



all lines and stubs are $\lambda/4$ long at 3 GHz. The calculated filter response is shown below:



(8.21) 3.00 - 3.50 GHz, $N=3$, B.P., 0.5dB E.R., $Z_0 = 50\,\Omega$

$f_0 = 3.25$ GHz ; $\Delta = 0.154$

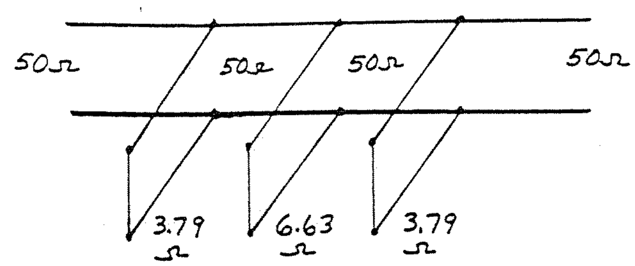
We find g_n from Table 8.4. Then the $\lambda/4$ short-circuit stub characteristic impedances can be found from (8.131):

n	g_n	Z_{0n} (Ω)
1	1.5963	3.79
2	1.0967	6.63
3	1.5963	3.79

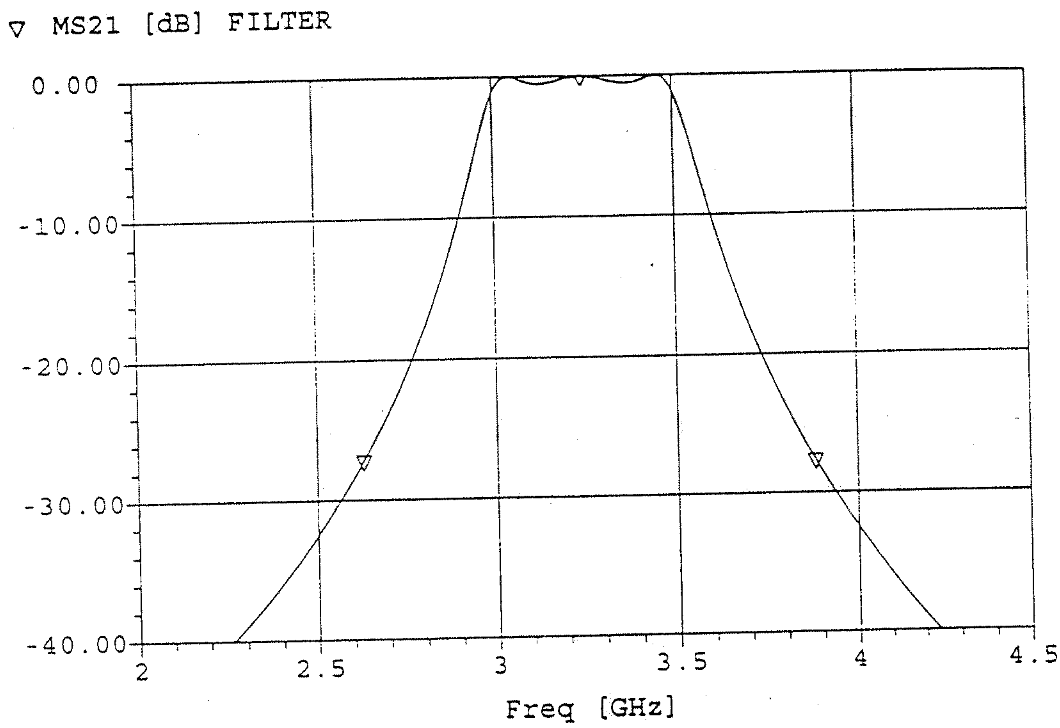
$$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}$$

(These impedances may be too low to be practical.)

The complete filter is shown below:



The calculated filter response is shown below.



(8.22) The bandpass $\lambda/4$ resonator filter uses short-circuited stubs, which have an input admittance of,

$$Y = -jY_{on} \cot\theta \quad , \text{ where } \theta = \pi/2 \text{ for } \omega = \omega_0 .$$

Let $\omega = \omega_0 + \Delta\omega$. Then $\theta = \pi/2\,(1 + \Delta\omega/\omega_0)$, and

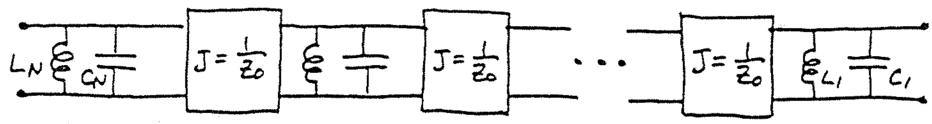
$Y = jY_{on} \tan \frac{\pi \Delta\omega}{2\omega_0} \simeq \frac{j\pi Y_{on}(\omega - \omega_0)}{2\omega_0}$. Now the admittance of a parallel LC resonator is, from Table 6.1,

$$Y \simeq 2jC_n(\omega - \omega_0).$$

So the characteristic impedance of the stub is,

$$Z_{on} = \frac{1}{Y_{on}} = \frac{\pi}{4\omega_o C_n}.$$

The filter circuit can be redrawn as follows:



This is the same as the circuit in part (e) of Figure 8.45, for coupled line bandpass filters (with $Z_o J_n = 1$). Correspondance with the lumped-element bandpass filter requires that,

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C_1'}{L_1'}}$$

$$Z_o^2 \sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{L_2'}{C_2'}}$$

where C_n' and L_n' are the lumped element filter values, and $L_n C_n = L_n' C_n' = 1/\omega_o^2$. Solving for C_n gives,

$$C_1 = C_1'$$

$$C_2 = \frac{1}{\omega_o^2 Z_o^2 C_2'}$$

Using Table 8.6 to transform back to LP prototype values gives,

$$C_1 = C_1' = \frac{g_1}{\Delta\omega_o Z_o}$$

$$C_2 = \frac{1}{\omega_o^2 Z_o^2}\left(\frac{\omega_o g_2 Z_o}{\Delta}\right) = \frac{g_2}{\Delta\omega_o Z_o}$$

So the characteristic impedances are,

$$Z_{on} = \frac{\pi}{4\omega_o}\left(\frac{\Delta\omega_o Z_o}{g_1}\right) = \frac{\pi\Delta Z_o}{4 g_n} \checkmark$$

which agrees with (8.131).

(8.23) $f_o = 4\ GHz$, $\Delta = 0.12$, B.P., M.F., $Z_0 = 50\Omega$

First transform 3.6 GHz to L.P. prototype form:

$$\omega \longleftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{.12}\left(\frac{3.6}{4} - \frac{4}{3.6}\right) = -1.76$$

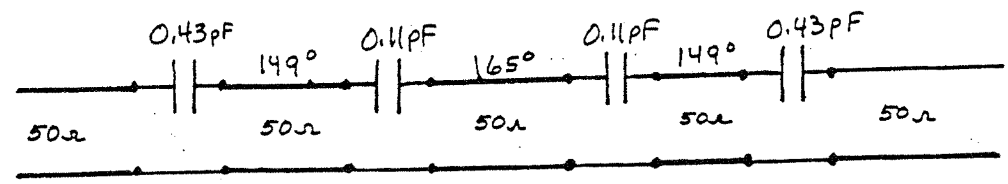
Then, $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.76$

So from Figure 8.26 we see that $N=3$ is required to achieve $\alpha > 12\ dB$ at 3.6 GHz. (or, analytically, $P_{LR} = 1 + (\omega/\omega_c)^{2N} = 14.9\ dB$ for $N=3$)

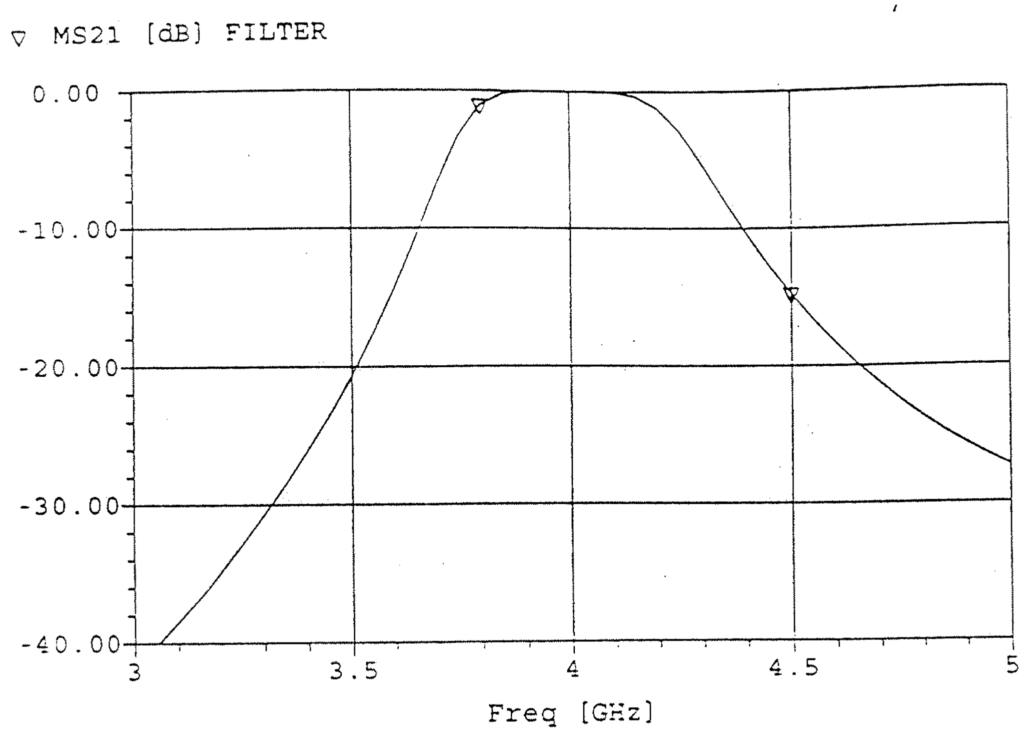
The prototype values are given in Table 8.3. Then $Z_0 J_n$ can be found from (8.121). Then, $Z_0 B_n = \frac{J_n Z_0}{1-(J_n Z_0)^2}$, and $\theta_n = \pi - \frac{1}{2}\left[\tan^{-1}(2Z_0 B_n) + \tan^{-1}(2Z_0 B_{n+1})\right]$. Also, $C_n = B_n/\omega_0$.

n	g_n	$Z_0 J_n$	$Z_0 B_n$	C_n (pF)	θ_n @ 4GHz
1	1.000	0.434	0.535	0.43	149°
2	2.000	0.133	0.135	0.11	165°
3	1.000	0.133	0.135	0.11	149°
4	1.000	0.434	0.535	0.43	—

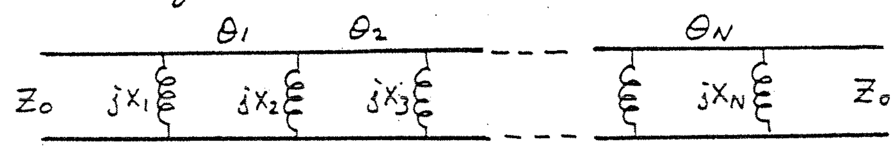
The filter circuit is shown below:



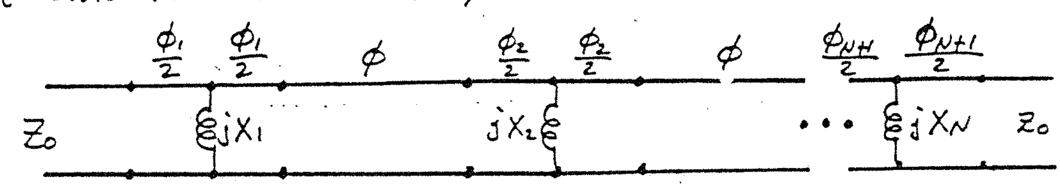
The calculated filter response is shown on the following page.



(8.24) The equivalent circuit is,

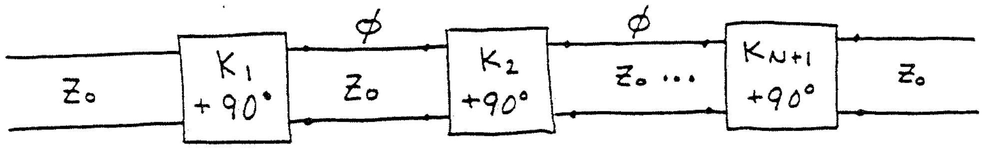


which can be redrawn as,



where $\theta_i = \pi + \frac{1}{2}\phi_i + \frac{1}{2}\phi_{i+1}$ for $i = 1, 2, \ldots N$

From Figure 8.38, the circuit can be redrawn using impedance inverters:



From the properties of the impedance inverter, we have,

$$\phi_i = -\tan^{-1}\left(\frac{2X_i}{Z_0}\right) \checkmark$$

$$X_i = \frac{K_i}{1-(K_i/Z_0)^2} \checkmark$$

Then,

$$\theta_i = \pi - \frac{1}{2}\left[\tan^{-1}(2X_i/Z_0) + \tan^{-1}(2X_{i+1}/Z_0)\right] \checkmark$$

The inverter constants are taken from (8.121):

$(J = K/Z_0^2)$

$$\frac{K_1}{Z_0} = \sqrt{\frac{\pi\Delta}{2g_1}}$$

$$\frac{K_n}{Z_0} = \frac{\pi\Delta}{2\sqrt{g_n g_{n-1}}}$$

$$\frac{K_{N+1}}{Z_0} = \sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$$

Note: Δ should be the fractional bandwidth in terms of guide wavelength:

$$\Delta = \frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g0}} \simeq \left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \left(\frac{\Delta\omega}{\omega_0}\right)$$

Chapter 9

9.1 $f = 10\,GHz$, $4\pi M_s = 1780\,G$

From (9.24)–(9.25),

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

where,

$$\mu = \mu_0\left(1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2}\right), \quad \kappa = \mu_0 \frac{\omega \omega_m}{\omega_0^2 - \omega^2}$$

1) $H_0 = M_s = 0 \implies \omega_0 = \omega_m = 0 \implies \mu = \mu_0 , \kappa = 0$

so,

$$[\mu] = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \checkmark$$

2) $H_0 = 1000\,Oe$

Then,

$$f_0 = 2.8 \frac{MHz}{Oe}(1000\,Oe) = 2.8\,GHz$$

$$f_m = 2.8 \frac{MHz}{Oe}(1780\,G)\left(\frac{1\,Oe}{1\,G}\right) = 4.98\,GHz$$

so,

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.849\,\mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.540\,\mu_0$$

$$[\mu] = \begin{bmatrix} 0.849 & -j0.540 & 0 \\ j0.540 & 0.849 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mu_0 \checkmark$$

9.2

$$B_x = \mu H_x + j\kappa H_y$$
$$B_y = -j\kappa H_x + \mu H_y$$
$$B_z = \mu_0 H_z$$

Then,
$$B^+ = \tfrac{1}{2}(B_x + jB_y) = \tfrac{1}{2}[\mu H_x + j\kappa H_y + \kappa H_x + j\mu H_y]$$
$$= \tfrac{1}{2}(\mu+\kappa)(H_x + jH_y) = (\mu+\kappa)H^+$$
$$B^- = \tfrac{1}{2}(B_x - jB_y) = \tfrac{1}{2}[\mu H_x + j\kappa H_y - \kappa H_x - j\mu H_y]$$
$$= \tfrac{1}{2}(\mu-\kappa)(H_x - jH_y) = (\mu-\kappa)H^-$$

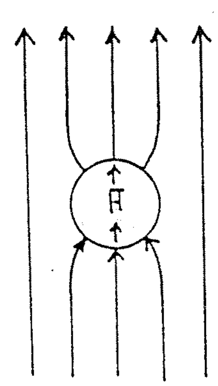
Thus,
$$\begin{bmatrix} B^+ \\ B^- \\ B_z \end{bmatrix} = \begin{bmatrix} (\mu+\kappa) & 0 & 0 \\ 0 & (\mu-\kappa) & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \begin{bmatrix} H^+ \\ H^- \\ H_z \end{bmatrix} \quad \checkmark$$

9.3

$$4\pi M_s = 1780\ \text{G}, \quad H_e = 1200\ \text{Oe}$$

From (9.41) the internal field is,
$$\bar{H} = \bar{H}_e - N\bar{M}$$

From Table 9.1 the demagnetization factors for a sphere are $N_x = N_y = N_z = 1/3$, so
$$H_z = H_{ze} - N_z M_z = 1200 - \tfrac{1}{3}(1780)$$
$$= 607\ \text{Oe}.$$



9.4 $4\pi M_s = 600\,G$

From Table 9.1 the demagnetization factors for a thin rod are $N_x = N_y = \frac{1}{2}$, $N_z = 0$. Then (9.46) gives the gyromagnetic resonance frequency as,

$$\omega_r = \mu_0 \gamma \sqrt{(H_a + \tfrac{1}{2}M_s)(H_a + \tfrac{1}{2}M_s)}$$

$$= \mu_0 \gamma (H_a + \tfrac{1}{2} M_s)$$

$$= \mu_0 \gamma (1300\,Oe)\left(\frac{1\,A/m}{4\pi \times 10^{-3}\,Oe}\right)$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{\mu_0 \gamma}{2\pi}\left(\frac{1\,A/m}{4\pi\times 10^{-3}\,Oe}\right)(1300\,Oe)$$

$$= 2.8\,\frac{MHz}{Oe}(1300\,Oe) = 3.64\,GHz \checkmark$$

9.5 $4\pi M_s = 1200\,G$, $\epsilon_r = 10$, $H_0 = 500\,Oe$, $f = 8\,GHz$

(FARADAY ROTATION)

$$f_0 = (2.8\,\frac{MHz}{Oe})(500\,Oe) = 1.4\,GHz$$

$$f_m = (2.8\,\frac{MHz}{Oe})(1200\,G) = 3.36\,GHz$$

$$k_0 = 167.6\,m^{-1}$$

Then from (9.25),

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.924\,\mu_0 \checkmark$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.433\,\mu_0 \checkmark$$

From (9.52) the propagation constants of the CP waves are,

RHCP: $\beta_+ = \omega\sqrt{\epsilon(\mu+\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924-0.433} = 371.4\,m^{-1}$ ✓

LHCP: $\beta_- = \omega\sqrt{\epsilon(\mu-\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924+0.433} = 617.4\,m^{-1}$ ✓

Then, $\Delta\beta = \beta_+ - \beta_- = -246.0\,m^{-1}$

From (9.57) the polarization rotation of an LP wave is,

$$\phi = -(\beta_+ - \beta_-)z/2$$

so,

$$z = \frac{2\phi}{\beta_- - \beta_+} = \frac{2(\pi/2)}{246\,rad/m} = 12.8\,mm \checkmark$$

9.6 $4\pi M_s = 1200\,G$, $\epsilon_r = 10$, $H_0 = 2000$ Oe, $f = 4$ GHz

(BIREFRINGENCE)

$$f_0 = (2.8\,\text{MHz/Oe})(2000\,\text{Oe}) = 5.6\ \text{GHz}$$
$$f_m = (2.8\,\text{MHz/Oe})(1200\,\text{Oe}) = 3.36\ \text{GHz}$$
$$k_0 = 83.8\ \text{m}^{-1}$$

Then from (9.25),

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 2.23\mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = 0.875\mu_0$$

The $\hat{x}$ polarized wave has $\bar{H} = \hat{y} H_y$, and is the extraordinary wave. From (9.64)–(9.65),

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 1.89\mu_0$$

$$\beta_e = \omega\sqrt{\epsilon\mu_e} = k_0\sqrt{\epsilon_r \mu_e/\mu_0} = 364.3\ \text{m}^{-1}$$

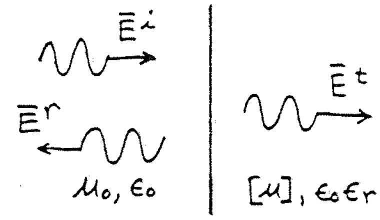
The $\hat{y}$ polarized wave has $\bar{H} = \hat{x} H_x$, and is the ordinary wave. Thus,

$$\beta_o = \sqrt{\epsilon_r}\, k_0 = 265.0\ \text{m}^{-1}$$

So the distance required for a differential phase shift of 270° is,

$$z = \frac{3\pi/2}{\beta_e - \beta_o} = 47.5\ \text{mm}.$$

(9.7)



The incident, reflected, and transmitted fields for a RHCP wave can be written as,

$$\bar{E}^i = E_0(\hat{x}-j\hat{y})e^{-j\beta_0 z} \qquad \bar{H}^i = \frac{E_0}{\eta_0}(\hat{y}+j\hat{x})e^{-j\beta_0 z}$$

$$\bar{E}^r = \Gamma^+ E_0(\hat{x}-j\hat{y})e^{j\beta_0 z} \qquad \bar{H}^r = \frac{-E_0}{\eta_0}\Gamma^+(\hat{y}+j\hat{x})e^{j\beta_0 z}$$

$$\bar{E}^t = T^+ E_0(\hat{x}-j\hat{y})e^{-j\beta_+ z} \qquad \bar{H}^t = \frac{E_0}{\eta_0}T^+(\hat{y}+j\hat{x})e^{-j\beta_+ z}$$

where, $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$, $\beta_+ = \omega\sqrt{\epsilon(\mu+\kappa)}$, $\eta_+ = \frac{1}{Y_+} = \sqrt{\frac{\mu+\kappa}{\epsilon}}$

Matching fields at $z=0$ gives (for both $\hat{x}$ and $\hat{y}$ components)

$$1+\Gamma^+ = T^+$$

$$\eta_+(1-\Gamma^+) = \eta_0 T^+$$

Solving gives,

$$\Gamma^+ = \frac{\eta_+ - \eta_0}{\eta_+ + \eta_0} \quad , \quad T^+ = \frac{2\eta_+}{\eta_+ + \eta_0} \quad ✓$$

Similarly, for a LHCP wave we obtain,

$$\Gamma^- = \frac{\eta_- - \eta_0}{\eta_- + \eta_0} \quad , \quad T^- = \frac{2\eta_-}{\eta_- + \eta_0} \quad ✓$$

where, $\eta_- = \frac{1}{Y_-} = \sqrt{\frac{\mu-\kappa}{\epsilon}}$.

9.8 $4\pi M_s = 1200\,G$, $\bar{H}_0 = \hat{z} H_0$, $f = 4\,GHz$, $\bar{E} = \hat{x} E_0$

$f_m = (2.8\,MHz/Oe)(1200\,G) = 3.36\,GHz$

This is a case of birefringence. From (9.64)-(9.65),

$$\beta_e = \omega\sqrt{\mu_e \epsilon}$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}$$

The wave will be cutoff when $\mu_e \leq 0$:

$$\frac{\mu^2 - \kappa^2}{\mu} < 0$$

$$\frac{\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right)^2 - \left(\frac{f f_m}{f_0^2 - f^2}\right)^2}{1 + \frac{f_0 f_m}{f_0^2 - f^2}} < 0$$

$$\frac{(f_0^2 - f^2)^2 + 2 f_0 f_m (f_0^2 - f^2) + f_m^2 (f_0^2 - f^2)}{(f_0^2 - f^2)\left[(f_0^2 - f^2) + f_0 f_m\right]} < 0$$

$$\frac{(f_0^2 - f^2) + 2 f_0 f_m + f_m^2}{f_0^2 - f^2 + f_0 f_m} < 0$$

If $f_0^2 - f^2 + f_0 f_m > 0$, then $(f_0 + f_m)^2 - f^2 < 0$

$$f_0 + f_m < f$$

$$f_0 < f - f_m = 4.0 - 3.36 = 0.64\,GHz \Rightarrow H_0 = 229\,Oe$$

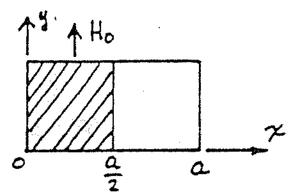
If $f_0^2 - f^2 + f_0 f_m < 0$, then $(f_0 + f_m)^2 - f^2 > 0$

$$f_0 = \frac{-f_m \pm \sqrt{f_m^2 + 4f^2}}{2} = -1.68 \pm 4.34 = 2.66\,GHz$$

$$H_0 = 950\,Oe$$

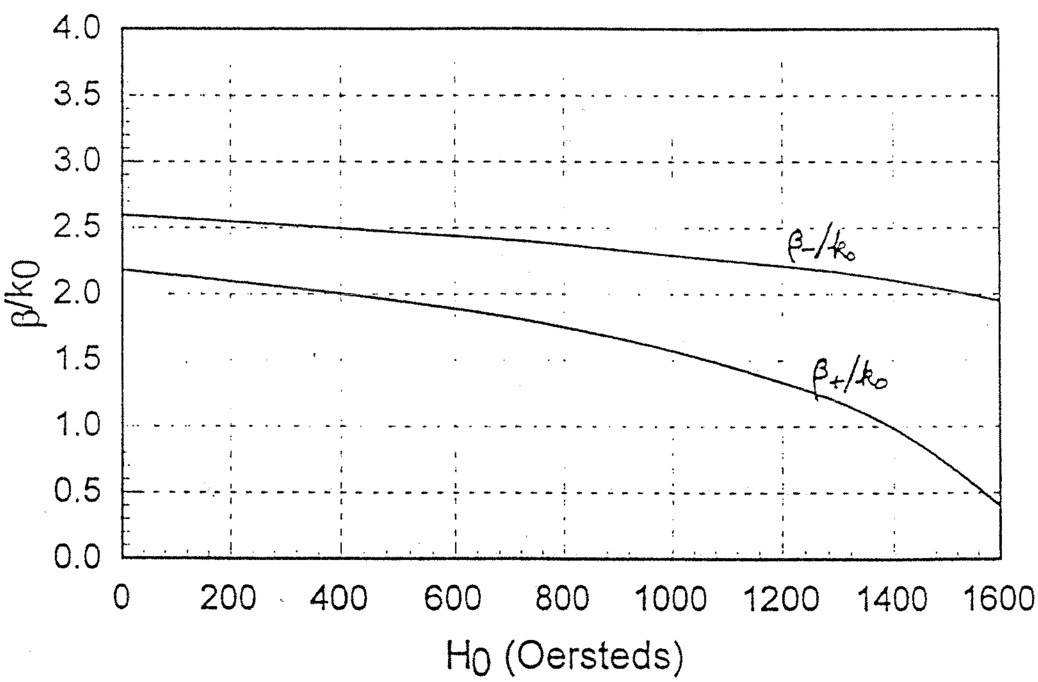
So the cutoff range is between 229 Oe and 950 Oe.

9.9



$a = 1.0$ cm
$f = 10$ GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants are found by solving (9.79) numerically. This was done using the computer program listed below, with the following results:

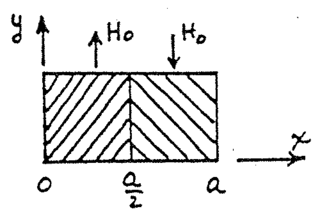


Computer program listing for the solution of Problem 9.9:

```
c plot cp propagation coefficients for ferrite
c loaded waveguide with one slab
c Ho in Oersteds
c 4PiMs in Gauss
c delH in Oersteds
c a is width of guide
c c is left-hand side of slab
c t is slab thickness
      real ms
      complex w0,cu,ck,cue,epsc,betp
      common a,c,t,cu,cue,ck,xk0,epsc
      external func
      fmc=10000.
      eps=13.
      tand=.001
      ms=1700.
      delh=.001
      a=.01
      c=1.e-8
      t=.5*a
      pi=3.1415926
c calculate constants
      xk0=2.*pi*fmc/300.
      w=2.*pi*fmc*1.e6
      epsc=eps*cmplx(1.,-tand)
      do 10 i=1,9
      h0=(i-1)*200.
      write(6,*) 'Ho=',h0
      do 10 k=1,2
      w0=2.8e6*(h0+(0.,1.)*delh/2.)*(2.*pi)
      wm=2.8e6*ms*(2.*pi)
      cu=1.+w0*wm/(w0*w0-w*w)
      ck=w*wm/(w0*w0-w*w)
      cue=(cu*cu-ck*ck)/cu
      if(k.eq.2) ck=-ck
      call root(func,xk0,(.3,-.00001)*xk0,(3.,0.)*xk0,betp)
      write(6,20) c/a,real(betp)/xk0,-aimag(betp)*8.7/100.
20    format(' c/a=',f7.3,'  bet/xk0=',f10.5,'  alp(dB/cm)=',e12.5)
10    continue
      stop
      end
      complex function func(bet)
      common a,c,t,cu,cue,ck,xk0,epsc
      complex cu,cue,ck,epsc,xka,xkf,ctc,ctd,ctt,bet
      xka=csqrt((1.,0.)*xk0*xk0-bet*bet)
      xkf=csqrt((1.,0.)*xk0*xk0*cue*epsc-bet*bet)
      if(aimag(xka).lt.0.) xka=-xka
      if(aimag(xkf).lt.0.) xkf=-xkf
      d=a-c-t
      ctc=ccos(xka*c)/csin(xka*c)
      ctd=ccos(xka*d)/csin(xka*d)
      ctt=ccos(xkf*t)/csin(xkf*t)
      func=(xkf/cue)**2+(ck*bet/cu/cue)**2
      func=func-xka*ctc*(xkf*ctt/cue-ck*bet/cu/cue)
      func=func-xka*xka*ctc*ctd-xka*ctd*(ck*bet/cu/cue-xkf*ctt/cue)
      return
      end
```

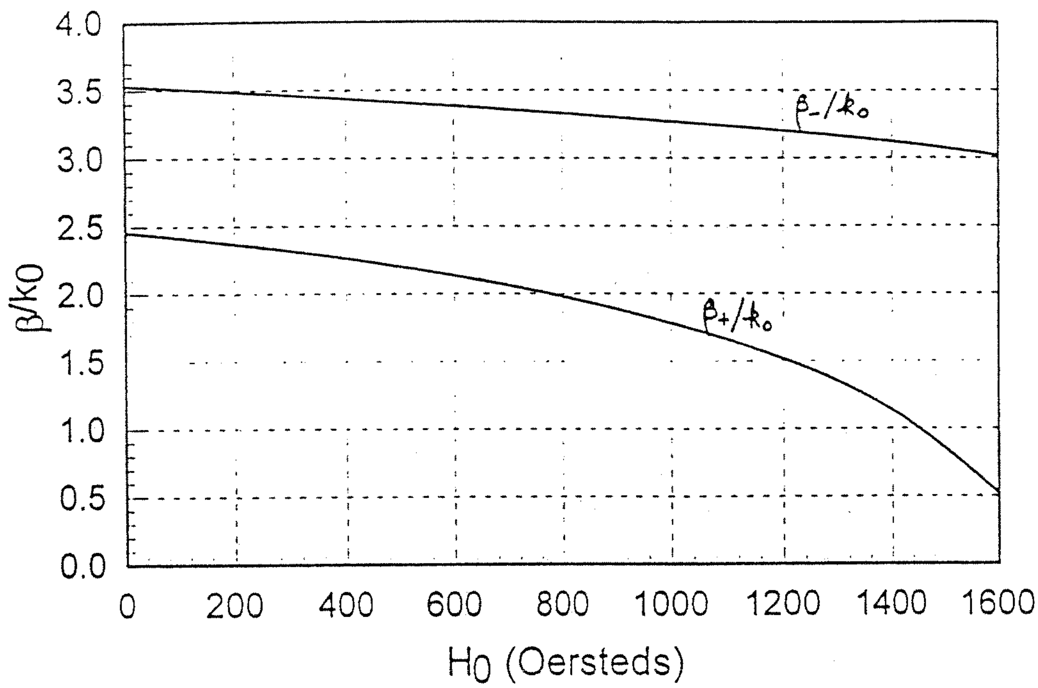
```
      SUBROUTINE root(func,xk0,BET1,BET2,broot)
      COMPLEX bet1,bet2,bc,bcn,bco,broot,f,fo,fn,fd,fp,func
      NDIV=600*real((BET2-BET1))/XK0
      DB=(BET2-BET1)/NDIV
      DO 30 I=1,NDIV+1
      BCN=1.00001*BET1+(I-1)*DB
      fn=func(bcn)
      IF(I.EQ.1) GOTO 20
      if(real(fo)*real(fn).gt.0. .and. aimag(fo)*aimag(fn).gt.0.)goto 20
C A ZERO CROSSING EXISTS BETWEEN BCO AND BCN
C USE INTERVAL HALVING AND INTERPOLATION TO FIND COMPLEX ROOT
      EPSILON=1.E-6
      DEL=.0001*XK0
      ITER=0
C COMPUTE NUMERICAL DERIVATIVE OF DENOMINATOR
      BC=BCN
      F=FN
40    fd=func(BC+DEL)
      FP=(FD-F)/DEL
      BC=BC-F/FP
      f=func(BC)
      IF(CABS(F/FP).LT.EPSILON*XK0) GOTO 70
      ITER=ITER+1
      IF(ITER.GT.100) WRITE(6,74)
74    FORMAT(' ITERATIONS EXCEEDED-NO ROOT FOUND-CONTINUING SEARCH')
      IF(ITER.GT.100) GOTO 20
      GOTO 40
C ROOT AT BC
70    continue
      broot=bc
      return
C RETURN TO DO LOOP
20    FO=FN
      BCO=BCN
30    CONTINUE
c no root found
      return
      end
```

(9.10)

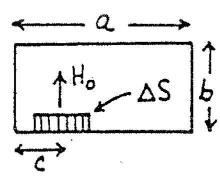


$a = 1.0$ cm
$f = 10$ GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants were found by solving (9.84) numerically using a computer program similar to the one listed above for problem 9.9. Results are shown on the following page.



(9.11)



$f = 10\,GHz$, $a = 2.286\,cm$, $b = 1.016\,cm$

$4\pi M_s = 1700\,G$, $c = a/4$, $\Delta S = 2\,mm^2$

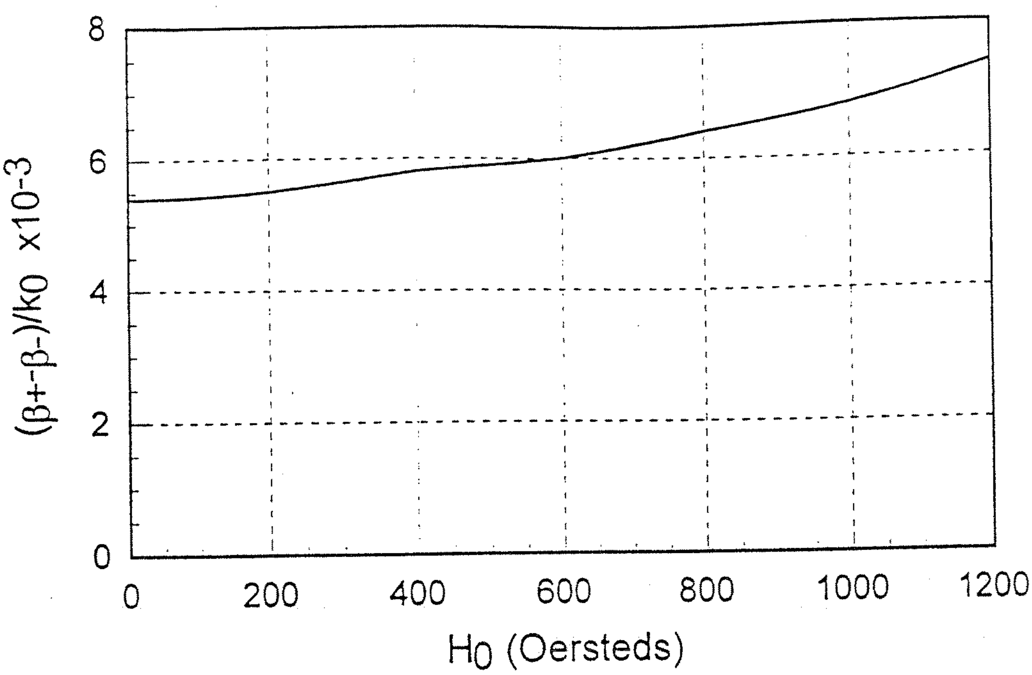
$k_0 = 209.4\,m^{-1}$, $S = 232.3\,mm^2$, $f_m = 4.76\,GHz$ ✓

From (9.80) the differential phase shift is,

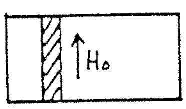
$$(\beta_+ - \beta_-)/k_0 = -2\frac{k_c}{k_0}\frac{\kappa}{\mu}\frac{\Delta S}{S}\sin 2k_c c = -0.0113\ \kappa/\mu$$

H_0 (Oe)	f_0 (GHz)	κ/μ_0	μ/μ_0	$(\beta_+-\beta_-)/k_0$
0	0	−0.476	1.000	0.0054 ✓
200	0.56	−0.477	0.973	0.0055
400	1.12	−0.482	0.946	0.0058
600	1.68	−0.489	0.918	0.0060
800	2.24	−0.501	0.891	0.0064
1000	2.80	−0.516	0.855	0.0068
1200	3.36	−0.537	0.820	0.0074 ✓

This data is plotted on the following page



(9.12)



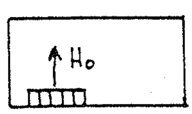
$f = 8\,GHz$, $4\pi M_s = 1500\,G$, $f_m = 4.2\,GHz$

Gyromagnetic resonance for this geometry is given approximately by (9.87): $(N_x = 1,\ N_y = N_z = 0)$

$$f = \sqrt{f_0(f_0 + f_m)}$$

Solve for f_0: $\quad f_0^2 + 4.2 f_0 - 64 = 0 \Rightarrow f_0 = -2.1 \pm 8.27 = 6.17\,GHz$

Thus, $\quad H_0 = \dfrac{6170\,MHz}{2.8\,MHz/Oe} = 2204\,Oe.$ ✓



Gyromagnetic resonance for this geometry is given by the condition,

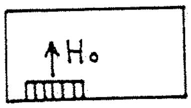
$f_0 = f = 8\,GHz \qquad (N_x = N_z = 0,\ N_y = 1)$

Thus, $\quad H_0 = \dfrac{8000\,MHz}{2.8\,MHz/Oe} = 2857.\,Oe$ ✓

(9.13)

$f = 10\,GHz$, $4\pi M_s = 1700\,G$, $\Delta H = 200\,Oe$

$\frac{\Delta S}{S} = 0.01$



Gyromagnetic resonance is given by the condition,

$$f_o = f = 10\,GHz \quad (\text{since } N_x = N_z = 0,\ N_y = 1)$$

$$H_o = \frac{10{,}000\,MHz}{2.8\,MHz/Oe} = 3571.\,Oe$$

The position of the ferrite slab is given by (9.86), since the RF magnetic fields, h_x and h_z, have demagnetization factors of zero. Thus,

$$\tan k_c x = \pm \frac{k_c}{\beta_o}$$

$k_o = 209.4\ m^{-1}$ ✓

$k_c = \pi/a = 137.4\ m^{-1}$.

$\beta_o = \sqrt{k_o^2 - k_c^2} = 158.\ m^{-1}$

$$s = x = \frac{1}{k_c}\tan^{-1}\frac{k_c}{\beta_o} = \underline{0.521\ cm}$$

The perturbation result of (9.81) must be used to find the attenuation constants, since this geometry cannot be analyzed exactly:

$$\alpha_\pm = \frac{\Delta S}{S\beta_o}\left(\beta_o^2 \chi''_{xx}\sin^2 k_c x + k_c^2 \chi''_{zz}\cos^2 k_c x \mp \chi''_{xy} k_c\beta_o \sin 2k_c x\right) =$$

where the susceptibilities are given by (9.39):

$f_o = f = 10\,GHz$

$f_m = 1700(2.8) = 4.76\,GHz$

$$\alpha = \frac{\Delta H}{2\omega\mu_o\gamma} = \frac{(200)(2.8\,MHz)}{2(10{,}000\,MHz)} = 0.028$$

$$\chi''_{xx} = \frac{\alpha f f_m\left[f_o^2 + f^2(1+\alpha^2)\right]}{\left[f_o^2 - f^2(1+\alpha^2)\right]^2 + 4f_o^2 f^2\alpha^2} = 8.50$$

$$\chi''_{zz} = \chi''_{xx} = 8.50$$

$$\chi''_{xy} = \frac{2 f_o f_m f^2 \alpha}{\left[f_o^2 - f^2(1+\alpha^2)\right]^2 + 4 f_o^2 f^2 \alpha^2} = 8.49$$

Then,
$$\alpha_\pm = 6.3\times10^{-5}\,(9.139\times10^4 + 9.136\times10^4 \mp 1.825\times10^5)$$

$$\alpha_+ = 0.0158 \text{ neper/m} = 0.137 \text{ dB/m}$$
$$\alpha_- = 23.0 \text{ neper/m} = 200 \text{ dB/m}$$

For 30 dB reverse attenuation, the required length is,
$$L = \frac{30\text{ dB}}{200\text{ dB/m}} = 0.15\text{ m} = 15.\text{ cm}$$

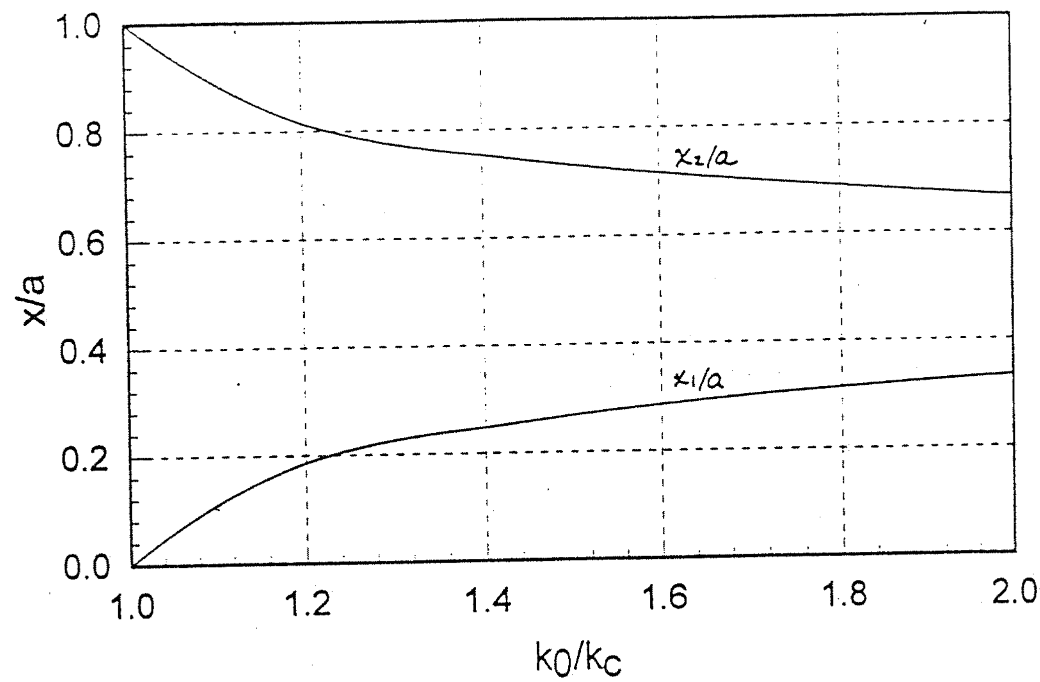
Then the forward insertion loss is,
$$IL = (0.137)(0.15) = 0.02\text{ dB}$$

Note: the calculation of $\alpha_\pm$ is numerically sensitive.

9.14 The magnetic fields for the TE_{10} waveguide mode can be written as,
$$H_x = \frac{j\beta A}{k_c} \sin k_c x\, e^{-j\beta z}$$
$$H_z = A \cos k_c x\, e^{-j\beta z}$$
with $k_c = \pi/a$, $\beta = \sqrt{k_0^2 - k_c^2}$. Circular polarization occurs when,
$$\frac{H_x}{H_z} = \pm j = \frac{j\beta}{k_c}\tan k_c x \,, \quad \text{or} \quad \tan k_c x = \pm\frac{k_c}{\beta} \qquad (9.86)$$

k_0/k_c	β/k_c	x_1/a	x_2/a
1.0	0	0	1.000
1.2	0.663	0.186	0.814
1.4	0.980	0.247	0.753
1.6	1.249	0.285	0.715
1.8	1.497	0.313	0.687
2.0	1.732	0.333	0.666

These two positions for circular polarization are plotted on the following page.



(9.15) $f = 6$ GHz, $\epsilon_r = 10$, $4\pi M_r = 1500$ G, $L = 2.78$ cm

So $f_m = 4.2$ GHz ✓, $k_0 = 125.7 \text{ m}^{-1}$

In the remanent state, $H_0 = f_0 = 0$. So,

$$\mu = \mu_0 \, , \; \kappa = \frac{-f_m}{f}\mu_0 = -0.70\mu_0 \; ✓$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 0.51\,\mu_0 \; ✓$$

In state #1, $\bar{m} = M_r \hat{x}$ and $\bar{H} = \hat{y} H_y$, so this is an extraordinary wave:

$$\beta_e = k_0 \sqrt{\epsilon_r \mu_e / \mu_0} = 283.9 \text{ m}^{-1} \; ✓$$

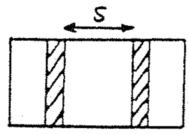
In state #2, $\bar{m} = M_r \hat{y}$ and $\bar{H} = \hat{y} H_y$, so this is an ordinary wave:

$$\beta_o = \sqrt{\epsilon_r}\, k_0 = 397.5 \text{ m}^{-1} \; ✓$$

Then the differential phase shift is,

$$\Delta\phi = (\beta_2 - \beta_1)L = (\beta_o - \beta_e)L = 181^\circ \; ✓$$

9.16



$S = 2\ mm$, $f = 10\ GHz$, $4\pi M_r = 1000\ G$

From Figure 9.18, maximum differential phase shift for $S = 2\ mm$ occurs for $t/a \cong 0.112$, so $t = 0.112a = 2.6\ mm$ ✓. Then $(\beta_+ - \beta_-)/k_0 = 0.24$, for $4\pi M_r = 1786\ G$. If we assume $(\beta_+ - \beta_-)$ is proportional to κ (and so M_r), then for $4\pi M_r = 1000\ G$ we have

$$\frac{(\beta_+ - \beta_-)}{k_0} = 0.24\left(\frac{1000}{1786}\right) = 0.134,$$

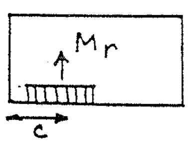
so, $(\beta_+ - \beta_-) = 0.134 k_0 = 16.1°/cm$ ✓

Then the slab lengths for 180° and 90° sections are,

$$L = \frac{180°}{16.1°/cm} = 11.2\ cm \checkmark$$

$$L = \frac{90°}{16.1°/cm} = 5.6\ cm \checkmark$$

9.17



$a = 2.286\ cm$, $b = 1.016\ cm$, $f = 9\ GHz$

$4\pi M_r = 1200\ G$, $c = a/4$, $\Delta S = 2\ mm^2$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a} \frac{\kappa}{\mu} \frac{\Delta S}{S} \sin \frac{2\pi c}{a}$$

Now for $H_0 = 0$,

$$\frac{\kappa}{\mu} = -\frac{f_m}{f} \cong \frac{-2.8(1200)}{9000} = -0.373$$

and,

$$S = ab = 232.3\ mm^2, \text{ so}$$

$$\beta_+ - \beta_- = 0.883\ rad/m = 0.506°/cm \checkmark$$

So the required length is,

$$L = \frac{22.5°}{0.506°/cm} = 44.5\ cm \checkmark$$

(a bit long!)

(9.18) $f = 9$ GHz, $4\pi M_s = 1700$ G, $\Delta S = 6$ mm^2
$a = 2.286$ cm, $b = 1.016$ cm, $H_a = 4000$ Oe

From (9.41) the internal bias field is ($N_z = 1$)

$$H_0 = H_a - NM_s = 4000 - 1700 = \underline{2300 \text{ Oe}}$$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a} \frac{\kappa}{\mu} \frac{\Delta S}{S} \sin \frac{2\pi c}{a}$$

Maximum phase shift will occur for $c = a/4 = \underline{0.572 \text{ cm}}$.
Then, $f_m = 2.8(1700) = 4.76$ GHz, $f_0 = 2.8(2300) = 6.44$ GHz.
So,

$$\mu = \mu_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.224 \mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -1.08 \mu_0$$

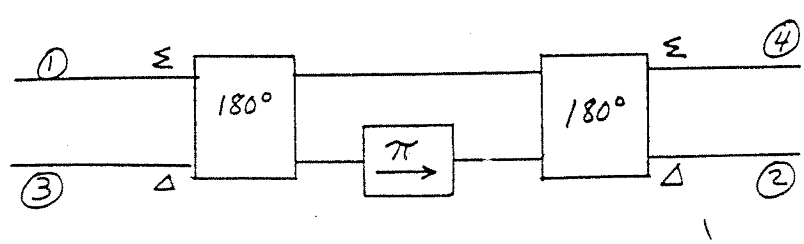
Thus,

$$\beta_+ - \beta_- = 0.342 \text{ rad/cm} = 19.6°/\text{cm}$$

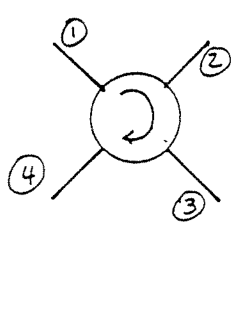
So the required length for a 180° phase shift is,

$$L = \frac{180°}{19.6°/\text{cm}} = \underline{9.2 \text{ cm}}$$

(9.19) a four-port circulator can be made using a gyrator and two 180° hybrid couplers:



a three port circulator can be obtained by shorting any one of the ports.



9.20 $RL = 10\,dB,\ 20\,dB.$

From (9.92) the scattering matrix of a mismatched circulator is,

$$[S] = \begin{bmatrix} \Gamma & \beta & \alpha \\ \alpha & \Gamma & \beta \\ \beta & \alpha & \Gamma \end{bmatrix}$$

Then $|\beta| \approx |\Gamma|$

$$|\alpha| \simeq 1 - |\Gamma|^2$$

For $RL = 10\,dB$, $I = |\beta| = 10\,dB$

For $RL = 20\,dB$, $I = |\beta| = 20\,dB$

Chapter 10

10.1 From (10.8) the ratio of power outputs is,

$$Y = \frac{T_1 + T_e}{T_2 + T_e} = \frac{300+250}{77+250} = 1.68 \checkmark$$

10.2

$$T_e = \frac{T_1 - Y T_2}{Y-1}$$

$$T_e + \Delta T_e = \frac{T_1 - (Y+\Delta Y)T_2}{(Y+\Delta Y)-1}$$

$$\Delta T_e = \frac{T_1 - (Y+\Delta Y)T_2}{(Y+\Delta Y)-1} - \frac{T_1 - Y T_2}{Y-1}$$

$$= \frac{T_1 - (Y+\Delta Y)T_2}{(Y-1)\left(1+\frac{\Delta Y}{Y-1}\right)} - \frac{T_1 - Y T_2}{Y-1}$$

$$\simeq \frac{[T_1 - (Y+\Delta Y)T_2]\left[1 - \frac{\Delta Y}{Y-1}\right] - (T_1 - Y T_2)}{Y-1}$$

$$\simeq \frac{-\frac{T_1}{Y-1} + \frac{Y T_2}{Y-1} - T_2}{Y-1}\Delta Y = \frac{(T_2 - T_1)}{(Y-1)^2}\Delta Y$$

$$\frac{\Delta T_e}{T_e} = \frac{(T_2 - T_1)Y}{(Y-1)^2 T_e}\frac{\Delta Y}{Y} = \frac{(T_1+T_e)(T_2+T_e)}{T_e(T_2 - T_1)}\frac{\Delta Y}{Y}$$

minimize with respect to T_e:

$$\frac{d}{dT_e}\left[\frac{\Delta T_e}{T_e}\right] = \frac{\left(\frac{T_1}{T_e}+1\right)\left(\frac{T_2}{T_e}+1\right) + T_e\left(\frac{-T_1}{T_e^2}\right)\left(\frac{T_2}{T_e}+1\right) + T_e\left(\frac{T_1}{T_e}+1\right)\left(\frac{-T_2}{T_e^2}\right)}{T_2 - T_1} = 0$$

Thus,

$$T_e = \sqrt{T_1 T_2} \checkmark$$

(10.3) Find attenuation for each line.

X-BAND W.G.: From Example 3.1 the attenuation of copper X-band guide at 10 GHz is $\alpha_c = 0.11$ dB/m. So the total loss is $L = 0.22$ dB.

RG-8/U: From appendix J, $\alpha_c = 35$ dB/100ft, or $\alpha_c = 1.15$ dB/m. So the total loss is $L = 2.30$ dB.

CIRCULAR W.G.: From Example 3.1, $R_s = 0.026\,\Omega$. From (3.133) the attenuation for TE_{11} mode propagation is, ($k = 209.\ m^{-1}$, $k_c = p'_{11}/a = 184.\ m^{-1}$, $\beta = 99.8$)

$$\alpha_c = \frac{R_s}{ak\eta\beta}\left(k_c^2 + \frac{k^2}{p'^2_{11}-1}\right) = 0.0172 \text{ neper/m} = 0.15 \text{ dB/m}$$

So the total loss is $L = 0.30$ dB. Thus, the rectangular waveguide is the best choice, since it has the lowest attenuation.

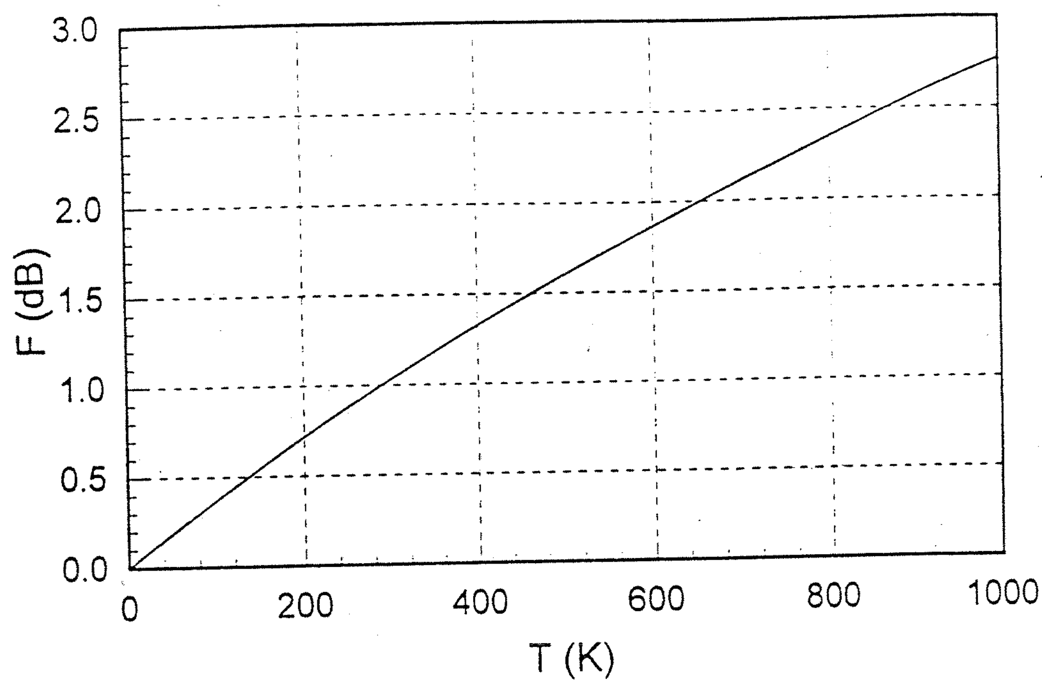
(10.4) From (10.16), the noise figure of a lossy line is,

$$F = 1 + (L-1)\frac{T}{T_0}.$$

When $T = T_0$, $F = L = 1$ dB $= 1.259$. Thus, $F = 1 + 0.259\left(\frac{T}{290K}\right)$

T(K)	F(dB)
0	0
250	0.88
500	1.60
750	2.23
1000	2.77

This data is plotted on the following page.



(10.5) The equivalent noise power input is,

$$P_{min} = kT_eB = (1.38\times10^{-23})(250)(10^9) = 3.5\times10^{-12}\ W$$

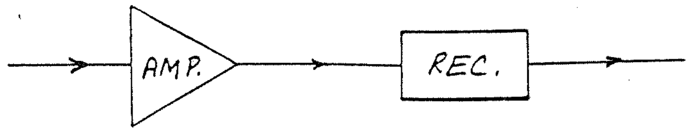
If we assume the upper limit is set by the 1 dB compression point, then

$$P_{max} = -10\ dBm = 0.1\ mW = 10^{-4}\ W.$$

Thus the dynamic range is,

$$10 \log \frac{P_{MAX}}{P_{MIN}} = 10 \log\left(\frac{10^{-4}}{3.5\times10^{-12}}\right) = 75\ dB$$

(10.6)



$G = 12\,dB = 15.8$
$BW = 150\,MHz$
$F = 4\,dB = 2.51$

$T_e = 900\,K$

The noise figure of the receiver is, from (10.11),

$$F_2 = 1 + \frac{T_e}{T_0} = 1 + \frac{900}{290} = 4.10$$

Then the noise figure of the cascade is, from (10.21),

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) = 2.51 + \frac{4.10-1}{15.8} = 2.71 = 4.3\,dB \checkmark$$

(10.7)

From (10.4) the equivalent noise temperature of the source is,

$$T_e = \frac{P_s}{kB} = \frac{(10^{-85/10})(10^{-3})}{(1.38\times10^{-23})(10^9)} = 229.\ K$$

From (10.11) the noise figure of the amplifier is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{180}{290} = 1.62 = 2.1\,dB \checkmark$$

From (10.16) the noise figure of the transmission line is,

$$F = 1 + (L-1)\frac{T}{T_0} = 1 + (1.41-1)\left(\frac{300}{290}\right) = 1.43$$

The noise figure of the transmission line and amplifier cascade is,

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) = 1.43 + (1.41)(1.62-1) = 2.30 = 3.6\,dB \checkmark$$

The total noise power output due to the line and amplifier is,

$$P_{cas} = kT_{e\,cas}BG_{cas} = k(F_{cas}-1)T_0BG_{cas}$$
$$= (1.38\times10^{-23})(2.30-1)(10^9)(10^{10.5/10})(290) = 5.84\times10^{-11}\ W \checkmark$$

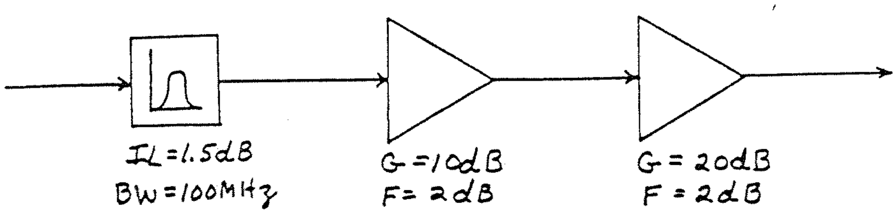
The noise power output due to the source is,

$$P_s = -85\,dBm - 1.5\,dB + 12\,dB = -74.5\,dBm = 3.55\times10^{-11}\ W \checkmark$$

Thus the total noise power output is,

$$P_n = 9.39\times10^{-11}\ W = -70.3\ dBm$$

(10.8)



From (10.23) the noise figure of the cascade is ($F_1 = IL = 1.5dB$)

$$F_{CAS} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + (1.41)(1.58-1) + \frac{1.41}{10}(1.58-1)$$

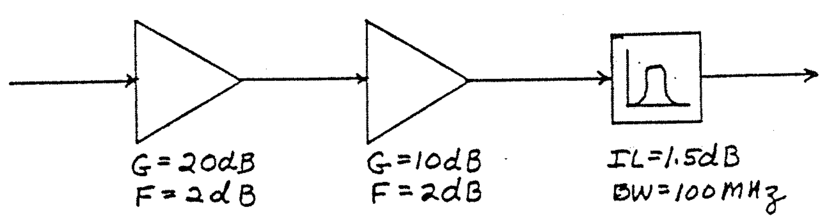
$$= 2.31 = 3.64dB$$

If $P_{in} = -90$ dBm, then $P_{out} = -90dBm - 1.5dB + 10dB + 20dB = -61.5dBm$.
The noise power output is then,

$$P_n = G_{cas} k T_{e_{cas}} B = k(F_{cas}-1)T_0 B G_{cas}$$
$$= (1.38\times10^{-23})(2.31-1)(290)(10^8)(10^{28.5/10}) = 3.71\times10^{-10}\ W$$
$$= -64.3\ dBm$$

Thus, $\frac{S_o}{N_o} = -61.5 + 64.3 = 2.8dB$

The best noise figure would be achieved with the arrangement shown below:



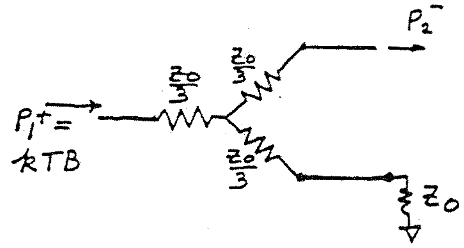
Then,

$$F_{CAS} = 1.58 + \frac{(1.58-1)}{100} + \frac{(1.41-1)}{1000} = 1.586 = 2.0dB$$

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)

10.9

a) RESISTIVE DIVIDER



$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

When the input noise power at port 1 is kTB, and the divider is at temperature T, the system is in thermodynamic equilibrium. Thus the output noise power at port 2 must be kTB. We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$P_2^- = kTB = \frac{kTB}{4} + \frac{N_{added}}{4}$$

$$\therefore N_{added} = 3kTB$$

The equivalent noise temperature is then,

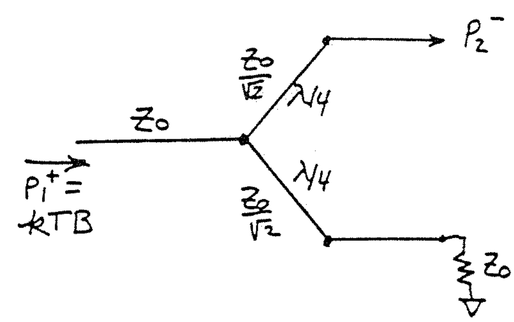
$$T_e = \frac{N_{added}}{kB} = 3T$$

And the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

at room temperature, $T = T_0$, so $F = 4 = 6\text{dB}$.
(this result checks with that obtained using the available gain method)

b) WILKINSON DIVIDER



$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$

In this case, if the input noise power is kTB, and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{4}kTB$, because of the mismatch of the output ports (1/4 of output power is reflected). Then we have,

$$P_2^- = \frac{3}{4}kTB = \frac{kTB}{2} + \frac{N_{added}}{2} \qquad (N_{added} \text{ ref. at input})$$

$$\therefore N_{added} = \tfrac{1}{2}kTB$$

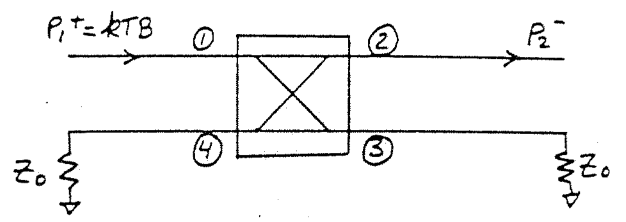
$$T_e = \frac{N_{added}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

If $T = T_0$, $F = \frac{3}{2} = 1.76\ dB$.

(Result verified with HP-MDS, calculations using available gain, and direct measurement)

c) QUADRATURE HYBRID



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Using the same thermodynamic arguments as above, the output noise power is kTB (outputs are matched). Thus,

$$P_2^- = kTB = \frac{kTB}{2} + \frac{N_{added}}{2}$$

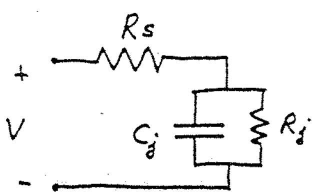
$$\therefore N_{added} = kTB$$

$$T_e = \frac{N_{added}}{kB} = T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{T_0}$$

If $T = T_0$, we have $F = 2 = 3dB$

10.10 The AC model of the diode is shown below:



The voltage sensitivity is, from (10.32)-(10.33),

$$\beta_v = \beta_i R_j = \frac{\Delta I_{dc}}{P_{in}} R_j \quad ,$$

where P_{in} is the RF input power and ΔI_{dc} is the change in DC current through the diode. If the RF voltage across the diode is V, then the RF input power is,

$$P_{in} = \frac{|V|^2}{2} Re\{Y_d\} = \frac{|V|^2}{2} Re\left\{\frac{1}{R_s + \frac{R_j(1/j\omega C_j)}{R_j + 1/j\omega C_j}}\right\}$$

$$= \frac{|V|^2}{2} Re\left\{\frac{1/R_j + j\omega C_j}{(1+R_s/R_j) + j\omega C_j R_s}\right\} = \frac{|V|^2}{2} \frac{\frac{1}{R_j}(1+R_s/R_j) + \omega^2 C_j^2 R_s}{(1+R_s/R_j)^2 + (\omega C_j R_s)^2}$$

From (10.31) the change in DC current is,

$$\Delta I_{dc} = \frac{|v_o|^2}{4} G_d' = \frac{|v_o|^2}{4} \frac{\alpha}{R_j}$$

where v_o is the peak RF junction voltage. This is the current when the junction is short-circuited. When the packaged diode is shorted, the effect of R_s must be included:

$$\Delta I_{dc} = \frac{|v_o|^2}{4} \frac{\alpha}{R_j} \frac{R_j}{R_j + R_s} = \frac{\alpha |v_o|^2}{4(R_j + R_s)}$$

The relation between v_o^2 and $|V|^2$ is,

$$v_o = V \frac{\frac{R_j/j\omega C_j}{R_j + 1/j\omega C_j}}{R_s + \frac{R_j/j\omega C_j}{R_j + 1/j\omega C_j}} = V \frac{1}{(1+R_s/R_j) + j\omega C_j R_s}$$

So,

$$|v_o|^2 = \frac{|V|^2}{(1+R_s/R_j)^2 + (\omega C_j R_s)^2}$$

Finally,

$$\beta_v = \frac{\alpha R_j}{2(1+R_s/R_j)\left[(1+R_s/R_j) + (\omega C_j)^2 R_s R_j\right]} \text{ V/W} \quad ✓$$

at $f = 10\text{GHz}$, $\omega C_j = 2\pi(10^{10})(0.1\times10^{-12}) = 0.0063$; $\alpha = 1/25\text{ mV}$

I_o (μA)	R_j (Ω)	β_v (V/mW)
0	2.5×10^5	33.
20	1.24×10^3	14.
50	4.99×10^2	3.

$$R_j = \frac{1}{\alpha(I_o + I_s)}$$

(10.11) The input signal can be written as,

$$v(t) = \cos\omega_1 t + \cos\omega_2 t$$

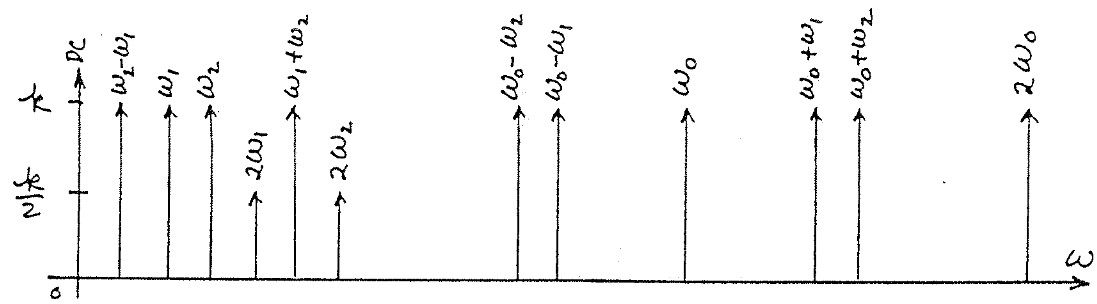
and the LO signal is,

$$v_{LO}(t) = \cos\omega_0 t$$

Then the v^2 mixer output will be,

$$i = k(\cos\omega_1 t + \cos\omega_2 t + \cos\omega_0 t)^2$$
$$= k[\cos^2\omega_1 t + \cos^2\omega_2 t + \cos^2\omega_0 t]$$
$$+ 2k[\cos\omega_1 t \cos\omega_2 t + \cos\omega_1 t \cos\omega_0 t + \cos\omega_2 t \cos\omega_0 t]$$
$$= \frac{k}{2}[3 + \cos 2\omega_1 t + \cos 2\omega_2 t + \cos 2\omega_0 t]$$
$$+ k[\cos(\omega_1-\omega_2)t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_0)t + \cos(\omega_1+\omega_0)t$$
$$+ \cos(\omega_2-\omega_0)t + \cos(\omega_2+\omega_0)t]$$

(all possible combinations of the sums and differences of ω_1, ω_2, and ω_0)



(ω_1, ω_2, and ω_0 are due to the linear term, not the v^2 term; they are shown here for reference only)

10.12 $\quad v_1 = v_0 \cos\omega t \quad ; \quad v_2 = v_0 \cos(\omega t + \theta)$

As in (10.42)-(10.45), the diode currents in a mixer using a quadrature hybrid will be,

$$i_1 = k v_0^2 [\cos(\omega t - \pi/2) + \cos(\omega t + \theta - \pi)]^2$$
$$= k v_0^2 [\sin\omega t - \cos(\omega t + \theta)]^2$$

$$i_2 = -k v_0^2 [\cos(\omega t - \pi) + \cos(\omega t + \theta - \pi/2)]^2$$
$$= -k v_0^2 [-\cos\omega t + \sin(\omega t + \theta)]^2$$

Low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2}\sin\theta)$$
$$i_2 = -k v_0^2 (1 - 1/2 \sin\theta)$$

So the output is $\quad i_1 + i_2 = k v_0^2 \sin\theta$ ✓

If a mixer with a 180° hybrid is used, the diode currents become,

$$i_1 = k v_0^2 [\cos\omega t + \cos(\omega t + \theta)]^2$$
$$i_2 = -k v_0^2 [\cos\omega t - \cos(\omega t + \theta)]^2$$

Then low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2}\cos\theta)$$
$$i_2 = -k v_0^2 (1 - \tfrac{1}{2}\cos\theta)$$

So the output is $\quad i_1 + i_2 = k v_0^2 \cos\theta$ ✓

(10.13) Retaining only the terms that give rise to the third order intermodulation products:

$$v_0 \sim k(v_1 \cos \omega_1 t + v_2 \cos \omega_2 t)^3$$

$$\sim k(v_1^2 v_2 \cos^2 \omega_1 t \cos \omega_2 t + v_1 v_2^2 \cos \omega_1 t \cos^2 \omega_2 t)$$

$$\sim k(v_1^2 v_2 \cos 2\omega_1 t \cos \omega_2 t + v_1 v_2^2 \cos \omega_1 t \cos 2\omega_2 t)$$

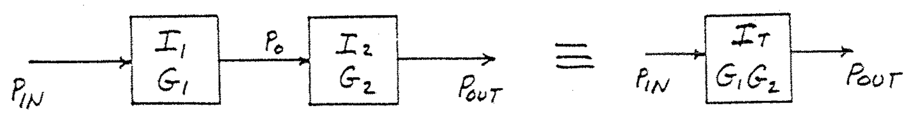
$$\sim \frac{k}{2}\left[v_1^2 v_2 \cos(2\omega_1 - \omega_2)t + v_1 v_2^2 \cos(2\omega_2 - \omega_1)t\right]$$

So the ratio of the powers in the two outputs is,

$$\left(\frac{v_1^2 v_2}{v_1 v_2^2}\right)^2 = \left(\frac{v_1}{v_2}\right)^2 = 6\,dB$$

Note that the individual output powers vary as P_{in}^3.

(10.14)



LINEAR BEHAVIOR: $P_o = G_1 P_{IN}$

THIRD-ORDER BEHAVIOR: $P_o = C P_{IN}^3$ (C is a constant)

at the third order intercept point, $P_o = I_1$, so

$$C = \frac{P_o}{P_{in}^3} = \frac{I_1}{P_{in}^3} = \frac{I_1 G_1^3}{I_1^3} = \frac{G_1^3}{I_1^2} \quad ✓$$

so the distortion power at the output of the first stage is, for input power P_{in},

$$P_{d1} = \left(\frac{G_1^3}{I_1^2}\right) P_{in}^3$$

Similarly, the distortion power at the output of the second stage (as caused by the second stage) is,

$$P_{d2} = \left(\frac{G_2^3}{I_2^2}\right) P_o^3 = \left(\frac{G_2^3}{I_2^2}\right) G_1^3 P_{IN}^3$$

The distortion voltages at the output of the cascade are,

$$V_{d1} = \sqrt{Z_0 G_2 P_{d1}} = \sqrt{Z_0 P_{in}^3} \sqrt{\frac{G_2 G_1^3}{I_1^2}}$$

$$V_{d2} = \sqrt{Z_0 P_{d2}} = \sqrt{Z_0 P_{in}^3} \sqrt{\frac{G_1^3 G_2^3}{I_2^2}}$$

Assuming these voltages add in phase, the total distortion voltage is,

$$V_d = \sqrt{Z_0 P_{in}^3} \left(\sqrt{\frac{G_1^3 G_2}{I_1^2}} + \sqrt{\frac{G_1^3 G_2^3}{I_2^2}} \right)$$

$$= \sqrt{Z_0 P_{in}^3 G_1^3 G_2} \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)$$

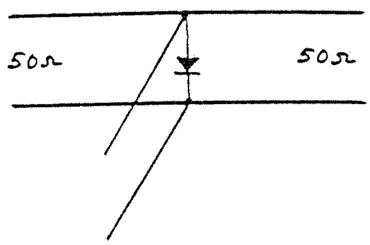
So the total distortion power output is,

$$P_d = \frac{V_d^2}{Z_0} = P_{in}^3 G_1^3 G_2^3 \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)^2 = \left(\frac{G_1^3 G_2^3}{I_T^2} \right) P_{in}^3$$

Thus,

$$I_T = \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)^{-1} \quad \checkmark$$

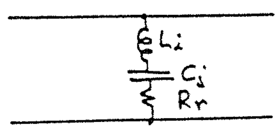
(10.15)



$\omega L_i = 7.5\Omega$

$1/\omega C_j = 79.6\Omega$

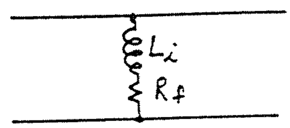
SWITCH ON: (DIODE OFF)



$Z_d = 0.5 - j72\Omega$

$Y_d = (0.096 + j13.9)\,mS$

SWITCH OFF: (DIODE ON)



$Z_d = 0.3 + j7.5\Omega$

$Y_d = (5.3 - j133.)\,mS$

To minimize the insertion loss for the ON state, let the stub susceptance by $Y_s = -j0.0139 = -j0.695/Z_0$. So the stub length should be $l = 0.403\lambda$.

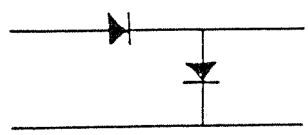
In the ON state, the shunt impedance is then, $Z = 1/0.096 \times 10^{-3} = 10{,}417\Omega$, so the insertion loss is, from (10.59b),

$$IL = -20 \log \left| \frac{2Z}{2Z + Z_0} \right| = 0.021\,dB$$

In the OFF state, the shunt impedance is $Z = 0.246 + j6.8\Omega$, so the insertion loss is,

$$IL = -20 \log \left| \frac{2Z}{2Z + Z_0} \right| = 11.7\,dB$$

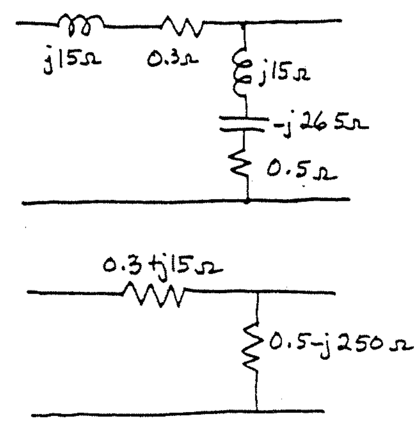
10.16



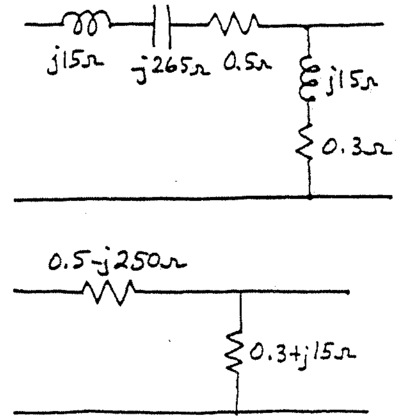
$\omega L_i = 15\Omega$

$1/\omega C_j = 265\Omega$

SWITCH ON:

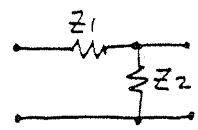


SWITCH OFF:



ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Z_1/Z_2 & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$



Convert to S_{21}:

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + Z_1/Z_2 + Z_1/Z_0 + Z_0/Z_2 + 1} = \frac{2}{2 + \frac{Z_1}{Z_2} + \frac{Z_1}{Z_0} + \frac{Z_0}{Z_2}}$$

ON STATE: $Z_1 = 0.3 + j15\Omega$, $Z_2 = 0.5 - j250\Omega$

$S_{21} = 0.995\angle{-14^\circ}$

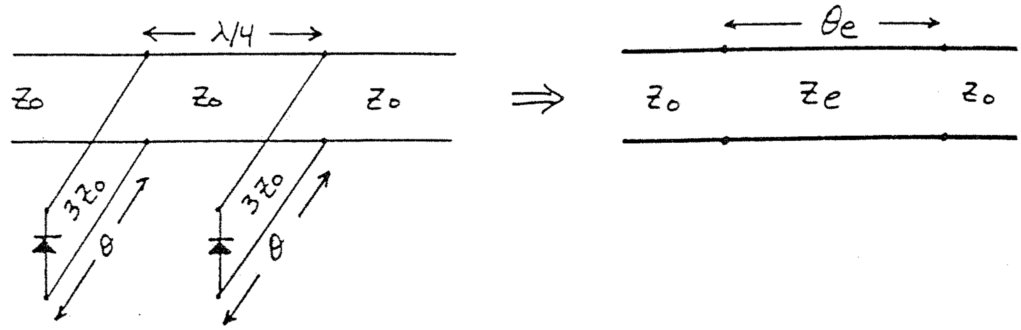
$IL = 0.044\,dB$

OFF STATE: $Z_1 = 0.5 - j250\Omega$, $Z_2 = 0.3 + j15\Omega$

$S_{21} = 0.118\angle{149^\circ}$

$IL = 18.6\,dB$

(10.17)



From (10.65), $\cos\theta_e = -b$

$Z_e = Z_0/\sqrt{1-b^2}$

where b is the normalized stub susceptance.

For diodes ON, $b = \frac{-1}{3}\cot\theta$

$\cos\theta_e = \frac{1}{3}\cot\theta$

For diodes OFF, $b = 1/3 \tan\theta$

$\cos\theta_e = \frac{-1}{3}\tan\theta$

So $\Delta\phi = 45° = \cos^{-1}(\frac{1}{3}\cot\theta) - \cos^{-1}(\frac{-1}{3}\tan\theta)$ (ON-OFF)

Solving this equation numerically:

θ	$\Delta\phi$
110°	73°
120°	46°
130°	40°
122°	44.3°
121°	45.2°

So we choose $\theta = 121°$. (Using $\theta = 31°$ gives $\Delta\phi = -45°$)

Insertion loss for $\theta = 121°$:

Using (10.64), $b = BZ_0$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-BZ_0 + j(1 - B^2Z_0^2) - BZ_0} = \frac{2}{-2b + j(2-b^2)}$$

$$|S_{21}|^2 = \frac{4}{4b^2 + (2-b^2)^2}$$

DIODES ON:

$b = \frac{-1}{3} \cot\theta = 0.20$

$|S_{21}|^2 = 0.9996$

$IL = 0.0017\,dB \sim 0\,dB$ ✓

DIODES OFF:

$b = \frac{1}{3} \tan\theta = -0.555$

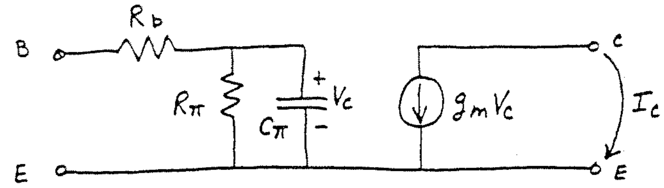
$|S_{21}|^2 = 0.977$

$IL = 0.102\,dB$ ✓

(SuperCompact analysis gives $IL_{ON} = 0\,dB$, $\phi_{ON} = -101.5°$, $IL_{OFF} = 0.10\,dB$, $\phi_{OFF} = -56.7°$, thus $\Delta\phi = 44.8°$)

Chapter 11

11.1 Unilateral bipolar transistor model:



From (11.1) the short-circuit current gain is,

$$G_i^{sc} = \left|\frac{I_c}{I_b}\right|\Bigg|_{V_{ce}=0} = \frac{g_m V_c}{I_b} = \frac{g_m I_b \left|\frac{R_\pi / j\omega C_\pi}{R_\pi + 1/j\omega C_\pi}\right|}{I_b}$$

$$= g_m \frac{R_\pi}{|1 + j\omega R_\pi C_\pi|} = \frac{g_m}{\left|\frac{1}{R_\pi} + j\omega C_\pi\right|} \simeq \frac{g_m}{\omega C_\pi} \quad \text{since } R_\pi \gg 1/\omega C_\pi$$

(e.g., if $R_\pi = 110\Omega$, $C_\pi = 18pF$, $f = 1GHz$, then $1/\omega C_\pi = 9\Omega$)

11.2 $R_i = 7\Omega$, $C_{ds} = 0.12pF$, $R_{ds} = 400\Omega$, $C_{gs} = 0.3pF$, $C_{gd} = 0$, $g_m = 30mS$, $f = 5GHz$.

$$y_{11} = \frac{j\omega C_{gs}}{1 + j\omega R_i C_{gs}} = \frac{j0.00942}{1 + j0.06597} = 0.0094\angle 86^\circ = 0.00062 + j0.0094 \checkmark$$

$$y_{21} = \frac{g_m}{1 + j\omega R_i C_{gs}} = \frac{.03}{1 + j0.06597} = 0.03\angle 4^\circ \checkmark \qquad y_{12} = 0 \checkmark$$

$$y_{22} = \frac{1}{R_{ds}} + j\omega C_{ds} = 0.0025 + j0.00377 = 0.00452\angle 56.5^\circ \checkmark$$

$$\Delta y = (Y_{11} + Y_0)(Y_{22} + Y_0) - Y_{12}Y_{21} = (.0226\angle 24.5^\circ)(.0228\angle 9.5^\circ) = 0.000515\angle 34^\circ$$

$$S_{11} = \frac{(Y_0 - Y_{11})(Y_0 + Y_{22})}{\Delta Y} = \frac{Y_0 - Y_{11}}{Y_0 + Y_{11}} = \frac{.0215\angle -25.9^\circ}{.0226\angle 24.5^\circ} = 0.951\angle -50^\circ \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{-2Y_{21}Y_0}{\Delta y} = \frac{(-.04)(.03\angle 4^\circ)}{.000515\angle 34^\circ} = 2.33\angle 150^\circ$$

$$S_{22} = \frac{Y_0 - Y_{22}}{Y_0 + Y_{22}} = \frac{.0179\angle -12^\circ}{.0228\angle 9.5^\circ} = 0.785\angle -22^\circ \checkmark$$

If conjugately matched, the unilateral transducer gain is,

$$G_{TU} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 148.8 = 21.7\,dB$$

The corresponding result from the circuit model, as given in (11.21), is

$$G_{TU} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = 21.6\,dB$$

(These results were verified via SuperCompact.)

11.3 The [S] matrix for a 3-dB matched attenuator is,

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

For $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0,\ \Gamma_s = 0,\ \Gamma_{out} = 0$

Then from (11.15), (11.16), and (11.11) we have

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)} = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

For $Z_L = 25\Omega$: $\Gamma_L = -1/3$, $\Gamma_s = \Gamma_{out} = 0$, $\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)} = 0.457 \checkmark$$

FOR $Z_S = 25\,\Omega$, $Z_L = 50\,\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = -1/3$, $\Gamma_{out} = S_{22}' + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = -1/6$

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)} = 0.457$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{1} = 0.444$$

$$G = |S_{21}|^2 = 0.5$$

11.4 From (11.28) - (11.29) the centers and radii of the stability circles are:

$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.117\angle{-50^\circ}$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 2.56\angle 28^\circ \checkmark$$

$$R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| = 1.37 \checkmark$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 3.77\angle 174^\circ \checkmark$$

$$R_L = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| = 2.53 \checkmark$$

From (11.131),

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}S_{21}|} = 1.35 > 1 \quad \checkmark$$

Since $K > 1$ and $|\Delta| < 1$, the transistor is unconditionally stable.

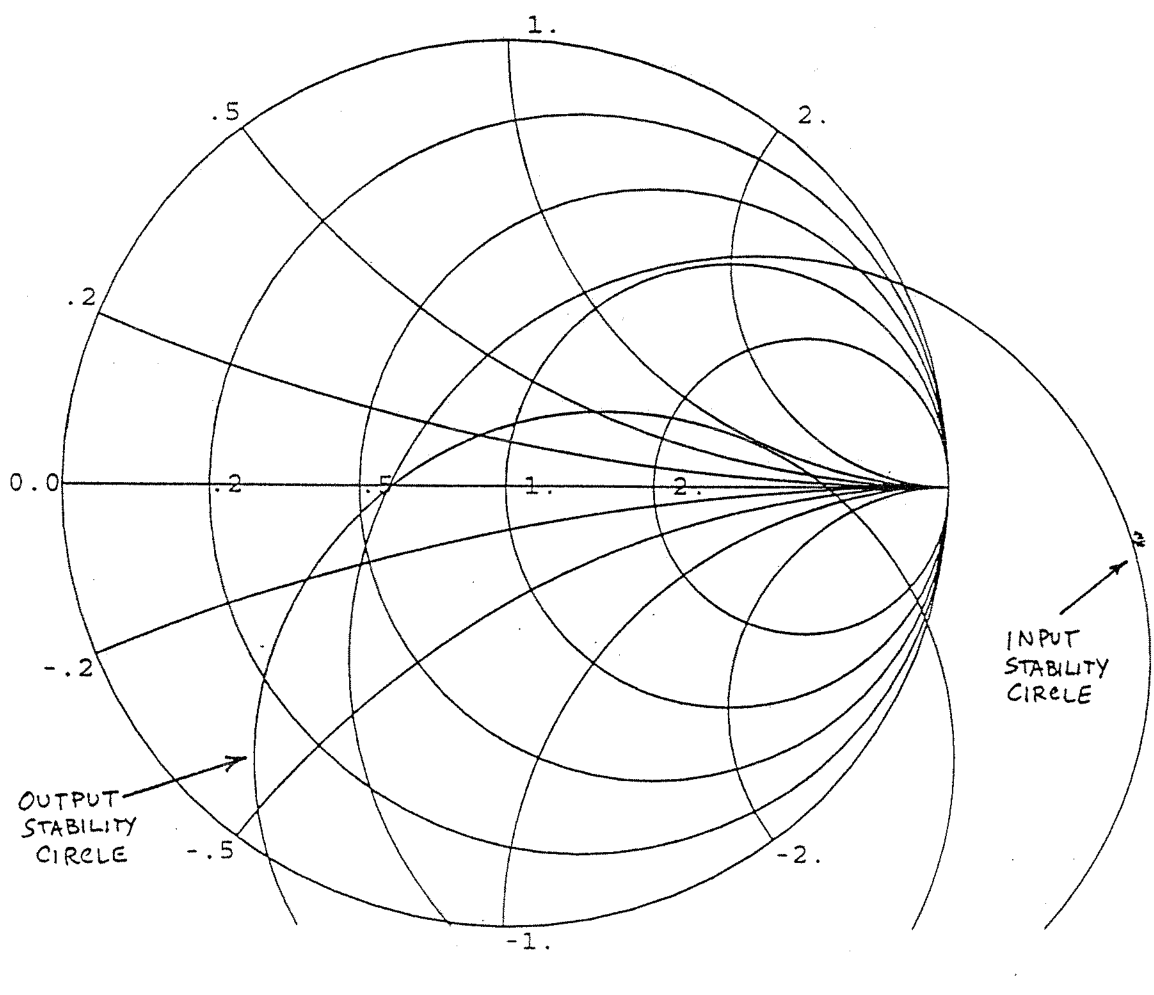
11.5 $S_{11} = 0.8\angle{-90°}$, $S_{12} = 0.3\angle{70°}$, $S_{21} = 5.1\angle{80°}$, $S_{22} = 0.62\angle{-40°}$

$\Delta = 1.52\angle{-49°}$

$C_L = 0.66\angle{-70°}$ $\quad$ $R_L = 0.79$

$C_S = 0.68\angle{-35°}$ $\quad$ $R_S = 0.91$ $\quad$ $K = 0.75$ ✓

Since $K<1$ the transistor is potentially unstable. The stability circles are plotted below.



11.6 Using (11.38) to compute μ:

DEVICE	S_{11}	S_{12}	S_{21}	S_{22}	μ	
A	$0.34\angle{-170°}$	$0.06\angle{70°}$	$4.3\angle{80°}$	$0.45\angle{-25°}$	1.193	UNC. STABLE
B	$0.75\angle{-60°}$	$0.2\angle{70°}$	$5.0\angle{90°}$	$0.5\angle{60°}$	0.283	POT. UNSTABLE
C	$0.65\angle{-140°}$	$0.04\angle{60°}$	$2.4\angle{50°}$	$0.7\angle{-65°}$	1.057	UNC. STABLE

Device A has the best stability.

11.7 From (11.38) the μ-parameter test is,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{21}S_{12}|} > 1$$

If $S_{12}=0$ (unilateral) then we have,

$$\Delta = S_{11}S_{22}$$

So,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2 S_{22}|} = \frac{1-|S_{11}|^2}{|S_{22}|\,|1-|S_{11}|^2|} > 1$$

Since the denominator is positive and μ is positive, the numerator must also be positive, thus $|S_{11}|<1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1 ,$$

So,

$$|S_{22}|<1 .$$

(11.8) $S_{11} = 0.65\angle{-140°}$, $S_{21} = 2.4\angle{50°}$, $S_{12} = 0.04\angle{60°}$, $S_{22} = 0.70\angle{-65°}$

First we check stability:

$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.393\angle{165°}$

$$K = \frac{1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2}{2|S_{12}S_{21}|} = 1.26$$

Since $|\Delta|<1$ and $K>1$ the transistor is unconditionally stable at 5 GHz. For maximum gain, the transistor should be conjugately matched: (using 11.43)

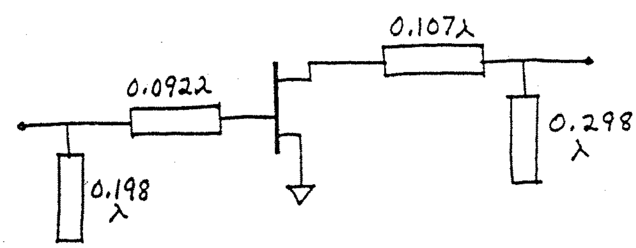
$\Gamma_S = \Gamma_{in}^* = 0.826\angle{147°}$ ✓

$\Gamma_L = \Gamma_{out}^* = 0.850\angle{71°}$ ✓

The gains can then be calculated as,

$G_S = 3.14$, $G_0 = 5.76$, $G_L = 1.64$

So the overall transducer gain is $G_T = 29.7 = 14.7\,dB$ ✓
Matching was done on a Smith chart. The final AC amplifier circuit is shown below:



SuperCompact analysis gives $|S_{11}| = 0.05$, $|S_{22}| = 0.04$, $G = 14.7\,dB$ ✓

(11.9) $S_{11} = 0.61\angle{-170^\circ}$, $S_{21} = 2.24\angle{32^\circ}$, $S_{12} = 0$, $S_{22} = 0.72\angle{-83^\circ}$

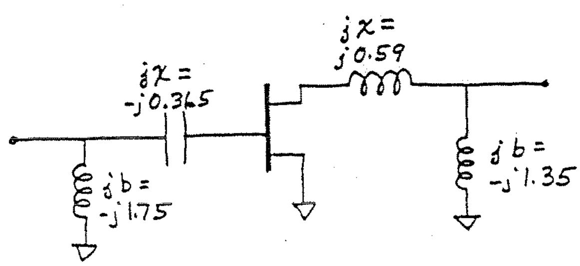
The transistor is unconditionally stable since $K = \infty$ and $|\Delta| < 1$.
Since the transistor is unilateral,

$$\Gamma_S = S_{11}^* = 0.61\angle{170^\circ} \checkmark \quad , \quad \Gamma_L = S_{22}^* = 0.72\angle{83^\circ} \checkmark$$

and the maximum gain is, from (11.45),

$$G_{TU_{MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 16.6 = 12.2\,dB$$

Matching was done with a Smith chart. The final circuit is:



The matching element values are, at 6 GHz,

$$C = \frac{-1}{\omega Z_0 x_c} = 1.45\,pF \checkmark \qquad L = \frac{Z_0 x_L}{\omega} = 0.78\,mH \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.76\,mH \checkmark \qquad L = \frac{-Z_0}{\omega b_L} = 0.98\,mH \checkmark$$

SuperCompact analysis gives $|S_{11}| = 0.035$, $|S_{22}| = 0.008$, and $G = 12.2\,dB$ ✓

11.10 $S_{11} = 0.61\angle{-170°}$, $S_{21} = 2.24\angle{32°}$; $S_{12} = 0$, $S_{22} = 0.72\angle{-83°}$

$G = 10dB$, $G_S = 1dB$, $G_L = 2dB$

Since $K = \infty$ and $|\Delta| < 1$, the transistor is unconditionally stable. From (11.48), we have

$$G_{S_{MAX}} = \frac{1}{1-|S_{11}|^2} = 1.59 \checkmark \quad , \quad G_{L_{MAX}} = \frac{1}{1-|S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1dB = 1.26$, and $G_L = 2dB = 1.58$, we have from (11.49),

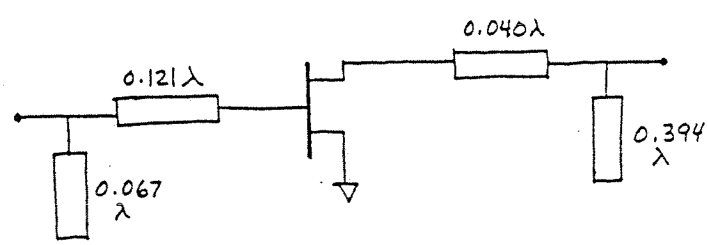
$$g_S = \frac{G_S}{G_{S_{MAX}}} = 0.792 \quad , \quad g_L = \frac{G_L}{G_{L_{MAX}}} = 0.760$$

Then the centers and radii of the constant gain circles can be found from (11.52)-(11.53):

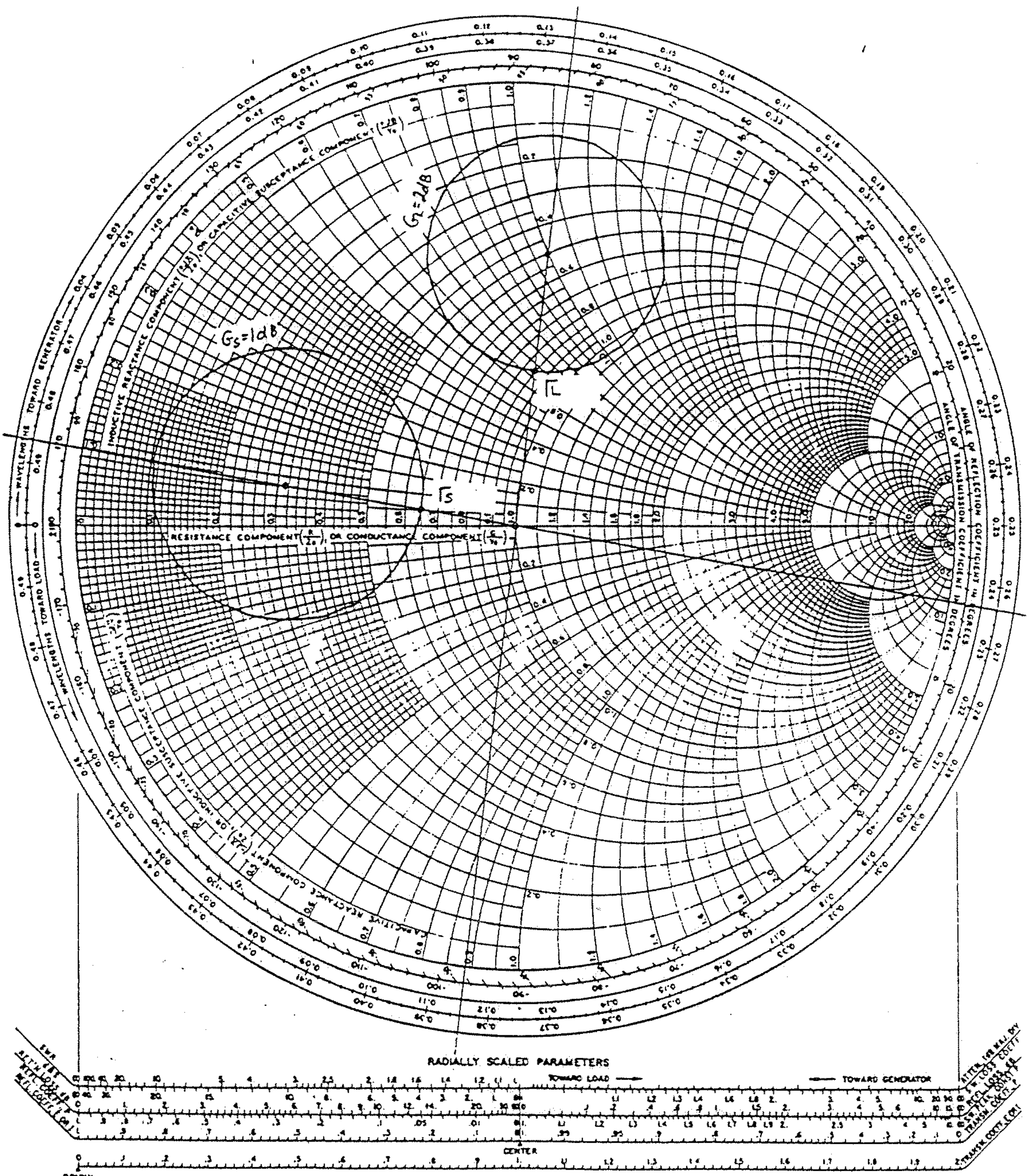
$C_S = 0.524\angle{170°}$ ✓ $\qquad C_L = 0.625\angle{83°}$ ✓

$R_S = 0.310$ ✓ $\qquad R_L = 0.269$ ✓

Since $G_0 = 10 \log |S_{21}|^2 = 7.0dB$, using the $G_S = 1dB$ and the $G_L = 2dB$ gain circles will give an overall gain of 10dB. We plot these circles on the Smith chart, and choose $\Gamma_S = 0.215\angle{170°}$ ✓ and $\Gamma_L = 0.361\angle{83°}$ ✓ to minimize the magnitude of these values. After matching, we have the following amplifier circuit:



SuperCompact analysis gives $|S_{11}| = 0.45$, $|S_{22}| = 0.48$, $G = 10.05dB$ ✓ (reflections at input and output serve to reduce the gain to 10dB). Smith chart shown on following page.



11.11 From (11.47) the unilateral figure of merit is,

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = \frac{(.06)(4.3)(.34)(.45)}{[1-(.34)^2][1-(.45)^2]} = 0.056$$

So from (11.46) this bounds the error in G_T/G_{TU} by,

$$0.897 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 1.122$$

or,

$$-0.47dB < G_T(dB) - G_{TU}(dB) < 0.5dB$$

11.12 From (11.48a) and (11.49a), when $G_S = 1$ we have,

$$g_s = \frac{1}{G_{S_{MAX}}} = 1 - |S_{11}|^2 \quad , \quad 1 - g_s = |S_{11}|^2$$

so (11.52) reduces to,

$$C_S = \frac{(1-|S_{11}|^2)\, S_{11}^*}{1 - |S_{11}|^4} = \frac{S_{11}^*}{1+|S_{11}|^2}$$

$$R_S = \frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4} = \frac{|S_{11}|}{1+|S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left| \Gamma_S - \frac{S_{11}^*}{1+|S_{11}|^2} \right| = \frac{|S_{11}|}{1+|S_{11}|^2}$$

one solution to this equation occurs for $\Gamma_S = 0$, so the circle must pass through the center of the Smith chart.

11.13 $S_{11} = 0.7 \angle -110°$, $S_{12} = 0.02 \angle 60°$, $S_{21} = 3.5 \angle 60°$, $S_{22} = 0.8 \angle -70°$

$F_{MIN} = 2.5dB$, $\Gamma_{OPT} = 0.7 \angle 120°$, $R_N = 15\Omega$

First check stability: $K = 1.07$, $|\Delta| = 0.53$

Since $K>1$ and $|\Delta|<1$ the device is unconditionally stable. Minimum noise figure occurs for $\Gamma_S = \Gamma_{OPT} = 0.7 \angle 120°$. Then we maximize gain by conjugate matching the output. From (11.41b),

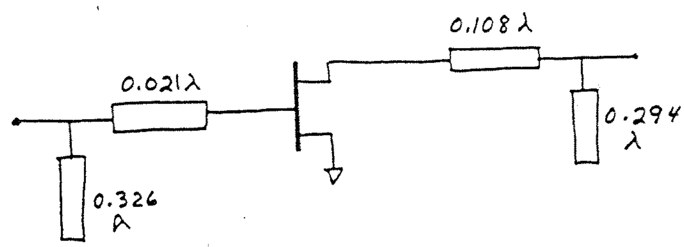
$$\Gamma_L = \left(S_{22} + \frac{S_{12} S_{21} \Gamma_S^*}{1 - S_{11}\Gamma_S} \right)^* = 0.873 \angle 74° \ ✓$$

So the noise figure will be $F = F_{min} = 2.5dB$, and the gain will be,

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|S_{22}|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= (1.85)(12.25)(3.81) = 86.3 = 19.4\,dB$$

Impedance matching is done with a Smith chart; the final amplifier circuit is shown below.



SuperCompact analysis of this amplifier gives $|S_{11}| = 0.33$, $|S_{22}| = 0.13$, $G = 19.7$ dB ✓, and $F = 2.5$ dB. ✓ The data file for this analysis is listed below. (The solution is a bit simpler if S_{12} is set to zero, resulting in $G = 18$ dB.)

```
Compact Software - SUPER-COMPACT PC V6.52  27-JUN-97  17:07:09
File: C:\COMPACT\PRB11_13.CKT

NOI
OST 1 0 z=50 e=117. f=5.GHZ
TRL 1 2 z=50 e=8.   f=5.GHZ
TWO 2 3 Q
TRL 3 4 z=50 e=39.  f=5.GHZ
OST 4 0 z=50 e=106. f=5.GHZ
AMP: 2POR 1 4
END
FREQ
5.GHZ
END
OUT
PRI AMP S
END
DATA
Q: S
5GHZ .7  -110.  3.5  60.  0.02  60.  .8  -70.
NOI RN
5.GHZ 2.5 .7 120. .3
END
```

(11.14) $S_{11} = 0.6\angle{-60°}$, $S_{21} = 2.\angle{181°}$, $S_{12} = 0$, $S_{22} = 0.7\angle{-60°}$

$F_{MIN} = 2.0\,dB$, $\Gamma_{OPT} = 0.62\angle{100°}$, $R_N = 20\,\Omega$

Since $S_{12}=0$ and $|S_{11}||S_{22}|<1$, the device is unconditionally stable. The overall gain is, $G_{TU} = G_S G_0 G_L$, where $G_0 = |S_{21}|^2 = 4 = 6dB$. ✓ So $G_S + G_L = 0\,dB$.

Plot noise figure circles for $F = 2.0, 2.05, 2.2$, and $3.0\,dB$:

F(dB)	N	C_F	R_F
2.05	0.0134	$0.61\angle{100°}$	0.09
2.20	0.055	$0.59\angle{100°}$	0.18
3.00	0.30	$0.48\angle{100°}$	0.40
2.00	0.	$0.62\angle{100°}$	0

Now plot constant gain circles for $G_S = G_L = 0\,dB$:

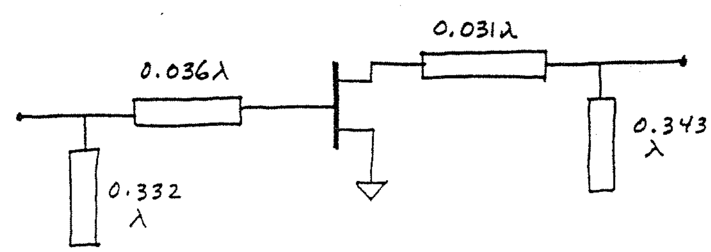
$G_{S_{MAX}} = 1.56$ ✓ $\qquad G_{L_{MAX}} = 1.96$ ✓

$g_S = 0.641 \qquad g_L = 0.510$

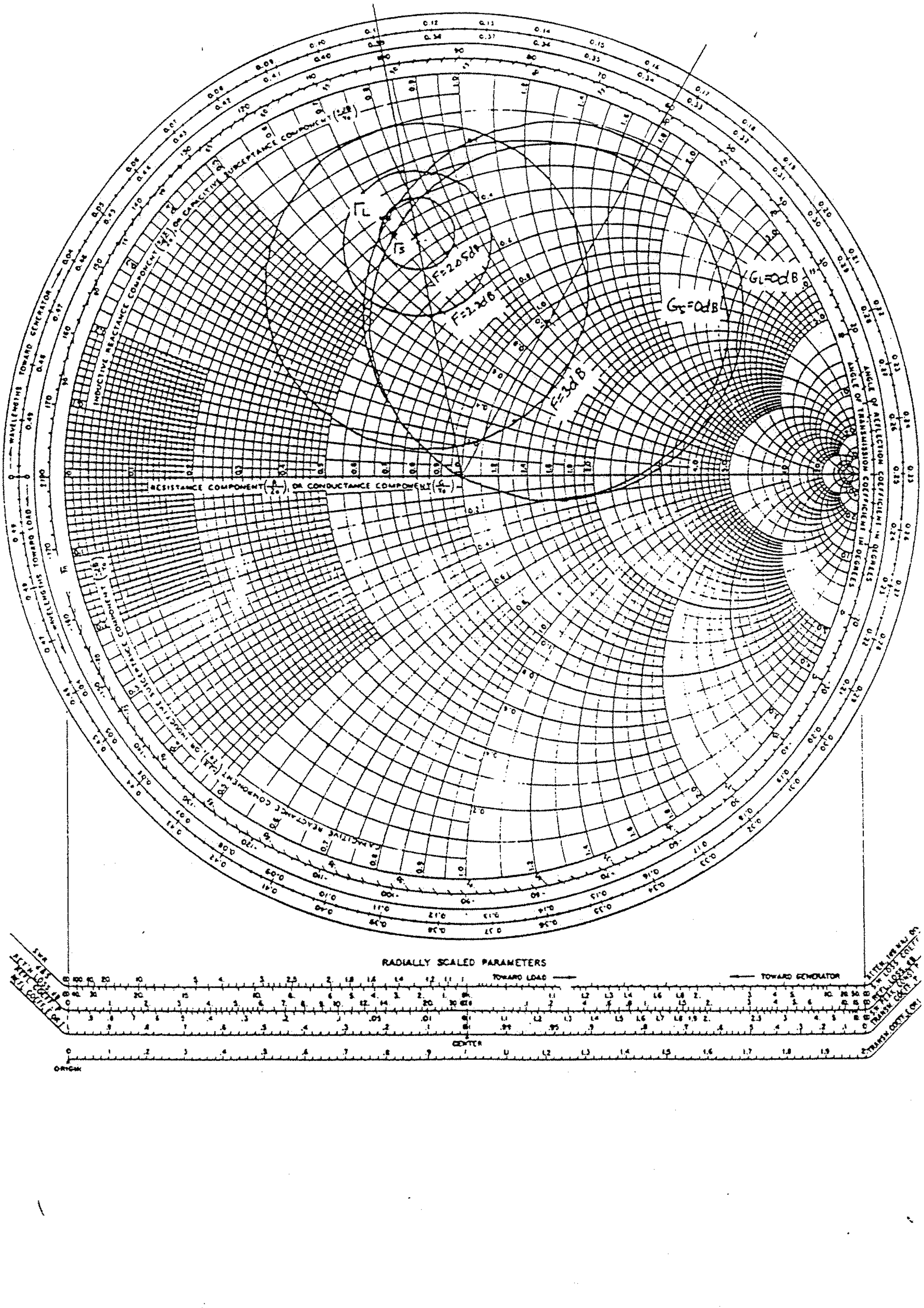
$C_S = 0.44\angle{60°} \qquad C_L = 0.47\angle{60°}$

$R_S = 0.44 \qquad R_L = 0.47$

These two circles are close together near the $F = 2dB$ point. We choose $\Gamma_L = 0.66\angle{105°}$, $\Gamma_S = 0.62\angle{105°}$. Then we should obtain $F \simeq 2.04dB$. The final AC amplifier circuit is:



Super Compact analysis gives $|S_{11}| = 0.62$, $|S_{22}| = 0.67$, $G = 6.1dB$, and $F = 2.04dB$ ✓ The gain and noise circles are shown below.

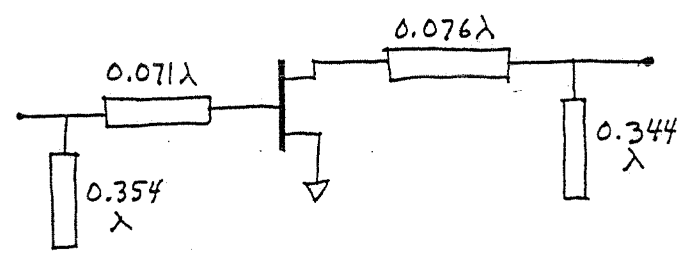

WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT
OR CAPACITIVE SUSCEPTANCE COMPONENT
RESISTANCE COMPONENT, OR CONDUCTANCE COMPONENT
CAPACITIVE REACTANCE COMPONENT
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
Γ_L
Γ_S
F=2.05dB
F=2.2dB
F=3dB
G_T=0dB
G_L=0dB
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN

11.15 S-parameters and noise parameters of Problem 11.14
Plot the $F = 2.5\,dB$ constant noise figure circle:
$N = 0.141$, $C_F = 0.543\angle 100^\circ$, $R_F = 0.286$

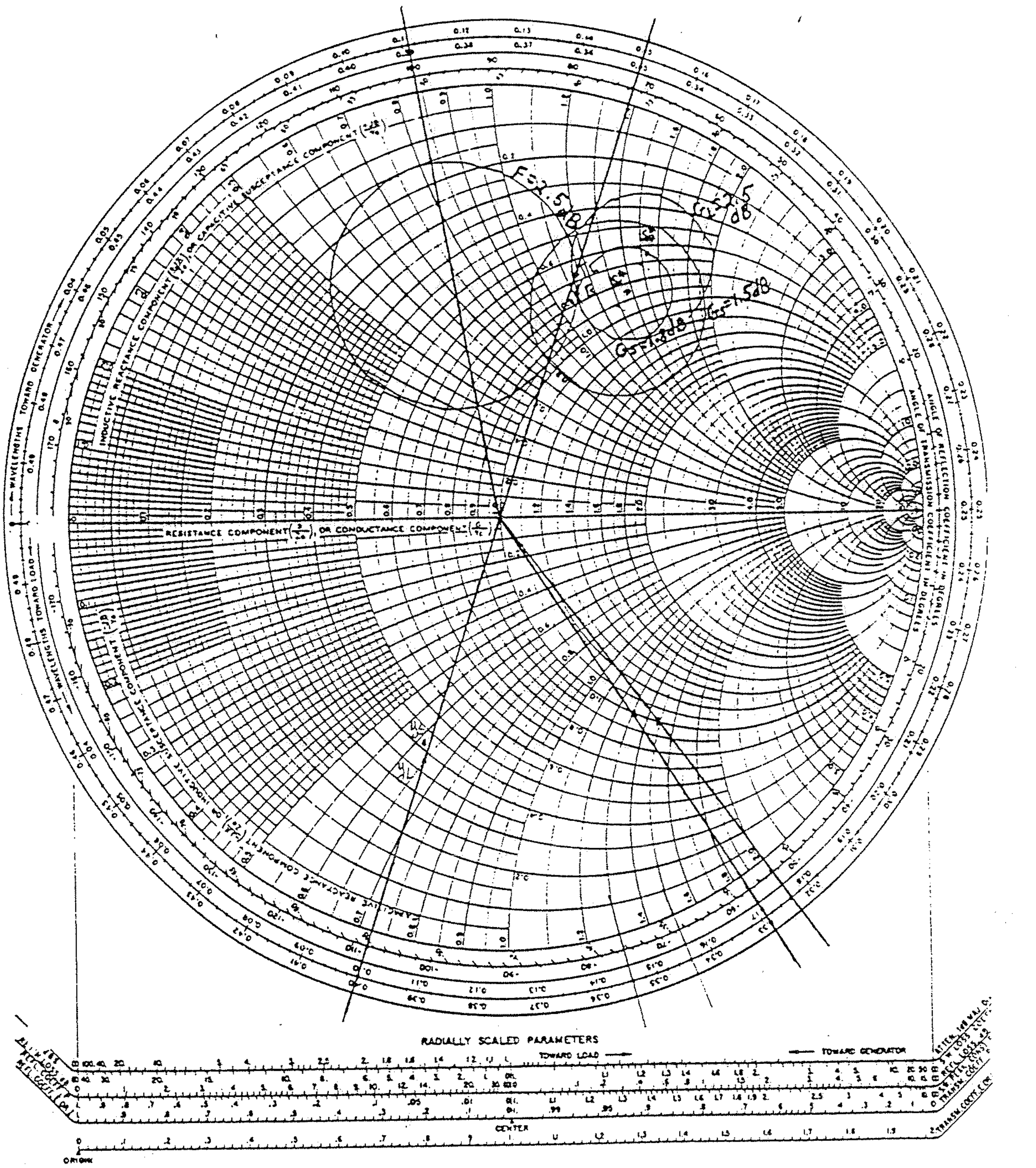
Now, $G_{S_{MAX}} = 1.56 = 1.93\,dB$, $G_{L_{MAX}} = 1.96 = 2.92\,dB$
But these points (S_{11}^*, S_{22}^*) do not lie on the $F = 2.5\,dB$ circle. We can plot some gain circles to just give intersections with the $F = 2.5\,dB$ noise circle:

$G_S = 1.5\,dB$	$g_S = 0.905$	$C_S = 0.56\angle 160^\circ$	$R_S = 0.204$
$G_L = 2.5\,dB$	$g_L = 0.907$	$C_L = 0.67\angle 160^\circ$	$R_L = 0.163$
$G_S = 1.7\,dB$	$g_S = 0.948$	$C_S = 0.58\angle 160^\circ$	$R_S = 0.149$
$G_S = 1.8\,dB$	$g_S = 0.970$	$C_S = 0.59\angle 160^\circ$	$R_S = 0.112$

The $G_S = 1.8\,dB$ and $G_L = 2.5\,dB$ circles are close to optimum (the $F = 2.5\,dB$ noise circle). Thus we have $\Gamma_S = 0.545\angle 70^\circ$, $\Gamma_L = 0.59\angle 72^\circ$, which will yield a gain of $G_T = 1.8 + 2.5 + 6 = 10.3\,dB$.
The final AC amplifier circuit is shown below:



SuperCompact analysis of this circuit gives $|S_{11}| = 0.20$, $|S_{22}| = 0.28$, $G = 10.3\,dB$ ✓, and $F = 2.4\,dB$ ✓. The noise and gain circles are shown on the following page.



WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT
CAPACITIVE SUSCEPTANCE COMPONENT
RESISTANCE COMPONENT
CONDUCTANCE COMPONENT
CAPACITIVE REACTANCE COMPONENT
INDUCTIVE SUSCEPTANCE COMPONENT
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN
Gs=1.5dB
Gs=1dB

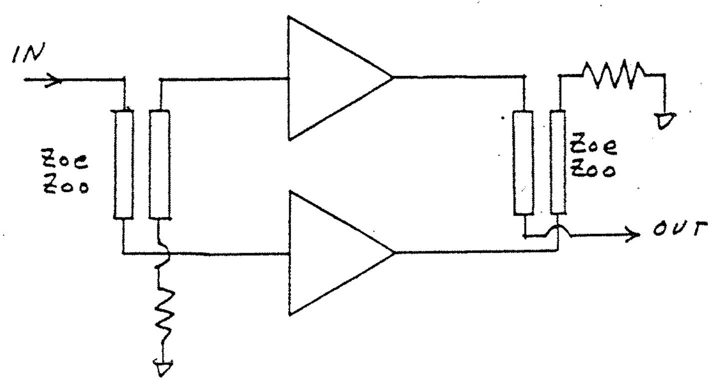
11.16

FOR THE COUPLED LINE COUPLERS:

$$C = 10^{-3/20} = 0.708$$

$$Z_{oe} = 50\sqrt{\frac{1+c}{1-c}} = 121\,\Omega$$

$$Z_{oo} = 50\sqrt{\frac{1-c}{1+c}} = 21\,\Omega$$

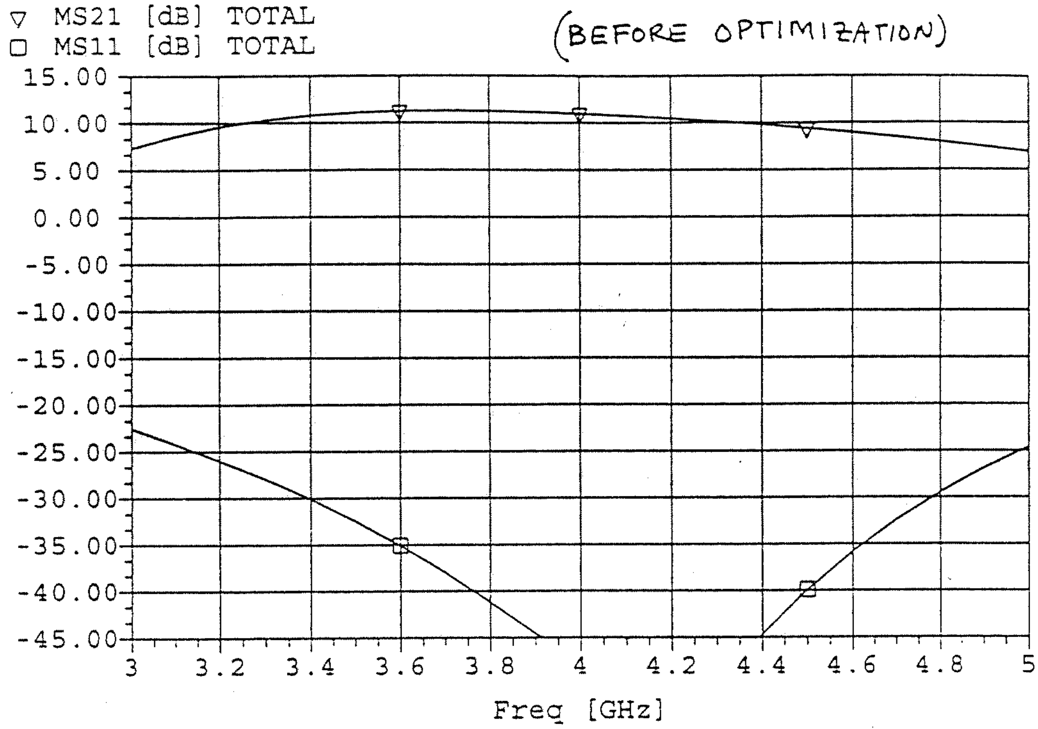


The amplifier circuit of Example 11.4 was used for both amplifiers here. As in Example 11.6, the amplifier matching networks were optimized using SuperCompact to give a flat 10dB gain response, with good input matching. Results of the optimization are given below, including the line and stub lengths before and after optimization, the SuperCompact data file, and the calculated gain and input return loss of the balanced amplifier before and after optimization. Results seem to be a bit better than those of Example 11.6.

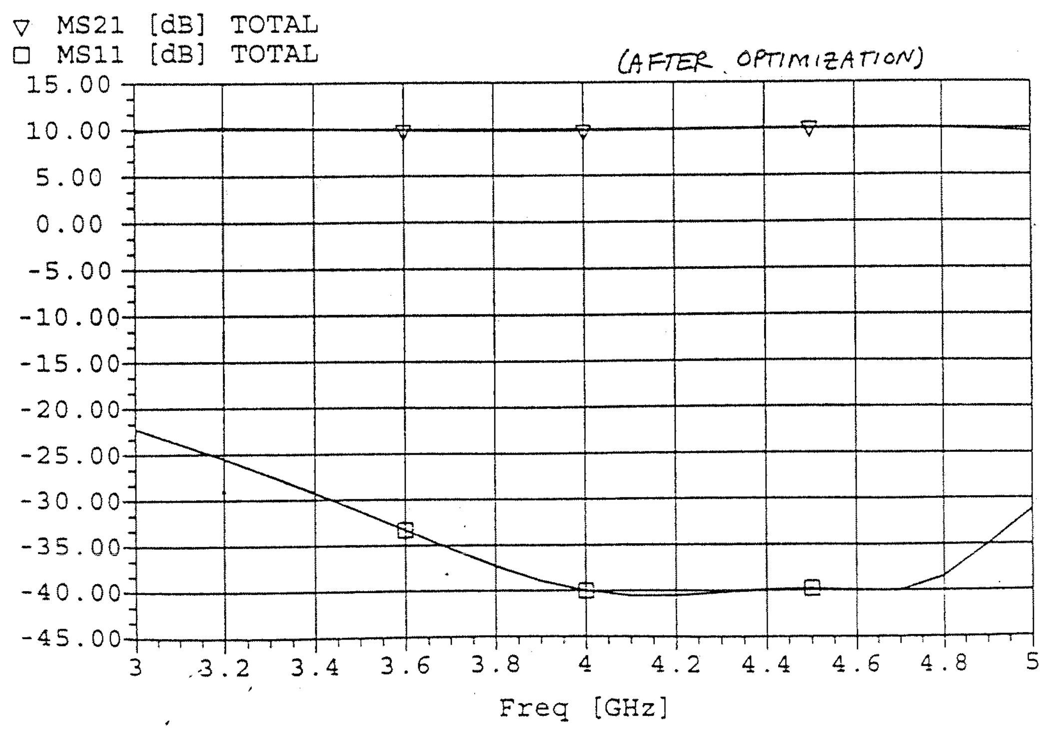
PARAMETER	BEFORE OPT.	AFTER OPT.
INPUT SECTION STUB LENGTH	0.100λ	0.125λ
INPUT SECTION LINE LENGTH	0.179λ	0.119λ
OUTPUT SECTION LINE LENGTH	0.045λ	0.089λ
OUTPUT SECTION LINE LENGTH	0.432λ	0.458λ

```
Compact Software - SUPER-COMPACT PC V6.52  16-MAY-97  16:25:15
File: c:\compact\prb11_16.ckt

LAD
OST 1 0 Z=50. E=?36.? F=4GHZ
TRL 1 2 Z=50. E=?64.? F=4GHZ
TWO 2 3 Q
TRL 3 4 Z=50. E=?16.? F=4GHZ
OST 4 0 Z=50. E=?156.? F=4GHZ
AMP: 2POR 1 4
END
BLK
cpl 1 2 3 4 ZE=121. ZO=21. E=90. F=4GHZ
clc: 4POR 1 2 3 4
END
BLK
clc 1 2 3 4
AMP 2 8
AMP 4 6
clc 5 6 7 8
RES 3 0 R=50.
RES 7 0 R=50.
TOTAL: 2POR 1 5
END
FREQ
STEP 3GHZ 5GHZ 100MHZ
END
OPT
total MS21=10DB MS11
END
OUT
PRI total S
END
DATA
Q: S
3GHZ .80  -90.  2.80  100.  .0  0.  .66  -50.
4GHZ .75  -120. 2.50  80.   .0  0.  .60  -70.
5GHZ .71  -140. 2.30  60.   .0  0.  .58  -85.
END
```

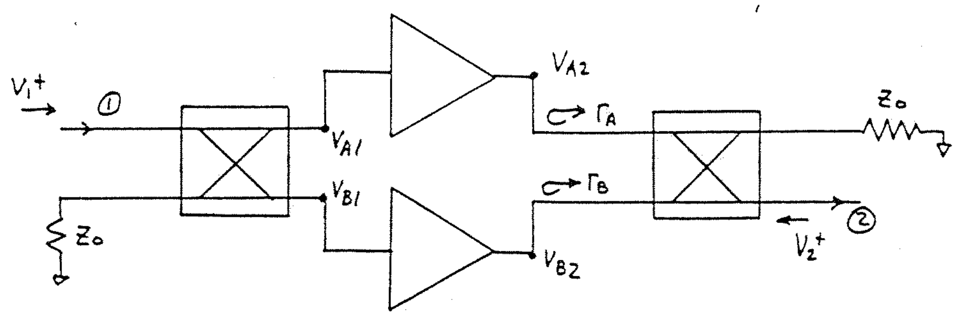


▽ MS21 [dB] TOTAL
□ MS11 [dB] TOTAL
(BEFORE OPTIMIZATION)
15.00
10.00
5.00
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
-45.00
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8
5
Freq [GHz]



▽ MS21 [dB] TOTAL
□ MS11 [dB] TOTAL
(AFTER OPTIMIZATION)
15.00
10.00
5.00
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
-45.00
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
4.8
5
Freq [GHz]

11.17



The analysis for S_{22} is identical to that for S_{11} in eqs (11.62)–(11.66), but with input V_2^+ at port 2.

Thus, if the input at port 2 is V_2^+, then the voltages incident at the amplifiers are,

$$V_{A2}^- = \frac{1}{\sqrt{2}} V_2^+$$
$$V_{B2}^- = \frac{-j}{\sqrt{2}} V_2^+$$

Then the reflected output voltage at port 2 is,

$$V_2^- = \frac{1}{\sqrt{2}} V_{A2}^+ + \frac{-j}{\sqrt{2}} V_{B2}^+ = \frac{1}{\sqrt{2}} \Gamma_A V_{A2}^- + \frac{-j}{\sqrt{2}} \Gamma_B V_{B2}^-$$
$$= \frac{1}{2} V_2^+ (\Gamma_A - \Gamma_B)$$

Thus, $$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{1}{2}(\Gamma_A - \Gamma_B) \checkmark$$

11.18 From (11.77), $$G = \frac{g_m^2 Z_d Z_g}{4} \frac{(e^{-N\alpha_g l_g} - e^{-N\alpha_d l_d})^2}{(e^{-\alpha_g l_g} - e^{-\alpha_d l_d})^2}$$

differentiating with respect to N and setting to zero gives,

$$\alpha_g l_g e^{-N\alpha_g l_g} - \alpha_d l_d e^{-N\alpha_d l_d} = 0$$
$$\ln \alpha_g l_g - N\alpha_g l_g = \ln \alpha_d l_d - N\alpha_d l_d$$
$$\ln \frac{\alpha_g l_g}{\alpha_d l_d} = N(\alpha_g l_g - \alpha_d l_d)$$
$$N = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} \checkmark$$

11.19 $R_i = 5\Omega$, $R_{ds} = 200\Omega$, $C_{gs} = 0.35\,pF$, $g_m = 40\,mS$

Assume $Z_g = Z_d = 50\Omega$.

We use (11.69) and (11.72) to find $\alpha_g l_g$, $\alpha_d l_d$. Then use (11.77) to find G:

f	G(N=4)	G(N=8)	G(N=16)
2	9.9dB	14.0dB	16.5dB
4	9.8	13.7	15.9
8	9.2	12.5	13.4
12	8.4	10.7	9.5
16	7.2	8.4	4.7
18	6.5	7.0	2.2
20	5.7	5.6	-0.2

Gain at f = 18 GHz vs. N:

N	G(dB)
5	7.1
6	7.3
7	7.2
8	7.0
9	6.7

So the optimum value of N is seen to be $N_{opt} = 6$.
Using (11.78) to find N_{opt} directly:

$$\alpha_g l_g = 0.1958$$

$$\alpha_d l_d = 0.125$$

$$N_{opt} = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} = 6.33$$

(11.20) Let $z = R + jX$, $R > 0$ (POSITIVE RESISTANCE)

Then $\mathfrak{z} = z/z_0 = r + jx$, $r > 0$

$$\Gamma = \frac{\mathfrak{z}-1}{\mathfrak{z}+1} = \frac{(r-1)+jx}{(r+1)+jx}$$

Now let $z = -R + jX$, $R > 0$ (NEGATIVE RESISTANCE)

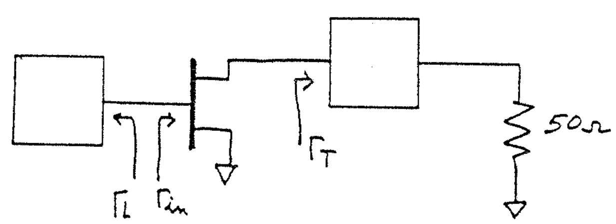
$\mathfrak{z} = -r + jx$, $r > 0$

Then,
$$\Gamma = \frac{\mathfrak{z}-1}{\mathfrak{z}+1} = \frac{-(r+1)+jx}{(-r+1)+jx} = \frac{(r+1)-jx}{(r-1)-jx}$$

So,
$$\frac{1}{\Gamma^*} = \frac{(r-1)+jx}{(r+1)+jx}$$

which has the same form as Γ for positive resistance. So we can read the resistance circles as negative, and interpret the "reflection coefficient" read from the Smith chart as $1/\Gamma^*$.

(11.21) $S_{11} = 0.9\angle{-150^\circ}$, $S_{21} = 2.6\angle{50^\circ}$, $S_{12} = 0.2\angle{-15^\circ}$, $S_{22} = 0.5\angle{-105^\circ}$



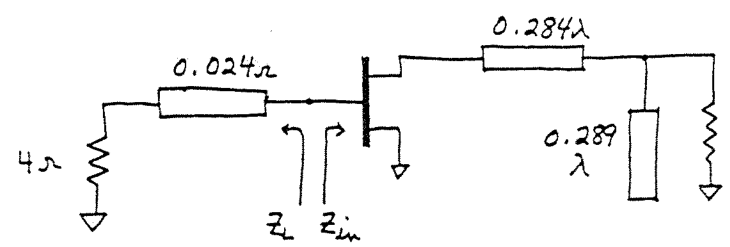
The output stability circle is,

$$C_T = 8.09\angle{-15} \quad , \quad R_T = 8.28$$

From (11.6a),

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{22}\Gamma_T}$$

Choose Γ_T so that $|\Gamma_T| < 1$ and Γ_{in} is large. By trial-and-error, we select $\Gamma_T = 0.9\angle{130^\circ}$. Then $\Gamma_{in} = 1.61\angle{-162^\circ}$, or $Z_{in} = -12. - j7.5\Omega$. So $Z_L = -R_{in}/3 - jX_{in} = 4 + j7.5\Omega = (0.08 + j0.15)Z_0$. Matching networks were designed on the Smith chart. The final AC circuit is shown below.



11.22 From (11.79), we know that $Z_L + Z_{in} = 0$.

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$, $\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$

$Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$, $Z_{in} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$

So,

$$\frac{1+\Gamma_L}{1-\Gamma_L} + \frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 0$$

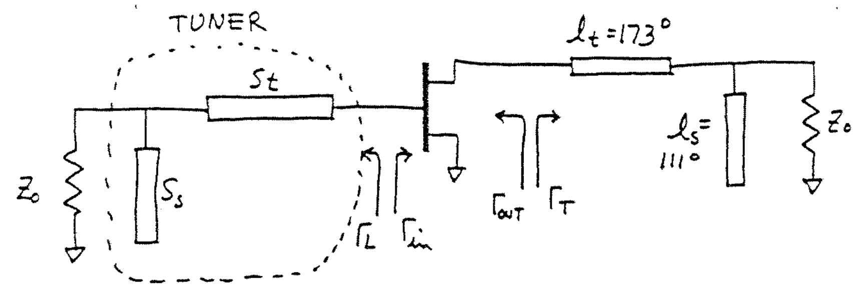
$$(1+\Gamma_L)(1-\Gamma_{in}) + (1+\Gamma_{in})(1-\Gamma_L) = 0$$

$$1 - \cancel{\Gamma_{in}} + \cancel{\Gamma_L} - \Gamma_L\Gamma_{in} + 1 - \cancel{\Gamma_L} + \cancel{\Gamma_{in}} - \Gamma_{in}\Gamma_L = 0$$

$$2 - 2\Gamma_{in}\Gamma_L = 0$$

$$\Gamma_{in}\Gamma_L = 1 \quad ✓$$

(11.23)

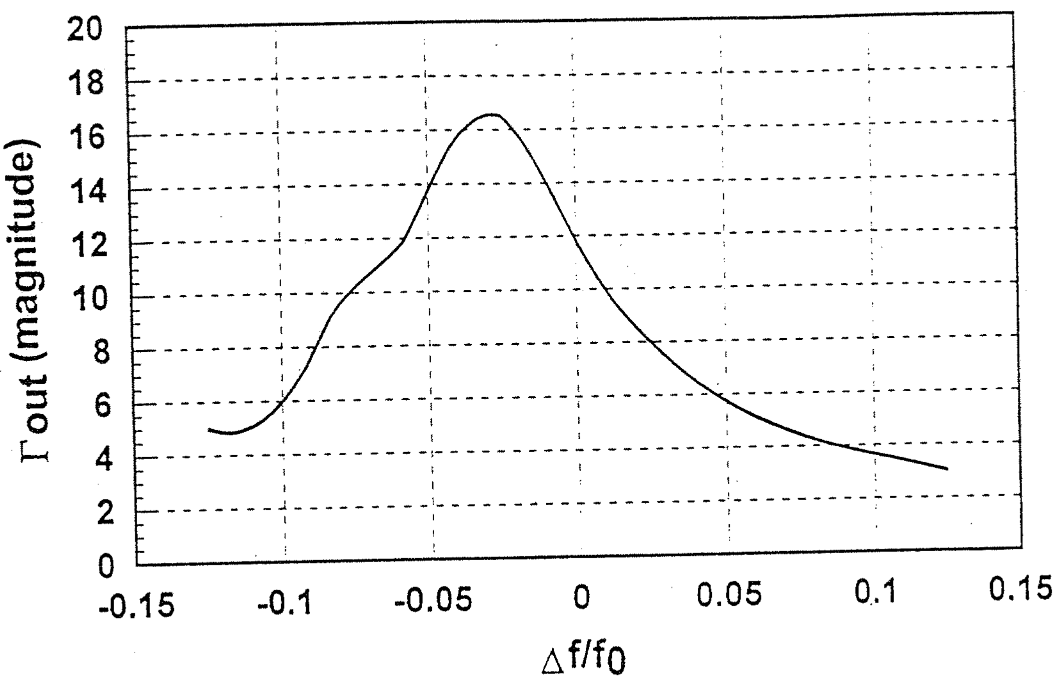


As in Example 11.10, choose $\Gamma_L = 0.6\angle{-130°}$. The Γ_{out}, Z_{out}, Z_T, ℓ_t, and ℓ_s are unchanged. Then we have the simple matching problem of using the stub tuner to match 50Ω to Γ_L. The stub susceptance is $jb_s = +j1.56$, or a stub length of $S_s = 0.158\lambda$. The line length is $S_t = 0.18 - 0.176 = 0.004\lambda$.

We then analyze the above circuit to compute Γ_{out} versus frequency:

f (GHz)	$\Delta f/f_0$	$\lvert\Gamma_{out}\rvert$
2.10	−0.125	5.0
2.18	−0.092	7.2
2.20	−0.083	9.1
2.26	−0.058	11.9
2.30	−0.042	15.4
2.34	−0.025	16.5
2.38	−0.008	13.6
2.40	0	11.8
2.42	0.008	10.2
2.46	0.025	7.9
2.50	0.042	6.3
2.60	0.083	4.1
2.66	0.110	3.4
2.70	0.125	3.0

The maximum of $\lvert\Gamma_{out}\rvert$ does not occur at $\Delta f = 0$ because the tuner is not resonant at f_0. The "Q" is much lower than in Example 11.10. This problem shows the advantage of using a high-Q resonator for the oscillator. $\lvert\Gamma_{out}\rvert$ vs. f is plotted on the following page.



(11.24) $S_{11} = 1.2\angle 150°$, $S_{12} = 0.2\angle 120°$, $S_{21} = 3.7\angle -72°$, $S_{22} = 1.3\angle -67°$

as in Example 11.10, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \simeq 1$, since

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8\angle -150°$ ✓. Then $\Gamma_{out} = 15.88\angle -99.3°$, and

$$Z_{out} = Z_0 \frac{1+\Gamma_{out}}{1-\Gamma_{out}} = -7.6 + j1.9\,\Omega$$

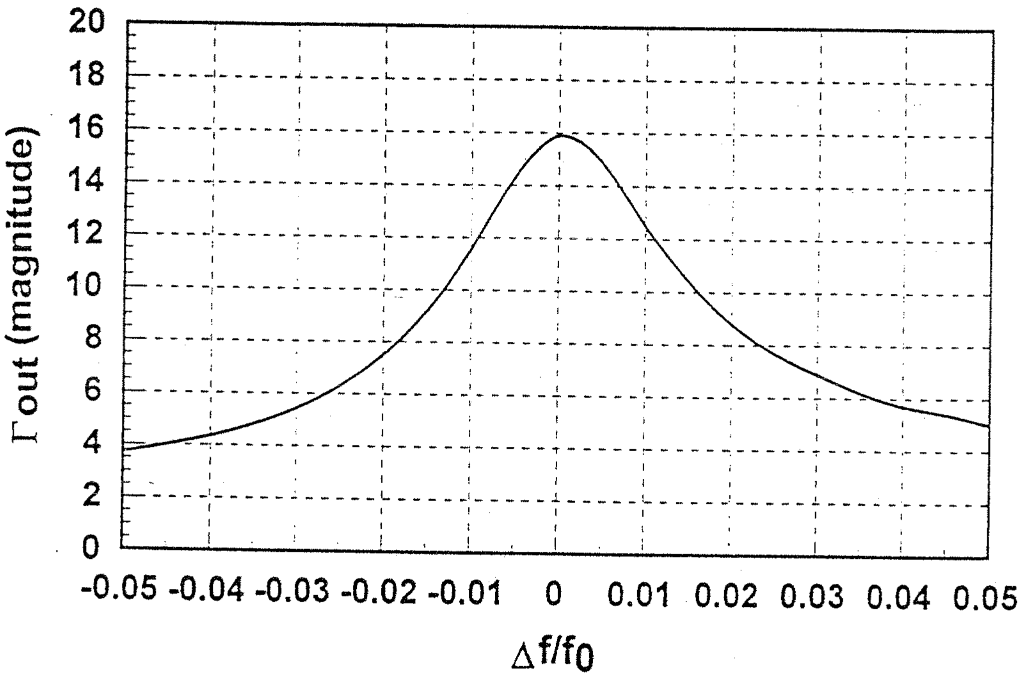
$$Z_T = \frac{-R_{out}}{3} - jX_{out} = 2.53 - j1.9\,\Omega \quad (z_T = 0.0506 - j0.038)$$

matching Z_T to the load impedance gives $l_t = 0.031\lambda$ with a required stub susceptance of $+j4$. Thus $l_s = 0.21\lambda$.

at the dielectric resonator, $\Gamma_L' = \Gamma_L e^{2j\beta l_r} = (0.8\angle -150°)e^{2j\beta l_r} = 0.8\angle 180°$. Thus $l_r = 0.4583\lambda$.

$$z_L' = z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'} = 5.55\,\Omega = N^2 R$$

$|\Gamma_{out}|$ vs f was calculated with SuperCompact, and is plotted below:



Chapter 12

12.1 From (12.10) and (12.11) the power density from an antenna with gain G is,

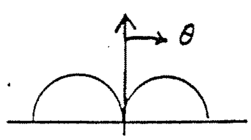
$$S = \frac{P_{in} G}{4\pi r^2} \quad W/m^2$$

So,

$$G = \frac{4\pi r^2 S}{P_{in}} = \frac{4\pi (5000)^2 (1.27\times10^{-6})}{100} = 4.0 = 6\,dB$$

12.2

$$E_\theta = \frac{C}{r} \sin\theta \, e^{-jk_o r} \quad \text{for } 0 \le \theta \le 90^\circ$$



From (12.2) and (12.1) the directivity is,

$$D = \frac{4\pi F_{MAX}}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi}$$

$$F(\theta,\phi) = r^2 S(\theta,\phi) = \frac{r^2}{\eta_o}|E_\theta|^2 = A\sin^2\theta \quad \text{for } 0 \le \theta \le \pi/2$$

Then

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi = 2\pi A \int_{\theta=0}^{\pi/2} \sin^3\theta \, d\theta = 2\pi A\left(\frac{2}{3}\right) = \frac{4\pi A}{3}$$

Thus,

$$D = \frac{4\pi A}{4\pi A/3} = 3 = 4.77\,dB \checkmark$$

12.3 From (12.5), (12.6), and (12.11),

$$P_\ell = A_e S = \frac{G\lambda^2 S}{4\pi} = \frac{(15.8)(.0375)^2(2\times10^{-6})}{4\pi}$$

$$= 3.54\times10^{-9}\,W \qquad (G = 15.8;\ \lambda = 0.0375\,m)$$

12.4 From (12.12), $T_e = \eta T_b + (1-\eta) T_p$

T_e = equivalent noise temperature of antenna

T_b = brightness temperature of background

T_p = physical temperature of antenna

Solving for η:

$$\eta = \frac{T_p - T_e}{T_p - T_b} = \frac{295 - 34.5}{295 - 5.} = 0.898$$

12.5 From the Friis power transmission formula of (12.15), the required transmitter power is, for $G_t = G_r = 10^{38/10} = 6310$, $\lambda = 0.061$ m, $P_r = 10^{-6}$ mW

$$P_t = \frac{(4\pi R)^2}{G_t G_r \lambda^2} P_r = \frac{(4\pi)^2 (27{,}000)^2}{(6310)^2 (0.061)^2} (10^{-6}) = 0.777 \text{ mW} = -1.1 \text{ dBm}$$

12.6 From (12.10) and (12.11),

$$S = \frac{P_{in} G}{4\pi r^2}$$

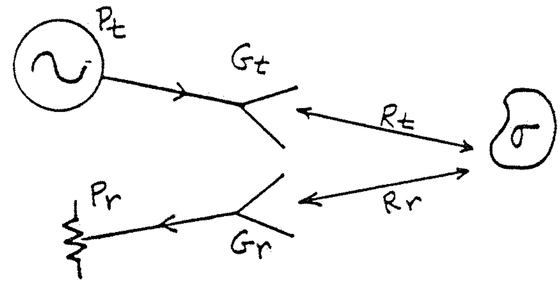
In the main beam of the antenna, $G = 10^{30/10} = 1000$, so,

$$S = \frac{10(1000)}{4\pi (30)^2} = 0.884 \text{ W/m}^2 = 0.088 \text{ mW/cm}^2$$

In the sidelobe region of the antenna, $G = 10^{15/10} = 31.6$, so,

$$S = \frac{10(31.6)}{4\pi (30)^2} = 0.028 \text{ W/m}^2 = 0.0028 \text{ mW/cm}^2$$

(12.7)



From (12.10)–(12.11) the power density incident on the target is,

$$S = \frac{P_t G_t}{4\pi R_t^2}$$

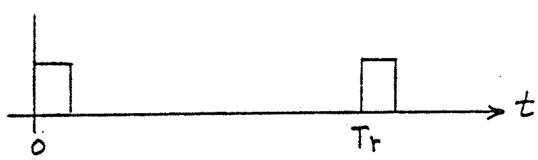
The scattered power density at the receiver is, from (12.26),

$$S_r = \frac{P_t G_t \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^2 R_t^2 R_r^2},$$

where $\sigma(\theta_t, \phi_t; \theta_r, \phi_r)$ is the radar cross-section of the target seen at θ_r, ϕ_r with an incident wave at θ_t, ϕ_t. Then the received power can be found using (12.14):

$$P_r = P_t \frac{G_t G_r \lambda^2 \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^3 R_t^2 R_r^2} \quad ✓$$

(12.8)



When a pulse is transmitted at $t=0$, the return pulse must come back before the next pulse in transmitted at $t=T_r$, to avoid an ambiguity in range. The round-trip time for a pulse return is,

$$T = 2R/c,$$

so the maximum unambiguous range is,

$$R_{MAX} = \frac{cT_r}{2} = \frac{c}{2f_r} \quad ✓$$

(12.9) From (12.29) the doppler frequencies are,

$$f_{d(MIN)} = \frac{2 v_{MIN} f_0}{c} = \frac{2(1\,\text{m/sec})(12\,\text{GHz})}{3\times 10^8\,\text{m/sec}} = 80\ \text{Hz}$$

$$f_{d(MAX)} = \frac{2 v_{MAX} f_0}{c} = \frac{2(20\,\text{m/sec})(12\,\text{GHz})}{3\times 10^8\,\text{m/sec}} = 1.6\ \text{kHz}$$

so the necessary passband is 80–1600 Hz.

(12.10) From (12.27) the received power is,

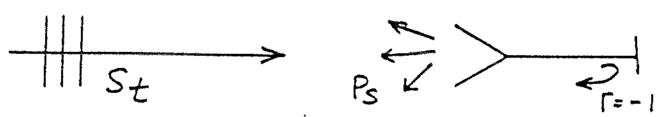
$$P_r = P_t \frac{G^2\lambda^2\sigma}{(4\pi)^3 R^4} = 1000\,\frac{(1000)^2(0.15)^2(20)}{(4\pi)^3(10^4)^4} = 2.27\times 10^{-11}\ \text{W}$$

$$= -76\ \text{dBm}$$

The transmitter power is $10\log(10^4)(10^3) = 70$ dBm. So the isolation between receiver and transmitter must be,

$$I = 70\ \text{dBm} - (-76\ \text{dBm}) + 10\ \text{dB} = 156\ \text{dB}$$

(12.11)



assume an incident plane wave with power density S_t. Then the received power of the antenna is, from (12.5) and (12.6):

$$P_\ell = S_t A_e = S_t \frac{\lambda^2 G}{4\pi}$$

Because of the short-circuit termination, all of this power is re-transmitted (assuming a lossless antenna), giving a radiated power in the main beam direction of, $P_s = G P_\ell$. Then the RCS can be found from (12.25):

$$\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2}{4\pi} \checkmark$$

(12.12) $G_j = 31.6\ ,\quad G_r = 1000.$

From (12.41) we have the following relation for the self-screening jammer case:

$$\frac{J}{S} = \frac{P_j}{P_t}\,\frac{4\pi R^2 G_j}{\sigma\, G_r}\left(\frac{B_r}{B_j}\right)$$

so,

$$1 = \left(\frac{100}{10{,}000}\right)\frac{4\pi R^2\,(31.6)}{5\,(1000)}\left(\frac{.5}{10}\right) = 3.97\times10^{-5} R^2$$

So the range for $J/S = 1$ is $R = 159\,m$. ($J/S > 1$ for $R > 159\,m$)

(12.13)

$$S = 1300\ W/m^2 = \tfrac{1}{2}|\bar{E}||\bar{H}| = \frac{1}{2\eta_0}|\bar{E}|^2 = \frac{\eta_0}{2}|\bar{H}|^2$$

$$|\bar{E}| = \sqrt{2\eta_0 S} = \sqrt{2(377)(1300)} = 990\ V/m$$

$$|\bar{H}| = \sqrt{\frac{2S}{\eta_0}} = \sqrt{\frac{2(1300)}{377}} = 2.6\ A/m$$

CHECK: $\tfrac{1}{2}|\bar{E}||\bar{H}| = \tfrac{1}{2}(990)(2.6) = 1300\ W/m^2$ ✓